Phase-Sensitive Amplification and Deamplification of Noise and the Noise Rise in Period-Doubling Systems near a Bifurcation

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We have observed in experiments with a driven nonlinear electrical resonator that the fluctuations in the "noisy precursor" to a period-doubling bifurcation are phase dependent and that the noise is deamplified in one of the phases. This is a classical effect analogous to "squeezing" which normally refers to deamplification of quantum noise. Preceding the period-doubling bifurcation a "noise rise" similar to that observed in Josephson-junction parametric amplifiers evolves and the noise deamplification ceases. This phenomenon and the "noise rise" appear to be related as successive stages in the development of a bifurcation in a nonlinear system in the presence of noise.

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It has recently been demonstrated that a dynamical system which is near a period-doubling bifurcation may serve as a small-signal amplifier.[1] The case which has been most thoroughly studied[2] is analogous to a parametric amplifier operated in the "three-photon" mode in which the signal frequency is near one-half of a "pump" frequency. This correspondence has allowed insights from dynamical-system theory to illuminate a problem which has severely limited the performance of Josephson-junction parametric amplifiers, the so-called "noise rise." The noise rise is characterized by a gain-dependent noise temperature; or equivalently, the noise at the amplifier output increases more rapidly than the output signal from a coherent input as the gain of the parametric amplifier is increased.[3] A center manifold analysis of the dynamics of a generic nonlinear system near a period-doubling bifurcation excited by a coherent signal plus random noise has shown that the noise rise is a consequence of the nonlinear response of the system to the input noise.[2]

In this Letter we report on experiments in which we studied the response to signals and random noise of a very simple driven nonlinear electrical circuit.[4] We have observed the noise rise and in addition we have found that under certain conditions the nonlinear system may deamplify input random noise. It has long been known that a three-photon parametric amplifier has the capability of amplifying signals of a given phase, relative to the parametric pump, while deamplifying signals in the quadrature phase.[5,6] Thus if our nonlinear circuit does indeed function as a three-photon parametric amplifier, we expect that the input noise will be squeezed. Of greatest interest is the relationship we have observed between the noise deamplification and the noise rise. We believe that a simple unified picture of noise deamplification and the noise rise has become apparent from these straightforward experiments.

The topic of noise squeezing has received a lot of attention lately. Both a fundamental interest in squeezed quantum states and the potential for applications in metrology and precision instruments have motivated a number of experiments with the goal of producing squeezed noise. Several groups have succeeded in producing squeezed quantum noise in optical experiments[7] and deamplification of 4.2-K thermal noise has been demonstrated in an experiment which uses a microwave Josephson parametric amplifier.[8] Our observation of noise deamplification in a third system, a rudimentary period-doubling electrical circuit, suggests that the ability to deamplify noise is a universal property of nonlinear systems near a period-doubling bifurcation. We further suggest that noise deamplification in period-doubling systems has not been widely observed before now because most measurement methods are phase insensitive and thus would overlook this inherently phase-sensitive phenomenon. To avoid confusion we reserve the term "squeezing" for reduced quantum fluctuations and call the related classical phenomenon "deamplification." We emphasize that the system we have studied was far in the classical regime.

The nonlinear system which we studied is a 1N4723 diode in series with a resistor and an inductor.[9] The circuit, shown in Fig. 1, was driven at its *LCR* resonant frequency of 21 kHz by a synthesizer with a 50-Ω output impedance. The output was taken as the voltage across the diode measured by a high-input-impedance amplifier. A signal approximately 120 Hz from one-half of the drive, or pump, frequency, $f_p/2$, and an adjustable amount of noise were injected into the same port as the pump. To monitor the two orthogonal phases of the output voltage near the frequency $f_p/2$, the output of the circuit was sent to a two-channel lock-in amplifier with a reference signal at $f_p/2$ which was phase coherent with the pump.

In Fig. 2 we show spectra of the two orthogonal phases of the output of the circuit for two cases. The spectra displayed in Fig. 2(a) were taken with a pump voltage of 9.00 V peak to peak which preceded the noise rise. The spectra in Fig. 2(b) were taken with a pump voltage of 9.15 V p.p. at which the noise rise was evident. The bi-

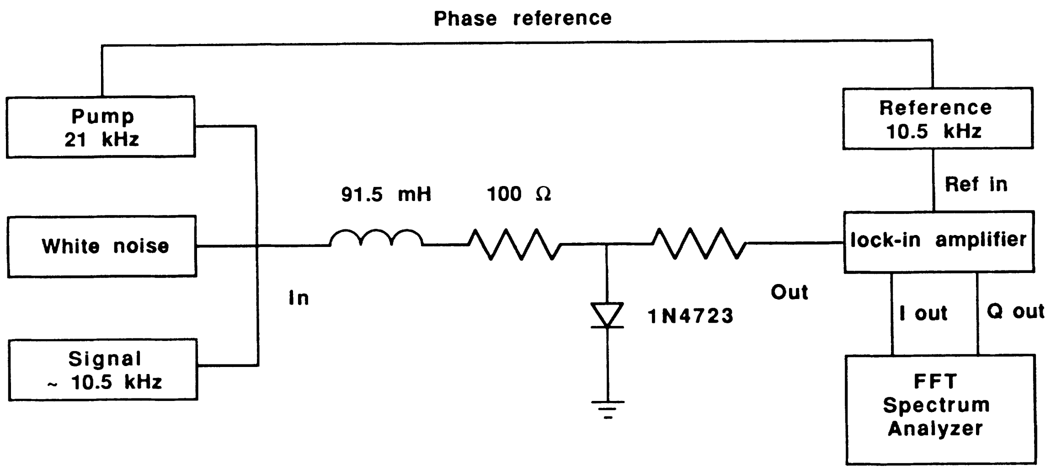

FIG. 1. A schematic of the nonlinear circuit studied and the block diagram of the phase-sensitive measurement scheme.

furcation was complete for a pump voltage of 20 V p.p. In both figures a spectrum of the output taken with the pump effectively turned off (1 mV p.p.) is shown to establish a noise base line; both phases have identical spectra with the pump turned off and so only one is shown.

In Fig. 2(a) we see that the noise in the "quiet" phase is reduced by approximately 8 to 10 dB in a band of frequen-

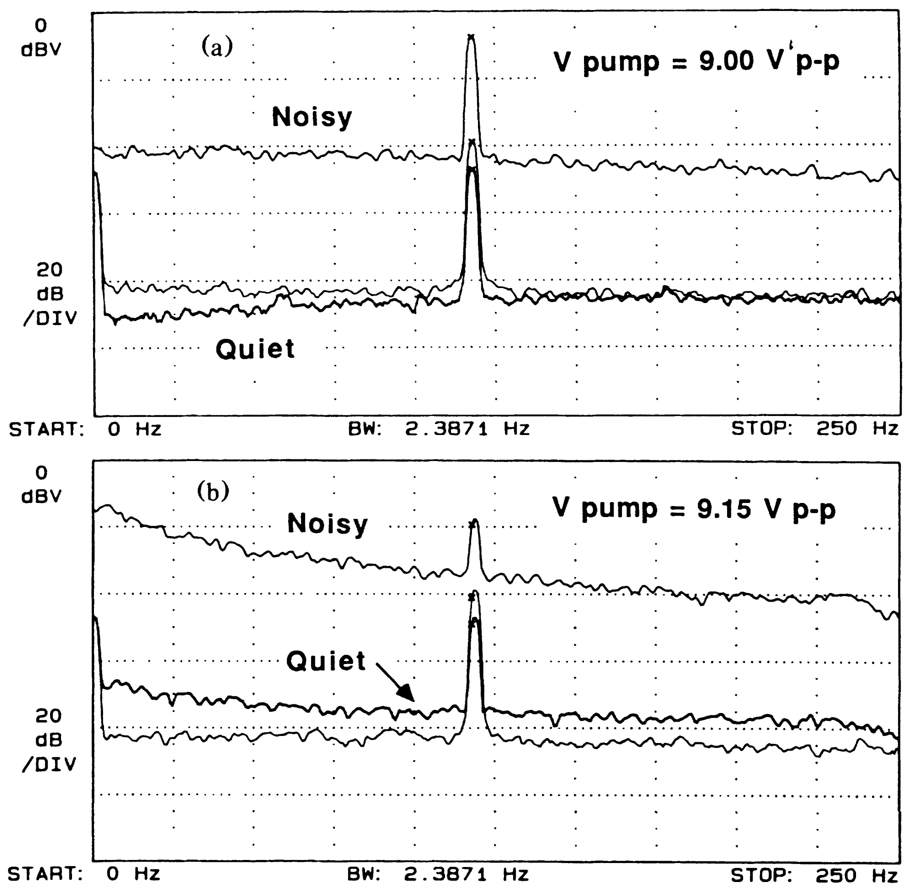

FIG. 2. Spectra of the two orthogonal phases of the nonlinear circuit's response at one-half the frequency of the ac drive voltage. The two phases are labeled "Noisy" and "Quiet." The third unlabeled spectrum in each figure is the noise base line taken with the drive voltage turned down to near zero. (a) The spectrum of the quiet phase lies below the noise base line which demonstrates squeezing. (b) The noise rise is apparent and both phases are more noisy than the base line.

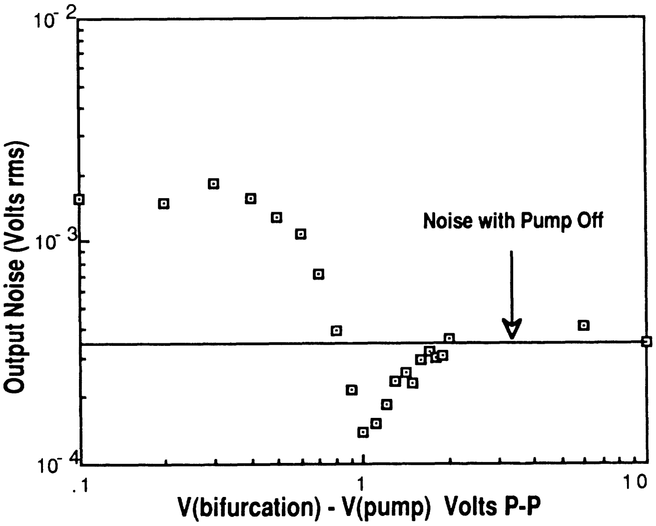

FIG. 3. The experimentally determined rms voltage in the quiet phase plotted vs $V(\text{bifurcation}) - V(\text{pump})$. The horizontal line is the noise base line and squeezing is present in the range $V(\text{bifurcation}) - V(\text{pump})$ from 0.8 to 2.0 V p.p.

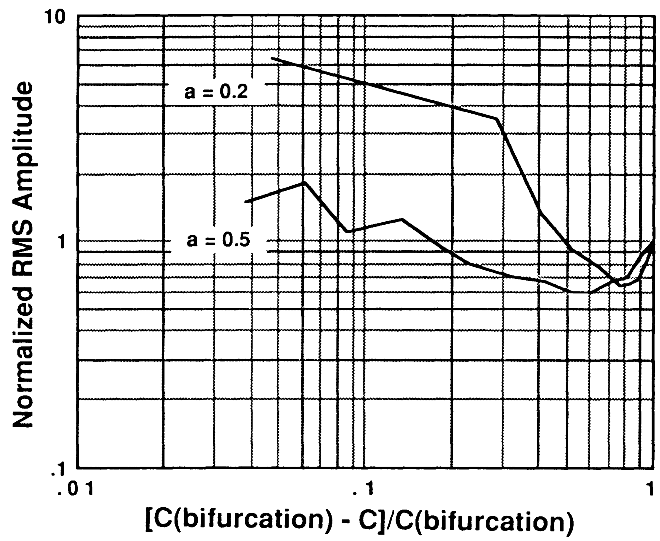

FIG. 4. Normalized rms amplitude of the noise-driven nonlinear circuit vs $[C(\text{bifurcation}) - C]/C(\text{bifurcation})$ from numerical simulations using Eq. (1); C(bifurcation) is the value of the drive parameter, C, at which the system undergoes a period doubling bifurcation. For the case $a = 0.2$, C(bifurcation) is 2.1, and for the case $a = 0.5$, C(bifurcation) is 2.6.

cies centered on one-half of the pump frequency and that the coherent signal is deamplified by approximately 10 dB. We now refer to the spectrum of the "noisy" phase in Fig. 2(a); one may observe that the noise is amplified by about 40 dB and that the coherent signal is amplified by approximately 33 dB. We note that the amplification of the noisy phase is much greater than the deamplification of the quiet phase which, according to the theory of degenerate parametric amplifiers, is true when there are losses present.[8]

The spectra shown in Fig. 2(b) were taken with a pump amplitude at which the noise rise was in evidence. First of all, we note that both phases of the output in Fig. 2(b) are more noisy than the base line. Near the signal in the spectrum of the noisy phase the noise is amplified by about 45 dB and the signal is amplified by approximately 20 dB. Thus the noise rise in this case is characterized by a 17-dB degradation of the signal-to-noise ratio. In the spectrum of the quiet phase in Fig. 2(b) we see that the signal remains to be deamplified by 10 dB, the same as in Fig. 2(a), but the noise is amplified by 6 to 8 dB. To obtain the spectra in Fig. 2 the phase of the lock-in reference was carefully adjusted to view the quiet output phase uncontaminated by the orthogonal noisy component of the output.

We determined the pump-amplitude dependence of the noise level in the quiet phase of the output. The pump voltage at which a stable bifurcation appeared was approximately 10 V p.p.; however, it was obvious that for pump amplitudes slightly less than 10 V p.p. there was noise-induced switching between adjacent branches of the system's attractor. The mean time between switching events became longer than the duration of the experiment when the pump voltage approached 10 V p.p. and for pump voltages very close to 10 V p.p. one could observe the switching as a 180° shift of the phase of the output signal $f_p/2$. Thus we adopted 10 V p.p. as the bifurcation voltage and in Fig. 3 we show a plot of the noise of the quiet phase as a function of the difference between the bifurcation voltage and the pump voltage. As the pump voltage was increased to approximately 90% of the bifurcation pump voltage the noise squeezing reached a maximum and thereafter the squeezing rapidly disappeared as the noise rise evolved. Experimentally it was impossible to discern if the noise rise occurred because the phase of the quiet quadrature fluctuated or if it was due to an actual increase of the noise in the quiet quadrature.

We theoretically investigated the behavior of this system by numerically integrating the equation of motion. A simplified equation of motion which describes the full dynamics of our diode oscillator is[9]

$$\ddot{q} + a\dot{q} + e^{q} - 1 = C\sin(2\pi f_p t) + D\sin(2\pi f_{\text{sig}} t) + \xi(t), \qquad (1)$$

where $q(t)$ is the diode-junction effective charge and $\xi(t)$ is a Gaussian-distributed random-noise source. A fourth order Runge-Kutta method of integrating this equation was employed and the response near $f_p/2$ was determined by our mathematically multiplying the solution $q(t)$ by $\sin(\pi f_p t)$ and $\cos(\pi f_p t)$, and digitally low-pass filtering the two modulated outputs.

In Fig. 4 we show the rms value of the simulated output versus the strength of the pump parameter, C, for two cases of the damping constant, $a = 0.2$ and $a = 0.5$. The horizontal axis is the normalized difference between the value of C at which the bifurcation occurred, which

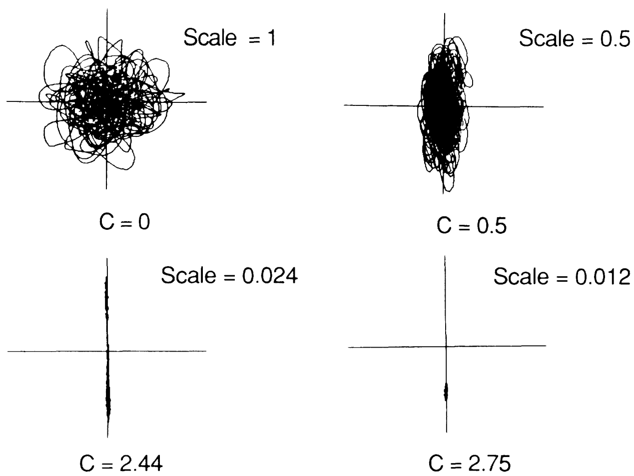

FIG. 5. A phase-plane portrait of the response of the noise-driven system for four values of the ac drive strength. The x and y coordinates are the amplitudes of the two orthogonal phases of the voltage at one-half the circuit's drive frequency. When $C=0$ the noise is not squeezed but as C is increased squeezing begins to appear. At $C=2.44$ the system spends most of the time near one of two points symmetrically located on the vertical axis above and below the horizontal axis and rarely passes through the horizontal axis. The squeezing has vanished by this stage, i.e., the quiet phase has become more noisy than the noise base line. When $C=2.75$ the bifurcation is complete and the system spends all of the time near the point on the attractor in the lower half-plane. See the text for a more complete explanation. Note that the scale is different for each plot.

we call C(bifurcation), and the pump parameter, C. Against this we have plotted the rms value of the output in the quiet phase normalized to its value when $C=0$. We note that the value of the pump parameter, C, which gives the maximum amount of squeezing is less for the lower value of the damping constant, $a=0.2$, but that the maximum amount of noise reduction is approximately the same in the two cases. The simulation reproduced the general trend shown by the data in Fig. 3 with one notable exception: The squeezing observed in the simulation appeared at low values of the pump parameter but the squeezing of noise displayed in the experimental data is apparent only when the pump voltage becomes approximately 80% of V(bifurcation).

To gain an intuitive understanding of the behavior of this system we show in Fig. 5 a plot of $q(t)\sin(\pi f_p t+\phi)$ vs $q(t)\cos(\pi f_p t+\phi)$ which was generated by the computer. The simulation output was digitally low-pass filtered to remove the strong system response at the pump frequency f_p. The phase, ϕ, was adjusted to make the noisy phase lie on the vertical axis. The attractor of the system will split into two points at the bifurcation which then will continue to separate along the vertical axis as the pump parameter is increased. The noise causes the system to jump between the two points unless the points become so widely separated that the noise is unable to provide an impulse of sufficient strength to cause a shift from one point to the other. Our experimental observation of random 180° phase shifts in the system output for pump values very close to bifurcation supports this picture; we were able to directly observe our system to jump from one point on the attractor to the other. When the pump amplitude is far below the value which causes the bifurcation, the noise is squeezed along the axis which will join the two points of the attractor after the bifurcation. The system motion is constrained along this axis with an actual reduction of the system's rms value along the orthogonal axis, i.e., the noise is squeezed. When the pump parameter approaches the value which causes the bifurcation, the system wanders less freely between the two points on the attractor and only occasionally jumps from one side of the horizontal axis to the other, spending most of the time near one of the two points. At this stage the noise ceases to be squeezed and the noise rise ensues.

In summary we have shown experimental and numerical simulations that a very simple nonlinear dynamical system is capable of squeezing noise and that this capability is lost with the evolution of a noise rise. The unification of these two phenomena as steps in the scenario of the bifurcation of a nonlinear system in the presence of noise is a very satisfying picture.

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