Wave Propagation in k Space and the Linear Ion-Cyclotron Echo

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When a magnetosonic wave packet crosses the second-harmonic gyroresonance layer, it linearly excites an ion-pressure–anisotropy wave packet, which propagates *only* in k space. The latter wave in turn excites a second magnetosonic wave packet which appears as a *time-delayed* reflection from the resonance layer. We call this phenomenon a linear ion-cyclotron echo.

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Although the Hamiltonian equations of ray optics describe orbits in (six-dimensional) ray phase space $(\mathbf{x},\mathbf{k})$, it has been traditional to think of the waves they represent as propagating in (three-dimensional) physical $\mathbf{x}$ space. In this Letter, we discuss a wave which propagates *only* in $\mathbf{k}$ space, and which remains localized in $\mathbf{x}$ space. Its linear conversion[1] with conventional magnetosonic waves gives rise to a new phenomenon, the *linear ion-cyclotron echo,* whose physical origin is quite different from the intrinsically *nonlinear* cyclotron echo studied earlier.[2]

The wave in question is an *ion-pressure–anisotropy* (IPA) wave, recently discovered by Friedland.[3] For a slab geometry with $\mathbf{B}=B_0(x)\hat{\mathbf{z}}$, and with perpendicular wave vector $\mathbf{k}$, its dispersion relation is $\omega(\mathbf{k},\mathbf{x})=2\Omega(x)+O(k^2)$ (with $\Omega\equiv eB_0/m_ic$), in the limit of $k\rightarrow 0$. (It is thus the $k\rightarrow 0$ limit of an ion Bernstein mode.[4]) The ray equations thus read $d\mathbf{x}/dt=\partial\omega/\partial\mathbf{k}=0$, $d\mathbf{k}/dt=-\partial\omega/\partial\mathbf{x}=-2\Omega'\hat{\mathbf{x}}$ $(\Omega'\equiv d\Omega/dx)$. Thus a wave packet remains stationary in $\mathbf{x}$ space, while in $\mathbf{k}$ space it propagates as $\dot{k}_x=-2\Omega'$.

One of the proposed methods[5] for the heating of a confined plasma is to launch an incident magnetosonic wave [Figs. 1(a) and 2(a)] across the magnetic field. [For $\mathbf{k}=k_x\hat{\mathbf{x}}$, the magnetosonic dispersion relation is $\omega^2=k_x^2c_A^2(x)$, with $c_A^2\equiv B_0^2/4\pi\rho_m$.] For a continuous wave with frequency ω_0, the incident magnetosonic wave has $k_x(x)=\omega_0/c_A(x)$. When this wave crosses the second-harmonic gyroresonance layer at x_R, where $2\Omega(x_R)=\omega_0$, it converts part of its energy to the IPA wave. That wave [Figs. 1(c) and 2(c)] then propagates in $\mathbf{k}$ space, until it crosses the curve $k_x(x)=-\omega_0/c_A(x)$ representing the *reflected* magnetosonic wave. The IPA wave there converts a fraction of *its* energy to the reflected wave [Figs. 1(d) and 2(d)], and proceeds [Figs. 1(e) and 2(e)] to larger $|k_x|$. (When k_x becomes appreciable for the IPA wave, it propagates in $\mathbf{x}$ space, and eventually damps by kinetic processes.[6])

In this Letter, motivated by the desire to understand this process, we consider an incident *wave packet.* It is

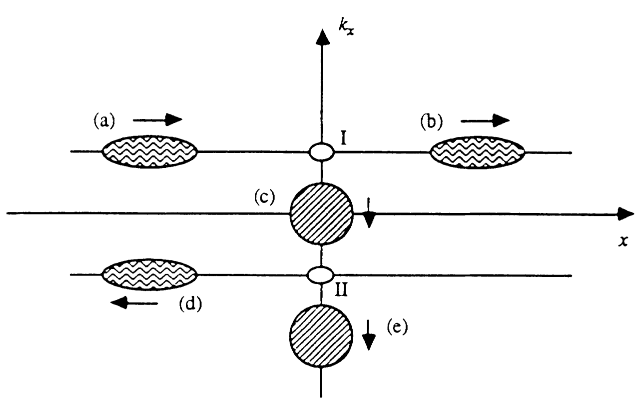

FIG. 1. Schematic diagram, in phase space, of mode-conversion process: (a) incident magnetosonic wave packet approaching mode-conversion point I; (b) transmitted magnetosonic wave packet; (c) ion-pressure–anisotropy wave packet between the two mode-conversion points; (d) reflected magnetosonic wave packet leaving mode-conversion point II; (e) ion-Bernstein wave packet leaving the mode-conversion point II.

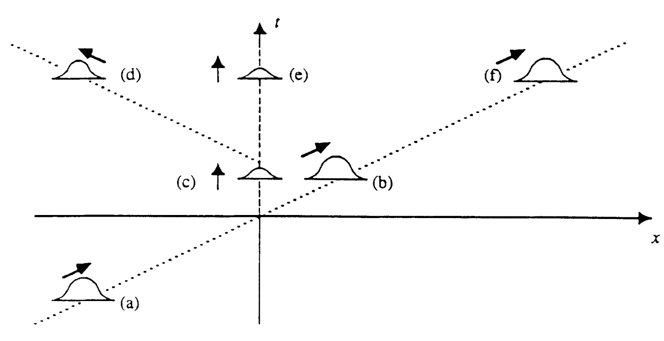

FIG. 2. Schematic diagram, in space-time, of wave packets of the electric field: (a) incident magnetosonic wave packet at $t<0$; (b) transmitted magnetosonic wave packet at $0<t<\Delta t$; (c) wave packet of electric field associated with ion-pressure anisotropy at $0<t<\Delta t$; (d) reflected magnetosonic wave packet at $t>\Delta t$; (e) ion-Bernstein wave packet at $t>\Delta t$; (f) transmitted magnetosonic wave packet at $t>\Delta t$.

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evident from Fig. 1 that, if the wave packet is sufficiently localized in phase space, then there is a *finite* time interval Δt between the crossing of the resonance layer by the incident wave and the subsequent emission of the reflected wave. (We call this phenomenon the *linear ion-cyclotron echo.)* Since the separation of the two linear conversions is $\Delta k_x = 2\omega_0/c_A$, the time interval is $\Delta t = \Delta k_x/|\dot{k}_x| = 2\Omega/c_A\Omega'$. We have qualitatively verified this echo phenomenon by computer simulation.[7]

We proceed to an analytic solution of this problem. By a kinetic analysis,[8] we derive coupled equations for the ion-pressure anisotropy $p \equiv p_{xy}(x,t)$ and the magnetic field perturbation $b \equiv \delta B_x(x,t)$:

$$[\omega - 2\Omega(x)]p(x,t) = \alpha b(x,t), \tag{1}$$

$$[\omega^2 - k_x^2 c_A^2] b(x,t) = \beta p(x,t). \tag{2}$$

The coupling constants α, β need not be specified in the present discussion. In (1) and (2), $\omega \equiv i\partial/\partial t$ and $k_x \equiv -i\partial/\partial x$. (We may treat c_A as constant in the resonance region.)

In the present Letter, we solve these equations iteratively, for the case of weak coupling (α, β small). On the right-hand side of (1), we take the incident magnetosonic wave packet, of width σ, as

$$b(x,t) = b_0 \exp[-(x - c_A t)^2/2\sigma^2] \exp[-i\omega_0(t - x/c_A)]. \tag{3}$$

We choose the x origin at the resonance layer for the wave frequency ω_0, i.e., $\omega_0 = 2\Omega(x=0)$ and $\Omega(x) = \omega_0/2 + x\Omega'$. Thus the wave packet crosses the resonance layer at $t=0$ (see Figs. 1 and 2).

Since Eq. (1) is a first-order ordinary differential equation in time, it is easily solved. After the wave packet has crossed the resonance layer, we have, for the IPA wave,

$$p(x,t) = p_0 \exp[-2\sigma^2\Omega'^2 x^2/c_A^2 - i2\Omega(x)(t - x/c_A)], \tag{4}$$

a wave packet whose envelope is not propagating in x space. On taking the Fourier transform of (4), we find (with $k_0 \equiv \omega_0/c_A$)

$$p(k_x,t) = p_1 \exp\{-[k_x - (k_0 - 2\Omega' t)]^2 c_A^2/8\sigma^2\Omega'^2\} \exp(-i\omega_0 t), \tag{5}$$

a pulse centered at $k_x = k_0 - 2\Omega' t$, i.e., propagating with $\dot{k}_x = -2\Omega'$. [To simplify the result (5), we have assumed $\sigma^2 \gg L_0/k_0$, where L_0 is the magnetic scale length $L_0 = \Omega/\Omega'$.]

The validity condition for our model is that the two linear conversion events are disjoint. This requires that the width, $\Delta k = 2\sigma\Omega'/c_A$, in k_x space of the IPA is small compared to the separation $2k_0$ of the two crossings. This simply requires $\sigma \ll L_0$, and is easily satisfied.

Since the incident and reflected magnetosonic waves are thus well separated in phase space, we can approximate the second-order differential operator in (2), for the reflected wave, by

$$\omega^2 - k_x^2 c_A^2 = (\omega + k_x c_A)(\omega - k_x c_A) \approx 2\omega_0(\omega + k_x c_A). \tag{6}$$

Equation (2) then reads, in k_x space,

$$2\omega_0(i\partial/\partial t + k_x c_A) b(k_x,t) = \beta p(k_x,t). \tag{7}$$

Again we have a first-order ordinary differential equation in time; we substitute (5) into (7), and obtain (after the IPA wave has passed the crossing)

$$b(x,t) = b_1 \exp\{-[x + c_A(t - \Delta t)]^2/2\sigma^2\} \exp[-i\omega_0(t + x/c_A)]. \tag{8}$$

Thus, as expected, the reflected magnetosonic wave packet leaves the resonance layer ($x=0$) at $t = \Delta t$.

In connection with the practical application to plasma heating, we have generalized this model in two directions.[8] First, we allow for finite k_x; then we must replace the full ion-pressure-anisotropy moment $p_{xy}(x,t)$ by the hybrid moment

$$p_{xy}(v_x; x,z,t) = \int\int dv_x\, dv_y (v_x - u_x)(v_y - u_y) f(v_x, v_y, v_z; x,z,t), \tag{9}$$

which is partially kinetic. The operator on the left-hand side of (1) becomes $\omega - 2\Omega(x) - k_z v_z$, so that in the weak-coupling limit, the IPA wave is replaced by a van Kampen-type mode representing gyroresonance guiding centers:

$$p_{xy}(v_z; x,z,t) \sim \delta[\omega - 2\Omega(x) - k_z v_z] \exp[i(k_z z - \omega t)]. \tag{10}$$

Since the dispersion relation is now $\omega = 2\Omega(x) + k_z v_z$, the propagation of k_x is unchanged; $dk_x/dt = -2\Omega'$, and so the time delay Δt for the echo is likewise unchanged.

Secondly, we allow the coupling to be strong. The coupled equations can still be solved to obtain self-consistent transmission and reflection coefficients. The portion of the absorption which resides in the collective ion-Bernstein mode is found by a projection technique. The results satisfy an energy conservation law.

It should be relatively straightforward to extend this idea further to general geometry. Work along this line will be reported later.

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