Traveling-Wave Convection in an Annulus

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We report experimental observations of traveling-wave convection in a binary fluid mixture in an annular container—a flow which approximates a large, one-dimensional dynamical system. In this geometry, the convective rolls align themselves radially and propagate azimuthally. We observe competition between several nonequilibrium states. This leads to a rich variety of dynamical behavior, including coexisting regions of conduction and convection, sources and sinks of convective rolls, and more complex spatiotemporal defects.

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The nature of turbulence in fluids is an open and intriguing question. While we are still far from an understanding of three-dimensional turbulence, recent years have witnessed the emergence of an understanding of model equations which exhibit complex spatiotemporal behavior in one dimension.[1] While many of the concepts associated with this behavior have been explored in numerical simulations, the need to produce a large experimental system with reduced spatial dimensionality has long been felt. This issue has been addressed to some extent by two recent experiments on convection in pure fluids confined in a large-aspect-ratio annular container, which is meant to approximate a one-dimensional, nonequilibrium dynamical system.[2,3] However, in order to produce dynamical convective states in pure fluids, it is necessary to operate at extremely high Rayleigh numbers. In this regime, comparison with theory is difficult, and three-dimensional effects can play an important role.

By contrast, the first state observed in convection in binary-fluid mixtures is intrinsically time dependent, consisting of traveling waves which arise from a subcritical Hopf bifurcation.[4-6] Since this dynamical state is seen at onset, one might expect its theoretical description to be simpler than that of the dynamics observed in pure fluids. The nature of this state is controlled by the Rayleigh number R, which is proportional to the temperature difference applied vertically across the fluid layer, and the separation ratio ψ, which is a measure of the stabilizing density gradient produced by the Soret effect and which can be varied by adjusting the solute concentration and the mean temperature.[4,7] In this Letter, we report experiments on traveling-wave convection in a large-aspect-ratio annulus. We find that complex dynamical behavior can be observed near onset in this quasi one-dimensional system.

Our experiments were conducted in an annular container with a large aspect ratio (the ratio of height to radial width to mean circumference is 1:1.63:105.3). The absence of walls perpendicular to the (azimuthal) direction of propagation of the rolls means that there are no reflections of rolls from end walls,[8,9] and that wave number selection due to the boundary conditions at such walls[10] is unimportant. We find that this system exhibits a rich variety of spatiotemporal behavior. In this Letter, we will present results concerning uniform states of slow traveling rolls, states of fast traveling rolls coexisting stably with regions of conduction, spatiotemporal defects, and transients resulting from the competition between the various states.

The apparatus consists of a mirror-polished, rhodium-plated copper bottom plate and a sapphire top plate. The vertical walls of the container are defined by two O rings which sit in concentric circular grooves in the bottom plate. The height of the cell is $d = 0.241$ cm. The bottom plate of the cell is heated by a resistive heater, and the top plate is cooled by flowing water. The temperature difference across the cell is regulated with a stability of $\pm 5\times 10^{-4}\,^{\circ}$C. Shadowgraph images of the flow, taken from above, can be simultaneously recorded on video tape and digitized with an annular photodiode array. The fluid is a solution of 8 wt.% ethanol in water at a mean temperature of 28.6 °C, for which $\psi = -0.247 \pm 0.005$.[7] For this mixture, the predicted oscillation period τ_0 of the linear instability is $0.54\tau_v$, where $\tau_v \equiv d^2/\kappa = 45.1$ s is the vertical thermal diffusion time, with κ the thermal diffusivity of the fluid. We normalize R to the critical value $R_c = 1708$ for the onset of convection in a homogeneous fluid with the same thermal properties as the mixture. A reduced Rayleigh number $r \equiv R/R_c = 1$ is achieved at a temperature difference of approximately 5.57 °C.

Special attention has been paid to the elimination of inhomogeneities in the apparatus. The uniformity of the spacing of the top and bottom plates was checked interferometrically and varied in some runs by only $\pm 0.05\%$ over the entire azimuthal extent of the cell. The fluid is injected by removable hypodermic needles, and so there are no filling holes to break the azimuthal symmetry or to otherwise pin the convective rolls. The cooling water circulates azimuthally above the top plate, so that lateral temperature gradients are minimized. We have also performed some experiments with the inner O ring replaced

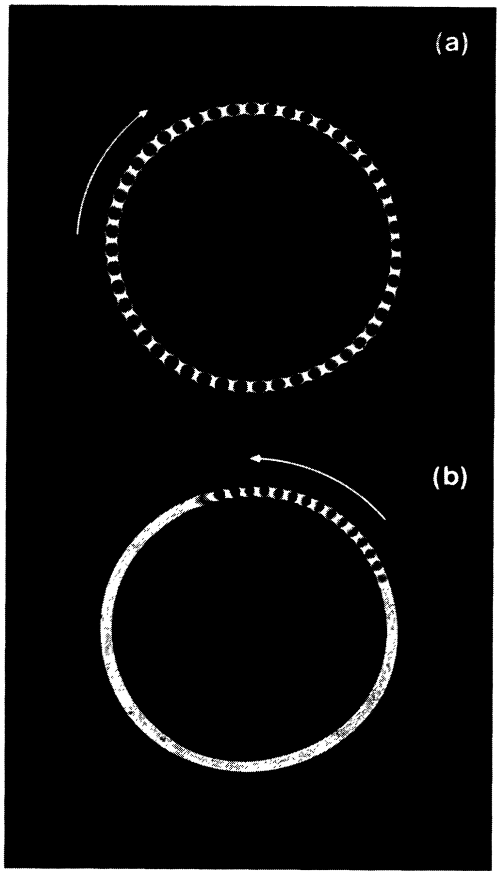

FIG. 1. Shadowgraph images of convection patterns in an annular container. (a) Uniform state of rolls which travel clockwise with a period of approximately $\tau_u = 80\tau_v$. (b) Confined state of rolls which travel counterclockwise with a period of approximately $\tau_c = 1.28\tau_v$.

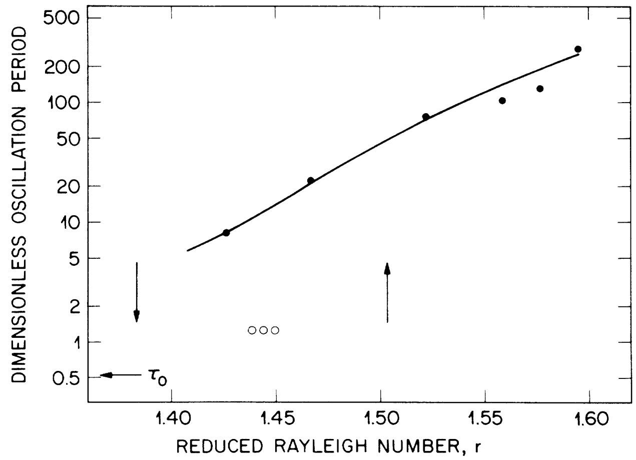

FIG. 2. The closed circles show the dimensionless period τ_u/τ_v with which convective rolls drift past a stationary point of observation in an image like the one in Fig. 1(a), as a function of reduced Rayleigh number r. The solid curve is a guide to the eye. The onset of convection is at $r_{c0} = 1.504$, and the system undergoes a transition back to the conductive state when r is reduced below $r_{\min} = 1.383$. These transitions are indicated by vertical arrows. The open circles show the oscillation period τ_c/τ_v in the confined states; the horizontal range of these symbols shows the band of r within which their boundaries are motionless. The horizontal arrow indicates the oscillation period τ_0/τ_v of the linear instability.

by a plastic disk with a straight, vertical edge, as well as in a second annular cell of aspect ratio 1:2.95:67, which has straight, vertical walls in place of the O rings. The results reported here do not seem sensitive to the shape of the vertical walls or to lateral inhomogeneities in the Rayleigh number or in the radial width of the cell.

Uniform states.— When the Rayleigh number is increased above $r_{c0} = 1.504$, convection begins with a complicated transient. In this flow, the rolls are oriented radially, but the rolls in different angular regions exhibit different spacings and travel in different azimuthal directions and at different velocities. Mediated by defect mechanisms such as the one described below, this transient "anneals" into a uniform, stable state of flow [Fig. 1(a)]. In this state, the cell is filled with uniformly spaced rolls which propagate azimuthally in a single direction with a period τ_u of about $50\tau_v$ at $r = r_{c0}$. If r is reduced below r_{c0}, the period decreases, as shown in Fig. 2, down to $\tau_u \approx 5\tau_v$ at $r_{\min} = 1.383$. Below $r_{\min}$, the pattern develops a localized nonconvecting region which grows to fill the entire cell, leading to a transition back to the uniform conducting state. These observations are qualitatively similar to those in experiments conducted in rectangular convection containers.[5,6,11] However, in the annulus, the variation of τ_u with r is about five times larger than in a rectangular container.[5,11]

Confined states.— As shown in Fig. 1(b), we are also able to produce stable confined states in which traveling waves are seen in one or more isolated angular sectors of the cell. A single confined region can be produced during the transition from the uniform convective state to the conducting state by our abruptly increasing r to a value within a "locking band" centered on $r = 1.444$, which has a width $\Delta r = 0.013$. This band is shown as a row of open circles in Fig. 2. Alternatively, a group of confined regions, with random locations and propagation directions, can be produced during the annealing phase at the onset of convection by rapid quenching of r to a value within the band [see Fig. 3(a)]. It is important to point out that, at any Rayleigh number within the locking band, these boundaries are motionless for days at a time, even if r is changed to a different value within the band. If r is increased (decreased) beyond the limits of the band, then the confined region slowly grows (shrinks) and can fill (empty) the entire cell.

The convection amplitudes A_u and A_c and the wave numbers k_u and k_c in the uniform and the confined states are comparable (i.e., $A_c/A_u \approx 1.3$, $k_c \equiv 2\pi d/\lambda_c = 3.64$, and $k_u = 2.86$ to 3.28). The wave number k_c is

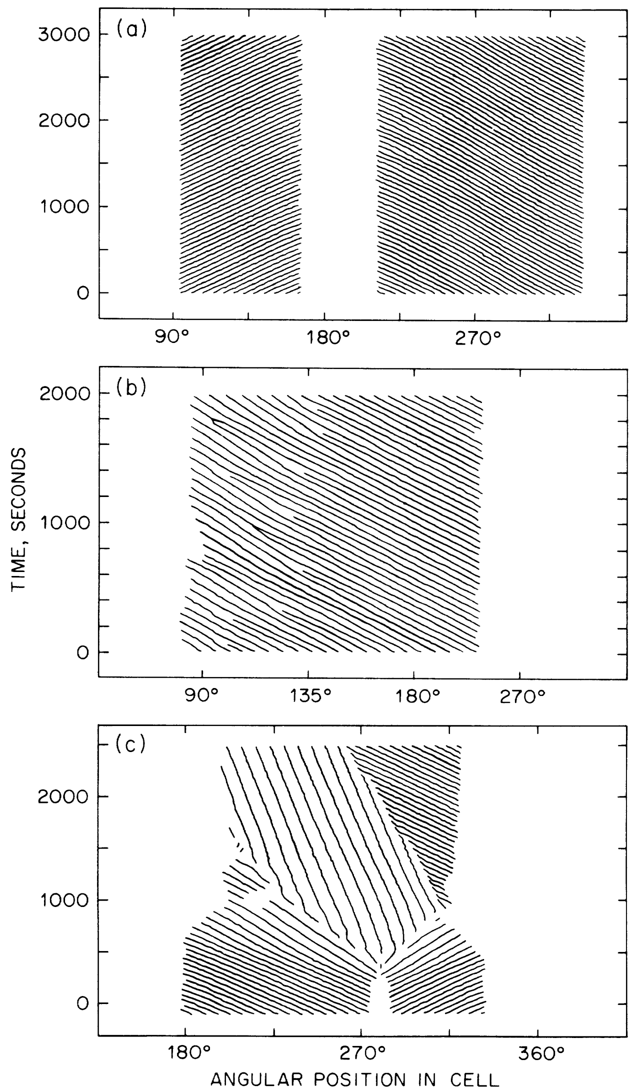

FIG. 3. Space-time representation of convective flow patterns. The lines show the positions of the downflow roll boundaries, which correspond to the bright stripes in Figs. 1(a) and 1(b). These are plotted as functions of time (vertical axis). (a) Stable confined state observed at a Rayleigh number in the middle of the band in Fig. 2. The oscillation period is $\tau_c = 1.28\tau_v$, and the boundaries of the convecting region are stationary. (b) Space-time dislocations observed within the locking band. The short-wavelength rolls on the right annihilate as they encounter a region containing rolls with a slightly longer wavelength on the left. The front between the two sets of rolls is roughly stationary, and this state is stable for days. (c) Transient state formed when nearby confined regions drift together to form sources of rolls. To produce this state, r was decreased, during the transition from conduction to convection, to just above the band of stable confined states.

approximately constant in space, decreasing by about 10% near the edges of the confined regions. Strikingly, the oscillation period τ_c in the confined state is quite fast: $\tau_c = 1.28\tau_v$, which is about a factor of 2 larger than the period $\tau_0 = 0.54\tau_v$ of the linear instability and a factor of 8 to 11 smaller than the period of the spatially uniform state observed at the same Rayleigh number. The high frequency in the confined state is strong evidence that the confinement is not just an artifact due to the pinning of rolls (which would otherwise fill the cell, producing the uniform state) by inhomogeneities in the cell. Another observation which argues against pinning by nonuniformities in the experimental cell is that the location of the confined regions, as well as the direction of propagation within them, appears to be random from run to run. The evidence presented here is convincing that regions of traveling waves can coexist stably with regions of conduction over a finite range of Rayleigh numbers in a one-dimensional system without boundaries.

Confined states of convection in binary-fluid mixtures have been observed previously in experimental[6] and theoretical[9] work involving rectangular containers. Our observations exhibit certain similarities to those in previous experiments, most notably in the fact that the phase velocity is much faster in the confined state than in the uniform state. However, there are important differences. In previous work, the leading edge of the confined region of traveling waves was observed at one of the short sides of the container, and the existence of confined states was thought to depend on the reflection of the waves from that wall.[9] This is not the mechanism responsible for our observations.

Spatiotemporal defects.— Within the locking band, a variety of stable defected structures associated with the fast traveling waves can be observed. For example, we have observed confined regions which contain two sets of counterpropagating waves separated by a sink of rolls. In this case, the space-time plot analogous to Fig. 3(a) consists of a chevron pattern or "spatiotemporal grain boundary." Another type of one-dimensional defect, reminiscent of the "spatiotemporal dislocations" seen in numerical simulations,[1] is illustrated in Fig. 3(b). Here, fast waves of one wavelength and period evolve into waves with a slightly longer wavelength and period. During this process, pairs of rolls annihilate at a somewhat random location in the cell. It is important to stress that states such as that shown in Fig. 3(b) are not transients but have been observed for many days, in different runs, and at different places in the cell.

Transients.— A variety of transient states have been observed in this system. For example, attempts to produce source defects have repeatedly led to transients of the type illustrated in Fig. 3(c). When the Rayleigh number is just above the locking band, nearby zones of convection may grow towards one another. If the waves in two such zones propagate away from the conducting region in between, then, when they meet, they merge to form a system of slow, unidirectional rolls, which invade the zone of fast traveling waves. The interface between the resulting confined state of slow rolls and the conduction state is unstable. Fast waves are created at this boundary, and they overtake the slow rolls, leading to a single confined state of unidirectional fast waves like that illustrated in Fig. 3(a). It is by mechanisms like this that the complicated flows produced at onset anneal to produce a system of uniformly spaced, unidirectional slow rolls. All of the defects just described are one dimensional, in the sense that there is no structure observed along the roll axes anywhere in the cell, even at the site of a defect.

Our observations raise a number of interesting issues. The existence of a *band* of Rayleigh numbers for which the fronts between regions of conduction and convection are stationary has been suggested by Pomeau[12] in the context of a subcritical bifurcation in stationary convection. However, in the case of subcritical bifurcation to *traveling waves*, it can be argued[13] that locking of these fronts requires the presence of a small standing-wave component, which we do not observe. Thus, the question of the specific mechanism responsible for the locking remains open for the present.

Finally, the transients and spatiotemporal defects reported here appear to result from the competition between different dynamical states. For example, Fig. 3(c) shows competition between fast traveling waves, slow traveling waves, and conduction. This behavior may be thought of in terms of the propagation and stability of the fronts between these states and might be illuminated by a study of model equations.[12]

In summary, we have studied convection in a binary fluid mixture in an annulus in order to understand the dynamics of traveling-wave states in a large one-dimensional system with periodic boundary conditions. To our knowledge, our results provide the first evidence that fronts between convecting and conducting regions can be motionless in a large system without boundaries. In addition, we have presented evidence that this system exhibits a variety of competing dynamical states. Thus, it is likely that this system will be extremely useful in the study of the complex spatiotemporal behavior of one-dimensional systems.

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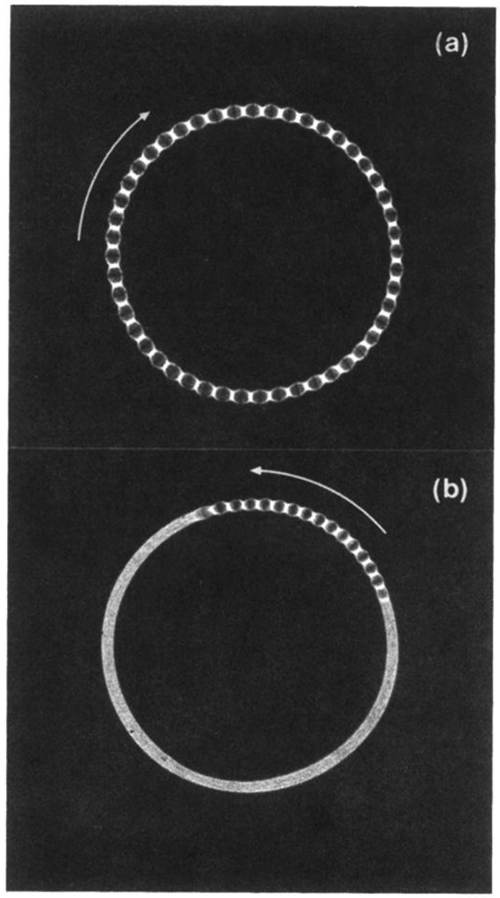

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