Orbital Magnetoconductance in the Variable-Range–Hopping Regime

U. Sivan and O. Entin-Wohlman

School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, Ramat Aviv, 69978 Tel Aviv, Israel

and

Y. Imry

Department of Nuclear Physics, Weizmann Institute, Rehovot 76100, Israel, and IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598

(Received 24 August 1987)

The orbital magnetoconductance (MC) in the variable-range–hopping (VRH) regime is evaluated by use of a model which approximately takes into account the interference among random paths in the hopping process. Instead of logarithmic averaging the MC is obtained by the critical percolating resistor method. The small-field MC is quadratic in H; it is positive deep in the VRH regime and changes sign when the zero-field conductivity is high enough. This behavior (except for the sign change) and the relevant magnetic field scale are in agreement with recent experiments. The calculated MC is always positive for strong fields and is predicted to saturate at sufficiently large fields.

PACS numbers: 72.20.Dp, 05.60.+w, 71.55.Jv, 72.20.My

The study of the magnetoconductance (MC) in disordered metals in the weak-localization regime has given valuable insights on the interference processes in such systems. Various electronic relaxation times have also been determined with this method. There is no such detailed understanding of the magnetotransport in the strongly localized regime. Recent studies[1-3] have focused on the magnetotransport in the mesoscopic range where the finite size of the sample is relevant. It is, however, also of interest to study the MC in a large, macroscopic, sample in the regime where thermal hopping dominates the transport.

A recent experimental study[4] of transport properties of indium oxide samples in the variable-range–hopping (VRH) regime reveals a positive MC. In the absence of magnetic field, the conductance of these specimens[4] obeys Mott's VRH law, $\sigma=\sigma_0\exp[-(T/T_0)^{1/(d+1)}]$, in two and three dimensions ($d=2,3$) with $200\leq T_0/T \leq 1000$. The hopping distance R_M extracted from the data is typically of the order of 5–10ξ, where ξ is the localization length. The behavior of the MC at low fields is as follows: After an initial, fast dependence (H^4?) at extremely small fields, the MC becomes quadratic in the magnetic field for a rather large range of the latter. Studies of the dependence of the MC on the magnetic field orientation relative to that of the film strongly indicate that is results from an orbital, rather than a spin, effect. The change in the conductance due to the magnetic field is characterized by the flux $\Phi_M\equiv HR_M^{3/2}\chi^{1/2}$ through an effective area of the order of $R_M^{3/2}\chi^{1/2}$, where χ is the microscopic length (i.e., the typical distance between impurities). The positiveness of the MC and the anisotropy with respect to the magnetic field orientation were also observed in earlier measurements on 2D Si-inversion layers.[5]

None of the theories known to us accounts for these experimental data. Shklovksii and Efros[6] and Suprapto and Butcher[7] predict a negative MC, due to the shrinkage of the wave functions in the presence of magnetic field. Nguyen, Spivak, and Shklovskii (NSS)[8,9] consider the effect of the interference among the various paths associated with the hopping between two sites at a distance R_M apart and a small energy separation of the order of a few k_BT. They find that the interference between all possible paths within a cigar-shaped domain of length R_M and width $(R_M\xi)^{1/2}$ might change considerably the hopping probability between two sites. Averaging numerically the logarithm of the conductivity over many random impurity realizations, in the presence of a magnetic field, they obtain under certain conditions a positive MC which is *linear* in the field in the whole relevant field range. This linearity is in qualitative disagreement with the experimental results in Ref. 4 (see below). Both positiveness and linearity emerge from the logarithmic nature of the averaging process.

Here we present a theory for the orbital MC in the VRH regime, which yields the sign of the MC and the *quadratic* field dependence over most of the weak-field range, where the field scale is determined by the parameter Φ_M/Φ_0 ($\Phi_0=hc/e$ being the quantum flux unit). Furthermore, our model predicts a saturation of the MC for $\Phi_M/\Phi_0\gg 1$. Spin effects have been treated in Ref. 2 and by Kamimura.[10] Rather than employing the logarithmic averaging, for which we know of no real justification, our calculations are based on the critical path analysis of Ambegaokar, Halperin, and Langer[11] (see also Pollak[12]) which we generalize to include the effects of interference and magnetic field. A major difference between the results of NSS[8,9] and our analysis is that we find an H^2 dependence at small fields rather than an $|H|$ one. We shall later explain this overestimate of the MC in the logarithmic averaging procedure. The experimental results[4] are certainly inconsistent with the $|H|$ behavior and are in reasonable agreement with the quadratic dependence over most of the weak-field range. The indicated faster-than-H^2 dependence at extremely small fields might be due to improbable long jumps whose treatment is beyond the present analysis.

The conductivity of a sample in the hopping regime may be analyzed in terms of an equivalent resistor network[13] in which two sites are connected by a conductivity $\sigma_{if}=\sigma_0\exp(-ar_{if}-\beta\epsilon_{if})$ where r_{if} is the distance between the two sites, $a=1/\xi$, ϵ_i and ϵ_f are the site energies, $\epsilon_{if}=(|\epsilon_i|+|\epsilon_f|+|\epsilon_i-\epsilon_f|)/2$, and σ_0 is some constant having the dimensionality of a conductivity. Following NSS[8,9] we model each resistor by a square (a cube in 3D) as shown in Fig. 1. The two sites i and f on the opposite corners of the square represent the sites between which the hopping occurs. The rest of the sites simulate all other scattering centers sampled by the hopping electron. For each square we assume, following NSS, a nearest-neighbor tight-binding Anderson model,[8,9] $H=\sum_\nu \epsilon_\nu a_\nu^\dagger a_\nu+\sum_{\nu\neq\mu}V_{\nu\mu}a_\mu^\dagger a_\nu$, where $V_{\nu\mu}=V$ for nearest neighbors and 0 otherwise. ϵ_ν is assumed to be randomly distributed with zero mean and standard deviation W. The localization length ξ $(\geq \chi)$ is determined by $(W/V)^2$. For all resistors in the critical network,[11] $|\epsilon_i|$ and $|\epsilon_f|$ are of the order of a few times k_BT, much smaller than the average energy spacing within a localization volume. The conductivity of the square is proportional to[8,9] $|I|^2$, where I is the effective matrix element between the states localized at sites i and f. The main contribution to I comes from the oriented paths between i and f, excluding backwards steps.[8,9] The corrections due to winding paths will be briefly considered later. The matrix element squared is given by

$$|I|^2=V^2(V/W)^{4N-2}\left|\sum_\gamma J_\gamma\exp(i\phi_\gamma)\right|^2. \tag{1}$$

Here

$$J_\gamma=\prod_{\nu\in\gamma}[W/(\epsilon_i-\epsilon_\nu)]\simeq\prod_{\nu\in\gamma}(-W/\epsilon_\nu)$$

is the contribution of the γth path, ν runs over the $2N$ points on the path (excluding i), and ϕ_γ is the phase acquired by the γth path in the presence of a magnetic field. Since the number of oriented paths, n, is exponential in N, it is possible to redefine a in the expression for σ_{if} such that

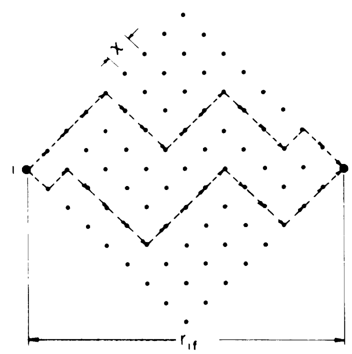

FIG. 1. NSS (Refs. 8 and 9) model for a hop. The electron hopping from site i to f samples all other scattering centers *en route*. The conductivity is proportional to the sum over all possible paths leading from i to f.

$$\sigma_{if}=\sigma_0(|J|^2/n)\exp(-ar_{if}-\beta\epsilon_{if}), \tag{2}$$

with $|J|^2=|\sum_\gamma J_\gamma\exp(i\phi_\gamma)|^2$ which is of the order of n. The quantity $|J|^2$ is a random variable. Its probability distribution leads to a probability distribution $P(\sigma/\sigma_0,r)$ for the dimensionless conductivity σ/σ_0 and a spatial separation r, and the dependence of the latter upon the magnetic field will yield the MC. Ambegaokar, Halperin, and Langer[11] argue that the percolation condition upon $P(\sigma/\sigma_0,r)$ determines the conductance of the sample. One should introduce the resistors into the network, one by one, in an increasing order of resistivity, and the conductivity of the sample would be given by the conductivity σ_c of the resistor which just makes the sample connected.[11] The percolation threshold condition for the case where the resistors have varying lengths is that the total volume occupied by the resistors is a certain finite fraction, Z_c, of the volume Ω of the system

$$\frac{\Omega}{\chi^{2d}}\int_{\sigma_c}^\infty d\sigma\int_0^\infty r^d d(r^d)P(\sigma/\sigma_0,r)=Z_c. \tag{3}$$

This is an implicit equation for σ_c. The dependence of σ_c on the magnetic field arises from the field dependence of $P(\sigma/\sigma_0,r)$.

The distribution $P(\sigma/\sigma_0,r)$ is given by

$$P(\sigma/\sigma_0,r)=\int_0^\infty d(y^2)P_1(y^2|r)\int_0^\infty dt\,P_2(t,r)\delta(\sigma/\sigma_0-y^2e^{-t}), \tag{4}$$

where $P_1(y^2|r)$ is the conditional probability of y^2 for a given r, $P_2(t,r)$ is the joint probability of t and r, $t=ar_{if}+\beta\epsilon_{if}$, and $y^2=|J|^2/n$. We calculate the probability distribution $P_1(y^2|r)$ for a NSS square having n oriented paths. By definition, J_γ is a random variable with zero mean and standard deviation of the order of 1. This implies at zero field (by the central limit theorem) a Gaussian distribution for y, regardless of the exact form of the probability distribution of J_γ. The MC is generated by the change in $P_1(y^2|r)$ due to the magnetic field. One may conjecture various probability distributions for J_γ, but for the sake of simplicity we assume $P(J_\gamma)=1/(2\pi)^{1/2}\exp(-J_\gamma^2/2)$ and neglect possible correlations among the paths. Setting $J=J'+iJ''$, one obtains

$$P_1(J',J''|r)=(2\pi)^{-n/2}\int_{-\infty}^{\infty}dJ_1\cdots\int_{-\infty}^{\infty}dJ_n\left\{\exp\left[-\sum_{\gamma=1}^{n}J_\gamma^2/2\right]\delta\left[J'-\sum_{\gamma=1}^{n}J_\gamma\cos\phi_\gamma\right]\delta\left[J''-\sum_{\gamma=1}^{n}J_\gamma\sin\phi_\gamma\right]\right\}. \quad (5)$$

If we choose a gauge in which for any path with a phase ϕ_γ there is also a symmetric path with a phase $\phi_{\gamma'}=-\phi_\gamma$, Eq. (5) yields

$$P_1(y^2|r)=\frac{\exp[-(y^2/4)(a^{-2}+b^{-2})]}{2ab}I_0\left[\frac{y^2}{4}\left(\frac{1}{b^2}-\frac{1}{a^2}\right)\right], \quad (6)$$

where $a^2=n^{-1}\sum_\gamma\cos^2\phi_\gamma$, $b^2=1-a^2$ and I_0 is the modified Bessel function of order zero.

In the weak field limit $\Phi_M/\Phi_0\ll 1$ and $y^2/(4b^2)\gg 1$. Consequently,

$$P_1(y^2|r)=\frac{\exp\{-y^2/2a^2\}}{(2\pi)^{1/2}a|y|}\left[1+\frac{b^2}{2}\left(1+\frac{1}{y^2}\right)\right]. \quad (7)$$

In the strong-field limit, $\Phi_M/\Phi_0\gg 1$ and $a^2\simeq b^2=\frac{1}{2}$, leading to

$$P_1(y^2|r)=\exp(-y^2). \quad (8)$$

Note that $P_1(y^2|r)$ is independent of the size of the square, both at zero and at strong magnetic fields.

The last ingredient needed in Eq. (4) is the probability distribution $P_2(t,r)$, which for the relevant range $t<\beta W\sqrt{3}$ is found to be

$$P_2(t,r)=(3/2\beta^2W^2\Omega)(t-ar)\Theta(t-ar), \quad (9)$$

where Θ is the step function. Both at zero and at strong magnetic fields, t and y^2 are independent random variables, while in the intermediate range they are correlated because of the dependence of a^2 and b^2 on r_{if}. To evaluate this dependence we notice that the electron executes a random walk along the direction perpendicular to the hopping direction. The number of steps is proportional to r_{if}/χ and the step size is proportional to χ. The effective area for the magnetic flux is therefore proportional to $r_{if}^{3/2}\chi^{1/2}$ and hence, for weak fields, $b^2=KH^2r_{if}^{3/2}\chi^{1/2}/\Phi_0^2$, where K is some constant. We point out that it is the microscopic length χ that appears in the relevant area and not ξ, as suggested by NSS.[8,9]

Substituting Eqs. (4), (7) (with $b^2=0$), and (9) into the percolation condition (3), one obtains the well-known Mott VRH law. In the presence of a weak magnetic field, the percolation conditions takes the form

$$\frac{3}{2\beta^2W^2\chi^{2d}}\int_{\sigma_c/\sigma_0}^{\infty}\frac{d\sigma}{\sigma_0}\int_0^\infty d(r^d)\int_0^\infty\frac{d(y^2)\exp[-y^2/(2a^2)]}{(2\pi)^{1/2}ya}\times\left[1+\frac{b^2}{2}\left(1+\frac{1}{y^2}\right)\right]\int_0^\infty dt(t-ar)\Theta(t-ar)r^d\delta\left(\frac{\sigma}{\sigma_0}-y^2e^{-t}\right)=Z_c, \quad (10)$$

which exhibits in general a quadratic dependence of $\sigma_c(H)$ upon the field. Differentiating Eq. (10) with respect to H^2 at $H=0$, one finds for the MC

$$\frac{\sigma_c(H)-\sigma_c(0)}{\sigma_c(0)}=\frac{H^2K\chi\xi^3}{\Phi_0^2}\frac{d(2d+1)}{(d+2)(2d+3)}\frac{[\sigma_0/\sigma_c(0)]I_{3/2}-I_{1/2}}{M_{1/2}}, \quad (11)$$

where

$$I_{n/2}=\int_1^\infty\frac{dz}{z^{n/2}}\ln^{2(d+2)}(z)\exp\left(\frac{-\sigma_c(0)z}{2\sigma_0}\right),\quad\text{and}\quad M_{1/2}=\int_1^\infty\frac{dz}{z^{1/2}}\ln^{2d+1}(z)\exp\left(\frac{-\sigma_c(0)(z)}{2\sigma_0}\right).$$

The sign of the MC is thus determined by the conductivity $\sigma_c(0)$ in the absence of the magnetic field. Numerical evaluation of $I_{1/2}$, $I_{3/2}$, and $M_{1/2}$ for $d=2$ shows that for $\sigma_c(0)/\sigma_0<10^{-4}$ or $T_0/T>750$, the MC is positive, while for higher conductivities it is negative. Note, however, that the latter limit might not be consistent with the neglect of winding paths.

At strong magnetic fields $\Phi_M/\Phi_0\gg 1$, $P_1(y^2|r)$ is given by Eq. (8) and the percolation conditions gives

$$\frac{3}{4\beta^2W^2(a\chi)^{2d}(2d+1)}\int_1^\infty\frac{dz}{z}\exp\left(-\frac{z\sigma_c}{\sigma_0}\right)\ln^{2d+1}(z)=Z_c. \quad (12)$$

We find then that the conductivity saturates at high fields to a value, $\sigma_c(\infty)$, which is of the same order of magnitude as the zero-field one, $\sigma_c(0)$. Solving Eq. (12) numerically for σ_c, one finds that the MC at strong fields is positive for all values of $\sigma_c(0)/\sigma_0$ which are relevant for the VRH regime and that $[\sigma_c(\infty)-\sigma_c(0)]/\sigma_c(0)$ is only a weak function of $\sigma_c(0)$. Our results are schematically summarized in Fig. 2. Within the percolation picture the conductivity of the sample is given by the critical conductivity $\sigma_c(H)$. Therefore, the sign of the MC at low fields is related to the change of the matrix element characteristic of $\sigma_c(0)$, by the magnetic field. Small values of $\sigma_c(0)$ give more weight to small matrix elements or realizations with destructive interference among the paths. The magnetic field will act as to improve the matrix element and hence to increase the conductivity. On the other hand, when $\sigma_c(0)$ is large (constructive interference) the magnetic field will usually act as to reduce it, resulting in a negative MC, i.e., it is the magnitude of the characteristic matrix element of $\sigma_c(0)$ relative to the average matrix element which determines the sign of the low-field MC. At high magnetic fields, the probability distribution of the interference factor $\langle y^2\rangle$ weighs more than the large y^2, and hence leads to a positive value for the MC regardless of the disorder. The reason for the overestimate of the positive MC in the logarithmic averaging procedure[8,9] is its emphasis of the bonds with very low conductivity. As discussed above, such bonds are characteristized by a destructive interference and hence a large positive MC. However, these bonds play no role whatsoever in the conductance of the sample[11] as they are shunted by the better conductors in the network.

In this paper, only the contribution of the forward-going paths was included. NSS[8,9] justified this by arguing that going back even once will introduce a path multiplied by the small factor $(V/W)^2$. Of course, in a critical evaluation of such a contribution, the number of possibilities to introduce backward steps or loops should be taken into account as well. It would thus appear that $V/W \ll N^x$, with $x>0$ some as yet unspecified exponent, should be a sufficient condition for the validity of the NSS picture (which should be certainly valid deep enough in the VRH regime). We are currently pursuing this issue and the role of the loops in the MC. We believe that the results presented here are more general than the specific NSS model used.

The authors are especially grateful to Z. Ovadyahu and O. Faran for many discussions. We also thank A. B. Fowler, D. S. Fisher, and J. Chalker for useful discussions. This research was partially supported by the fund for basic research administered by the Israeli Academy for Sciences and Humanities. The hospitality of the Institute for Theoretical Physics at Santa Barbara, where part of this work was done, is gratefully acknowledged.

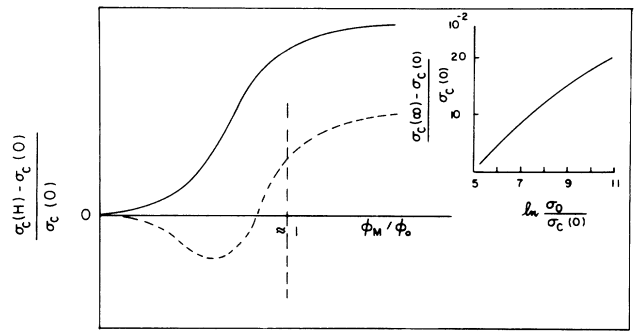

FIG. 2. The MC in the VRH (schematically) at weak (dashed line) and strong (solid line) disorder vs the flux through the area $R_M^{3/2}\chi^{1/2}$. Notice the quadratic dependence on the flux for $\Phi_M \ll \Phi_0$. Inset: $[\sigma_c(\infty)-\sigma_c(0)]/\sigma_c(0)$ vs $\ln[\sigma_0/\sigma_c(0)]$.

The work there was supported by National Science Foundation Grant No. Phy82-17853, supplemented by funds from the National Aeronautics and Space Administration.

[1]R. A. Webb, A. B. Fowler, A. Hartstein, and J. J. Wainer, Surf. Sci. **170**, 14 (1986); A. B. Fowler, G. L. Timp, J. J. Wainer, and R. A. Webb, Phys. Rev. Lett. **57**, 138 (1986); R. A. Webb, J. J. Wainer, and A. B. Fowler, to be published, and in the Proceedings of the Seventh International Conference on Electronic Properties of Two-Dimensional Systems, Santa Fe, New Mexico, 1987, Surf. Sci. (to be published).

[2]R. K. Kalia, W. Xue, and P. A. Lee, Phys. Rev. Lett. **57**, 1615 (1986).

[3]Ya. B. Poyarkov, V. Ya. Kontarev, I. P. Krylov, and Yu. V. Sharvin, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 291 (1986) [JETP Lett. **44**, 375 (1986)].

[4]Z. Ovadyahu, Phys. Rev. B **33**, 6552 (1986); O. Faran and Z. Ovadyahu, to be published.

[5]A. Hartstein, A. B. Fowler, and K. C. Woo, Physica (Amsterdam) **117 & 118B**, 655 (1983).

[6]B. I. Shklovskii and A. L. Efros, *Electronic Properties of Doped Semiconductors*, Springer Series in Solid-State Sciences Vol. 45, edited by M. Cardona *et al.* (Springer-Verlag, Berlin, 1984), pp. 210–216.

[7]B. B. Suprapto and P. N. Butcher, J. Phys. C **8**, L517 (1975).

[8]V. I. Nguyen, B. Z. Spivak, and B. I. Shklovskii, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 35 (1985) [JETP Lett. **41**, 42 (1985)].

[9]V. I. Nguyen, B. Z. Spivak, and B. I. Shklovskii, Zh. Eksp. Teor. Fiz. **89**, 1770 (1985) [Sov. Phys. JETP **62**, 1021 (1985)].

[10]H. Kamimura, Suppl. Prog. Theor. Phys. **72**, 206 (1982).

[11]V. Ambegaokar, B. I. Halperin, and J. S. Langer, Phys. Rev. B **4**, 2612 (1971).

[12]M. Pollak, J. Non-Cryst. Solids **11**, 1 (1972).

[13]A. Miller and E. Abrahams, Phys. Rev. **120**, 745 (1960).