Experimental Determination of Fractional Charge e/q for Quasiparticle Excitations in the Fractional Quantum Hall Effect

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The Laughlin-Haldane prediction that the charge e^* of quasiparticles excited across the energy gap of fractional quantum Hall-effect ground states at $\nu=p/q$ is $e^*=\pm e/q$, a new fundamental quantum of nature, is found to be consistent with experiment. The experimental probe of e^* is σ_{xx}^c, obtained from the extrapolated $1/T=0$ intercept of the activated region of the Arrhenius plot, which is shown to be constant for p/q fractions of the same q and scale as $1/q^2$ for $q=3$, 5, 7, and 9.

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One of the most striking theoretical predictions of the fractional quantum Hall effect (FQHE) observed at Landau-level filling factors[1,2] $\nu=p/q$ is the existence of excited-state quasiparticles of fractional charge e^*. Two different values of e^* have been proposed, namely $e^*=\pm e/q$,[3,4] and $e^*=\pm(p/q)e$.[5] As discussed by Laughlin,[3] the integral QHE is exact in the limit of low temperature and large sample size because it is a measurement of the electron charge, a fundamental quantum of nature. If the FQHE is also exact in the macroscopic limit, the implication of the theory is that e^* is also a fundamental quantum of nature. Although fractional quantization of the Hall resistance has been measured to be exact to 3 parts in 10^5,[6] there is to date no experimental measurement that probes the predictions for the quasiparticle charge.

We report a systematic study of $\sigma_{xx}^c=\sigma_{xx}(1/T=0)$ for a range of GaAs-GaAlAs heterojunctions and p/q states with $q=3$, 5, 7, and 9 obtained from the relation

$$\sigma_{xx}^c=\frac{\rho_{xx}^c}{(\rho_{xx}^c)^2+\rho_{xy}^2}=\frac{\rho_{xx}^c}{(\rho_{xx}^c)^2+[(q/p)h/e^2]^2} \tag{1}$$

which is valid at exactly $\nu=nh/eB=p/q$. In Eq. (1), ρ_{xx}^c is defined by $\rho_{xx}=\rho_{xx}^c e^{-\Delta/kT}$ where Δ is the energy gap between the fractional ground state and the mobile (nonlocalized) elementary quasiparticle excitations. It is shown that within experimental errors, σ_{xx}^c is constant for p/q fractions of the same q and scales as $1/q^2$. This observation provides an experimental probe of e^* which confirms that $e^*=\pm e/q$ and raises profound questions for the theory of localization in 2D in the presence of a magnetic field. A summary is first given of the main results followed by a presentation of the experimental data.

In our highest quality sample G139 we find that for the thirteen fractional states $p=2,4,5$ with $q=3$, $p=2,3,7,8$ with $q=5$, $p=3,4,9,10$ with $q=7$, and $p=4,5$ with $q=9$,

$$\sigma_{xx}^c=c(e/q)^2/h, \tag{2}$$

where, if we take an *average* of all thirteen fractions, the numerical constant is $c=0.91$ with standard deviation ± 0.11. The data can also be analyzed by the determination of the best fit to $\log\sigma_{xx}^c$ vs $\log q$ which gives $\sigma_{xx}^c=(1.07/q^{2.1})e^2/h$, consistent with Eq. (2) within error bars of the logarithmic plot. The validity of Eq. (2) in other samples is verified for $q=3$ by our σ_{xx}^c results for fractions $\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, and $\frac{5}{3}$ in four different high-quality heterojunctions,[7] where for nine of some sixteen activation studies $\sigma_{xx}^c=(0.82\pm 0.12)(e/3)^2/h$. Experimental errors in this preliminary work were greater than the G139 data, for which the measurement procedure has been optimized. In the FQHE regime, ρ_{xx} is proportional to σ_{xx} of the quasiparticles, which has the units $(e^*)^2/h$. Consequently our observation of a constant value of σ_{xx}^c for fractions of the same q that scales as $1/q^2$ is entirely consistent with the prediction that $e^*=\pm e/q$. This also agrees with the proposed FQHE σ_{xx} vs σ_{xy} scaling diagram[8] and our scaling data,[7] for which the high-temperature limit of σ_{xx} is progressively lower for "flow" to p/q states of increasing q and fixed points which determine this flow scale as $(e^*)^2\Gamma$ where Γ is a dimensionless disorder parameter. For a given q, Eq. (2) is valid for fractional states in which p^2 varies by up to an order of magnitude and hence a relation $e^*=\pm(p/q)e$ is incompatible with our data.

The numerical constants 0.91 and 1.07 in Eq. (2) and the $\log\sigma_{xx}^c$ vs $\log q$ best fit are close to unity and the provoking identity $\sigma_{xx}^c=1.0(e/q)^2/h$ provides a better fit to the $q=3$, G139 data at the expense of a slightly worse fit to the $q=5$, 7, and 9 results. This may be simply fortuitous or, on our noting that the quantized *Hall* conductivity in the integer QHE is $\sigma_{xy}=ie^2/h$ where i is an integer, might suggest a fundamental connection between the fractional and integer QHE. However, our data are presently limited to the $N=0$ Landau level and the overall situation may be more complex.

In our experiments the samples were mounted at field center in the dilute phase of a dilution refrigerator and thermometry was provided by a calibrated 220-Ω Speer resistor, corrected for magnetoresistance. Great care was taken to ensure thermal equilibrium between the sample-thermometers combination and the ^{3}He-^{4}He mixture. An "in-phase" current of accurately 20.0 nA was used in the ac lock-in measurements and the Hall bar length-to-width ratios were defined by precise sample lithography.

In Fig. 1(a) we show a full-field (10 T) trace of ρ_{xx}, ρ_{xy} data for the modulation-doped heterojunction G139 (1600-Å space layer) with $n=0.95\times10^{11}$ cm^{-2} and $\mu=1.04\times10^{6}$ cm^2/V s. The development of fractional ρ_{xx} minima and ρ_{xy} plateaus at selected temperatures down to 50 mK for the regions $1<\nu<2$ and $\frac{1}{2}<\nu<1$ is summarized in Figs. 1(b) and 1(c). The 50-mK data of Fig. 1(c) show a well-developed hierarchical sequence[4] of fractional states $p/q=\frac{3}{5}$, $\frac{4}{7}$, $\frac{5}{9}$, and $\frac{6}{11}$ deriving from the $\frac{2}{3}$ parent, with corresponding quantization of the Hall resistivity at $(q/p)h/e^2$. The daughter states $\frac{2}{5}$, $\frac{3}{7}$, and $\frac{4}{9}$ of the $\frac{1}{3}$ parent are equally well resolved as can be seen from Fig. 1(a). Activation plots, $\ln\rho_{xx}$ vs $1/T$, are presented in Figs. 2(a) and 2(b) for fractional states $\nu<1$ and $1<\nu<2$, respectively. We extrapolate the activated straight-line region at intermediate temperatures to $1/T=0$ as shown to determine $\rho_{xx}^c=\rho_{xx}(1/T=0)$ and σ_{xx}^c is then obtained from Eq. (1). At low temperature the Fig. 2 results exhibit curvature identified with the hopping regime.[9] While it is shown in Ref. 9 that allowance for the hopping contribution in the activated region leads to Δ values for $q>3$ $\simeq 25\%$ larger than those obtained from the straight line through the high-T data (for $q=3$ the difference is negligible), this consideration does not affect the determination of ρ_{xx}^c. From the functional dependence of the hopping which falls to zero at $1/T=0$, straight-line extrapolation of "raw" high-T data or of corrected data after subtraction of the hopping component leads to identical intercepts.

The invariance of σ_{xx}^c for fractions of the same q is not immediately obvious from the ρ_{xx}^c result. If we set $c=1$ in Eq. (2), which is within the error bars of the data, it follows from Eq. (1) that

$$\rho_{xx}^c=(h/2e^2)\{q^2-(q/p)[p^2q^2-4]^{1/2}\}.$$

However, a more transparent relation is obtained if we note that ρ_{cc}^x in units of h/e^2 ($\simeq 25.8$ kΩ) is considerably smaller than $(q/p)^2$ in the denominator of Eq. (1) and hence $\rho_{xx}^c\simeq(1/p^2)h/e^2$. Consequently, the consistency of the Fig. 2 data with the form of Eq. (2) can be observed *directly* on our noting that ρ_{xx}^c is approximately the same for p/q fractions of the same p and scales as $\simeq 1/p^2$.

Values for Δ (without hopping subtraction), ρ_{xx}^c, and σ_{xx}^c obtained from Fig. 2 and Eq. (1) are presented in Table I. The σ_{xx}^c values are plotted against $1/q$ in Fig. 3 and compared with curves a and b corresponding to Eq.

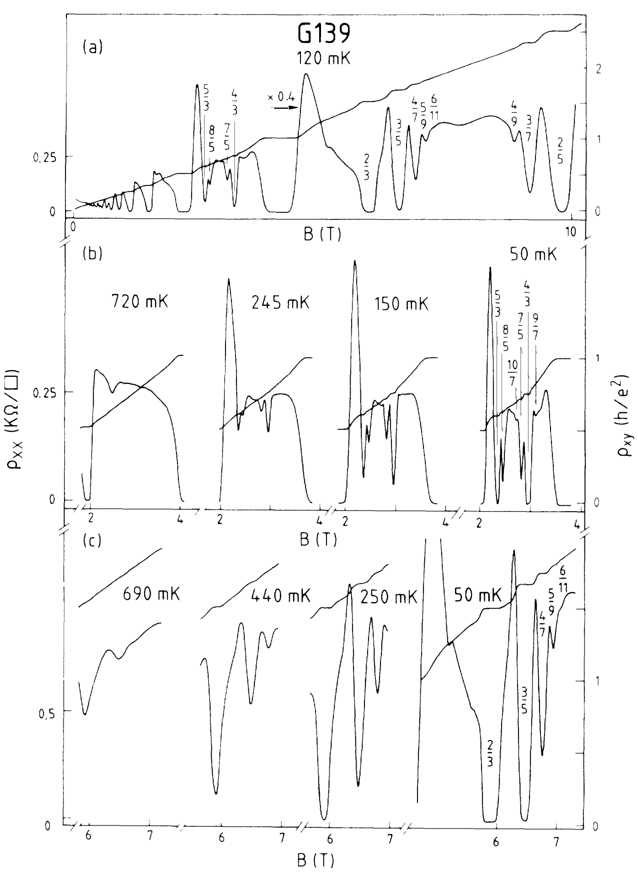

FIG. 1. ρ_{xx} and ρ_{xy} data for the GaAs-Ga$_{0.68}$Al$_{0.32}$As sample G139; $n=0.95\times10^{11}$ cm^{-2} and $\mu=1.04\times10^{6}$ cm^2/V s.

(2) with $c=0.91$ and $c=1.0$. In the inset of Fig. 3 we also show a $\log\sigma_{xx}^c$ vs $\log q$ plot and the straight-line best fit (curve c) discussed above. The fits provide convincing evidence that $\sigma_{xx}^c=c(e/q)^2/h$ where c is a constant close to 1.0. We note that "Δ" for the $\frac{9}{7}$, $\frac{10}{7}$, and $\frac{5}{9}$ states is $\simeq$20–30 mK and the quasiparticle pair-creation energy 2Δ is below the range of the exponential analysis. However, the weak temperature dependence of ρ_{xx} down to 50 mK for these fractions places an upper limit on the true activation energy and our extrapolation provides a reasonable estimate of ρ_{xx}^c.

Our observation that σ_{xx}^c is a constant value for fractional p/q ground states of the same q in our high-quality samples is a new result. There are several $q=3$ activation studies in the literature for high-mobility samples which we have extrapolated and found to agree with the Table I data.[9,10] In general however there is substantial variation, to higher σ_{xx}^c values, which reflects the distinction between sample-dependent "ideal" and "non-ideal" behavior found in the early activation measurements on Si inversion layers.[11]

The results also raise questions for localization. The existence of a universal minimum conductivity $\sigma_{\min}$ and mobility edge in 2D has remained controversial.[11] Exper-

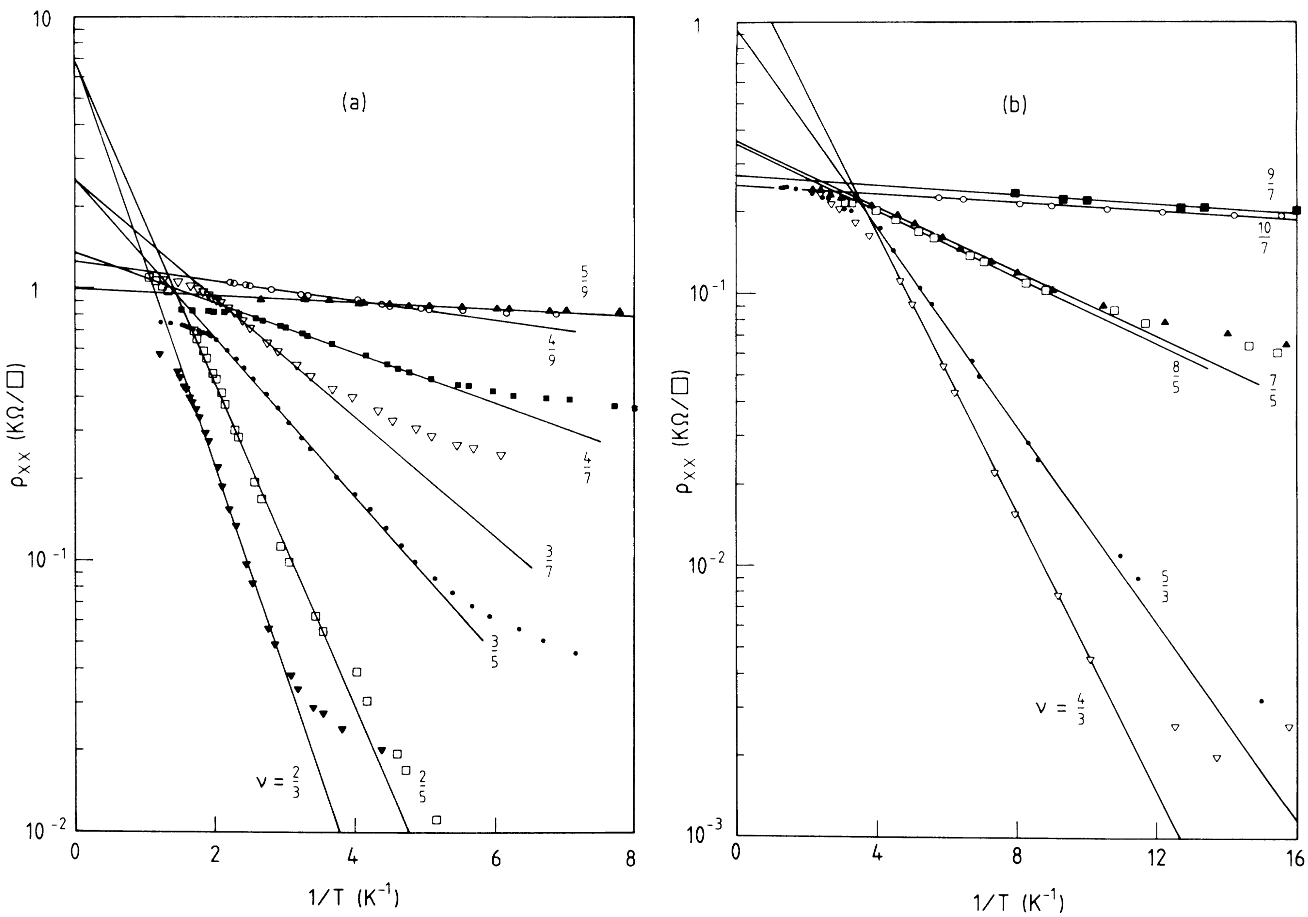

FIG. 2. Activation plots for (a) $p/q < 1$ and (b) $1 < p/q$ ground states in sample G139.

iments on Si inversion layers provided support for this concept and the theoretical estimate for noninteracting 2D electrons in zero magnetic field was $\sigma_{\min} = 0.75e^2/h$. However, following the development of scaling theory,[12] it was concluded that there is no sharp mobility edge in 2D, but there is a universal crossover from logarithmic to exponential behavior for the conductivity. The situation in a magnetic field is unclear, however, and it is argued[11] that since the integer QHE shows that all states in a Landau level cannot be localized, the theory of Abrahams *et al.*[12] is not applicable. Consequently, σ_{xx}^c could be interpreted as a minimum quasiparticle conductivity which, within the understanding that FQHE quasiparticles behave analogously to electrons in Landau levels, has far-reaching implications. Support for this interpretation is provided by the activation study of Chang *et al.*[13] for $\nu = \frac{2}{3}$ who argue that the variation of Δ between $\nu = 0.6$ and 0.7 can be understood in terms of quasiparticle/quasihole bands where disorder broadens the bands and gives rise to "a sharp mobility edge" separating extended and localized regions. Adopting their model, we have shown that Δ can be obtained from the temperature-dependent *widths* of ρ_{xx} minima,[7] in good agreement with values from $\ln\rho_{xx}$ vs $1/T$ plots. A constant σ_{xx}^c for fractions of the same q is also obtained in the width analysis.

The concept of a quasiparticle excitation of charge

TABLE I. Δ, ρ_{xx}^c, and σ_{xx}^c values for p/q states in sample G139.

p/q	B(T)	Δ(K)	ρ_{xx}^c (kΩ/□)	σ_{xx}^c (e^2/h)
$\frac{2}{3}$	5.9	1.72	6.8	1.02/9
$\frac{4}{3}$	2.9	0.59	1.8	1.11/9
$\frac{5}{3}$	2.4	0.42	0.94	0.91/9
$\frac{2}{5}$	9.8	1.37	6.8	1.04/25
$\frac{3}{5}$	6.5	0.67	2.5	0.87/25
$\frac{7}{5}$	2.8	0.138	0.36	0.68/25
$\frac{8}{5}$	2.5	0.14	0.354	0.88/25
$\frac{3}{7}$	9.2	0.50	2.5	0.87/49
$\frac{4}{7}$	6.9	0.21	1.35	0.84/49
$\frac{9}{7}$	3.1	0.019	0.27	0.85/49
$\frac{10}{7}$	2.8	0.018	0.25	0.97/49
$\frac{4}{9}$	8.8	0.084	1.26	0.78/81
$\frac{5}{9}$	7.1	0.027	0.99	0.96/81
Average				$0.91/q^2$

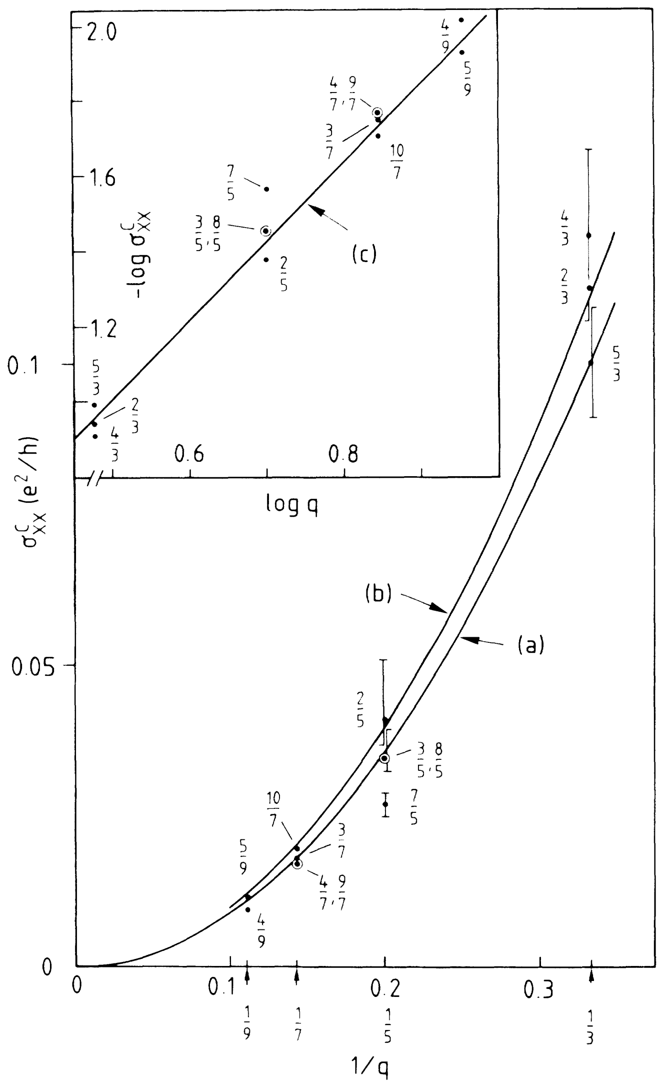

FIG. 3. σ^c_{xx} vs $1/q$ for p/q fractional states in sample G139. Inset: $\text{Log}\sigma^c_{xx}$ vs $\log q$. Fits a, b, and c correspond to Eq. (2) with $c=0.91$ and 1.0 and the relation $\sigma^c_{xx}=(1.07/q^{2.1})e^2/h$.

$\pm e/q$ at $\nu=1/q$, due to Laughlin,[3] has been extended to a hierarchical picture of fractional states $\nu=p/q$ by Haldane[4] who shows that $e^*=\pm e/q$ (independent of p) consistent with the Laughlin gauge-invariance argument. In a Wigner-crystal approach, Kivelson *et al.*[14] conclude that "$e^*=\pm \nu e$ where ν denotes one of the preferred rational filling fractions." Their theory, which has now been shown to be consistent with Laughlin's,[15] is essentially developed for $\nu=1/q$, however, and it is not clear how far it should be generalized to $\nu=p/q$. Tao[5] on the other hand clearly predicts that if a fermion or a boson system has a quantized Hall step at $\nu=p/q$ with $\sigma_{xy}=(p/q)e^2/h$ where e is the charge of the fermion or boson, identical quasiparticles of fractional charge $(p/q)e$ can be produced. Our data provide the first means of distinguishing between these theories and confirm the Laughlin-Haldane result.

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