Depression of the Superfluid Transition Temperature in ^{4}He by a Heat Current

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We report experimental results for the depression of the superfluid transition temperature $T_\lambda(Q)$ in ^{4}He by a heat current Q. The data were obtained by use of thermometry with a resolution of 10 nK, and cover the range $0.4 \lesssim Q \lesssim 10\ \mu\text{W/cm}^2$. They can be represented by $1-T_\lambda(Q)/T_\lambda(0)=(Q/Q_0)^x$ with $Q_0=568\pm200\ \text{W/cm}^2$ and $x=0.813\pm0.012$, and are in good agreement with theoretical predictions.

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The problem of phase transitions in nonequilibrium systems is an evolving field which is as yet relatively unexplored from both an experimental and a theoretical viewpoint.[1] One particular example which has been studied both theoretically and experimentally is that of a phase-separating binary mixture in the presence of shear flow. Another well-studied case is that of a superconductor carrying an electric current. In that case, however, fluctuations are unimportant at the transition and a mean-field theory provides an adequate description of the observed phenomena. A related and less trivial case is the superfluid transition in liquid ^{4}He in the presence of a heat current. Here a mean-field theory is inadequate because fluctuations dominate the behavior near the phase transition. The hydrodynamics of ^{4}He is well developed,[2] and very precise measurements near the transition are possible. Thus, this system is a prime candidate for experimental and theoretical study of a nonequilibrium phase transition in the presence of fluctuation. This work also will provide a deeper understanding of the range of applicability of two-fluid hydrodynamics.

The hydrodynamics of superfluid ^{4}He is well understood, and is based upon the assumption that a description of the fluid motion requires the specification of *two* velocity fields $\mathbf{v}_s$ and $\mathbf{v}_n$, with the mass-flux density $\mathbf{j}$ given by

$$\mathbf{j}=\rho_s\mathbf{v}_s+\rho_n\mathbf{v}_n, \tag{1}$$

and $\rho_s+\rho_n$ equal to the fluid density ρ.[2] The thermohydrodynamic equations then involve a new pair of conjugate variables in addition to the (temperature, entropy) and (pressure, density) pairs of ordinary thermodynamics, namely $(\frac{1}{2}w^2,\rho_n/\rho)$ where $\mathbf{w}=\mathbf{v}_n-\mathbf{v}_s$. Therefore, in general, ρ_n (and also ρ_s) will depend upon w^2. At temperatures well below the superfluid transition temperature T_λ (say below about 1.8 K at vapor pressure) the derivative $(\partial\rho_n/\partial w^2)_{P,T}$ can be calculated from the spectrum of elementary excitations,[2] and measurements consistent with this result have been obtained.[3] However, at higher T we do not know of any definitive experiments, and theoretical predictions[4-6] are less firmly based.

One way of creating a counterflow velocity $\mathbf{w}$ is to impose a heat current $\mathbf{Q}$.[2] In the superfluid the entropy flux is associated with $\mathbf{v}_n$, and $\mathbf{Q}=\rho ST\mathbf{v}_n$. If the total mass flux is equal to zero, Eq. (1) gives $\mathbf{v}_s=-(\rho_n/\rho_s)\mathbf{v}_n$, and thus

$$\mathbf{Q}=\rho_s ST\mathbf{w}. \tag{2}$$

Near T_λ, ρ_s vanishes approximately as $\rho_s/\rho=kt^\nu$, with $\nu=0.672$ and $t=1-T/T_\lambda$.[7] Therefore, sufficiently close to T_λ Eq. (2) predicts that even a modest Q will yield a large value of w. If ρ_s is depressed by this current, then T_λ (and possibly the nature of the transition) will be altered.[8] Theoretical discussions of this effect have been presented by several authors.[5,6] As noted above, the phenomenon is somewhat analogous to the depression of the transition temperature in a superconductor by an electric current or a magnetic field.[6] However, the fluctuations, which are important in the helium case but not in the superconductor, initially have been neglected in the theory.[5,6] An attempt has been made to include their effect *a posteriori* by use of scaling arguments and known values for the correlation length and the thermal conductivity.[6] As we shall see, this approach has yielded a remarkably accurate prediction; but a more fundamental treatment which includes the fluctuations at the beginning would nontheless be desirable.

Measurements of $\Delta T_\lambda=T_\lambda(0)-T_\lambda(Q)$ are difficult for two reasons. First, the heat current must be kept sufficiently small to avoid the inadvertent inclusion of significant temperature differences ΔT_{II} across the superfluid sample.[9] For our geometry, we found ΔT_{II} to be negligible compared to ΔT_λ for $Q\lesssim10\ \mu\text{W/cm}^2$; but for larger currents ΔT_{II} became significant and thus measurements of ΔT_λ became less reliable.[10] Consequently, the effect to be measured was less than 1 μK, and ultrahigh-resolution thermometry was required for quantitative determinations. The second experimental obstacle is due to the singular and Q-dependent boundary (Kapitza) resistance between He II and solids.[11] In our cell, a current density of 10 $\mu\text{W/cm}^2$ for instance yields a temperature difference ΔT_b across our bottom solid-liquid boundary of 11 μK, which is an order of

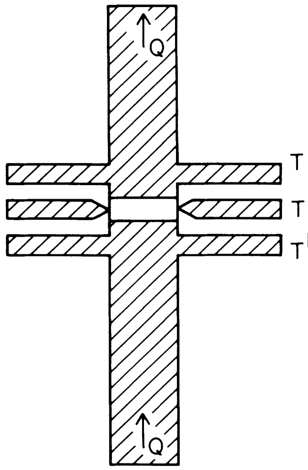

FIG. 1. Schematic diagram of the experimental cell.

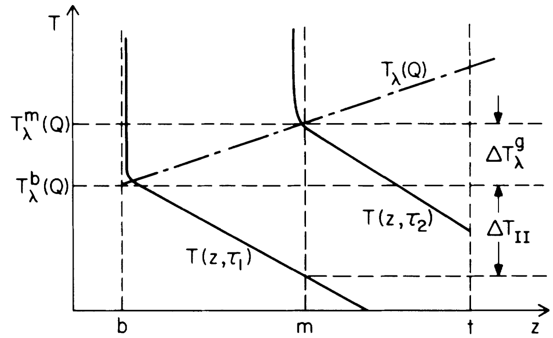

FIG. 2. Schematic diagram of the temperature distribution in the liquid-helium sample. The locations of the bottom, midplane, and top thermometers are indicated along the abscissa by b, m, and t, respectively. The dash-dotted line indicates the λ temperature, which in the presence of gravity depends upon z. The solid lines give the sample temperature at two values of the time τ during a ramping experiment.

magnitude larger than ΔT_λ. The fluid temperature thus could not be sensed by our monitoring the bottom and top cell ends where the current entered and left the cell. Instead, the temperature was determined at the cell side wall at half height.

A schematic drawing of the experimental cell is shown in Fig. 1. The temperatures T^t and T^b of the top and bottom cell ends, as well as the temperature T^m of a midplane, were monitored with susceptibility thermometers in ac bridges.[12] The current Q was generated by a heater at the bottom cell end. A total current of about 300 μW left the cell top, and the cell temperature was controlled by a regulator circuit which dissipated heat with a second heater mounted on the cell top. The temperature could be varied continuously with time by the provision of an offset voltage ramp to the regulator. Either T^t or T^m could be controlled with the regulating loop. Typical ramp rates were in the range 3×10^{-10} to 5×10^{-8} K/s. The sample was ^{4}He with a ^{3}He impurity concentration of 5×10^{-10}, had a circular cross section with a radius of 1.27 cm, and was 0.575 cm thick. The cell was installed in a cryostat described previously.[11,13]

The evolution of the temperature profile in the cell as the cell-top temperature is ramped from below T_λ through the transition is illustrated schematically in Fig. 2. When the entire sample is superfluid, any difference $\Delta T_{\rm II}(Q)$ across the fluid (excluding the boundary temperature drop) between T^b and T^m is small. Normal fluid (He I) first forms at the bottom of the sample because of the Earth's gravitational field.[14] Because of the finite conductivity of He I, the current entering the cell bottom then establishes a large temperature difference across the normal layer. As the temperature of the superfluid top portion of the sample is increased further and the He I layer thickness grows, the temperature difference grows dramatically. The evolution of the temperature profile is illustrated schematically in Fig. 2 by the solid curves corresponding to two particular times τ_1 and τ_2 in the ramp.

In Fig. 3 we show experimental data for T^b and T^m versus time in a run where T^t (not shown) was used in the temperature-control loop and was ramped from below to above the transition. For $T < T_\lambda^b$, the two temperatures essentially track each other (as well as T^t). Beyond $T_\lambda^b(Q)$, T^b increases dramatically because of the finite conductivity of the He I layer which begins to form at the cell bottom. From this point on an increasing (but small) portion of Q is used to establish the large temper-

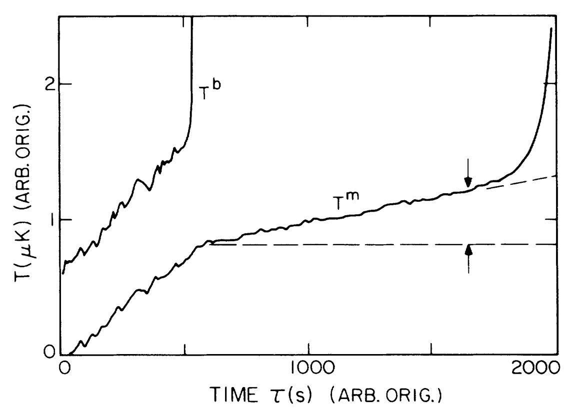

FIG. 3. Experimental data for the bottom temperature $T^b(\tau)$ and the midplane temperature $T^m(\tau)$ during a ramping experiment where the top temperature $T^t(\tau)$ (not shown) is changed linearly in the time τ. The data for $T^b(\tau)$ are displaced vertically relative to those for $T^m(\tau)$ by approximately -10 μK. The horizontal position of the two arrows corresponds to the time when T_λ^m was reached, and the distance between them is equal to $\Delta T_\lambda^g + \Delta T_{\rm II}$ (see Fig. 2). The data are for $Q = 9.79$ μW/cm^2. They show that the temperature drop $\Delta T_{\rm II}$ across the superfluid sample is negligible at this power.

ature gradient which develops in the normal layer as the He I–He II interface moves away from the bottom plate at an average velocity of 3 μm/s. Therefore, a slightly lesser current flows across the top liquid-solid boundary, thereby reducing the boundary temperature drop. This permits the midplane temperature to increase at a reduced rate while T^t is continuing to increase (under control of the temperature regulator) at the same rate. Finally, when T^m reaches T_λ^m, the midplane thermometer indicates the dramatic temperature rise associated with the arrival of the He I–He II interface.

As can be seen in Fig. 3, the temperature $T_\lambda^b(Q)$ is easily mapped onto the midplane thermometer. The temperature difference $T_\lambda^m(Q) - T^m(T_\lambda^b(Q))$, as indicated by the midplane thermometer at the two λ points, is equal to $\Delta T_\lambda^g + \Delta T_{\rm II}(Q)$, where $\Delta T_\lambda^g = 0.370$ μK is the gravity-induced difference in the λ points.[14] Since ΔT_λ^g is known, the measurement yields $\Delta T_{\rm II}(Q)$. The data in Fig. 3, which are for $Q = 10$ μW/cm^2, give $\Delta T_\lambda^g + \Delta T_{\rm II} = 0.380$ μK, thus implying that $\Delta T_{\rm II} = 10 \pm 20$ nK for this power level.[15] Thus within our resolution the thermal gradient in the superfluid is negligible for $Q \leq 10$ μW/cm^2.

Measurements of $T_\lambda(Q)$ were made at many values of Q by our determining $T^m(T_\lambda^b(Q))$. Each determination was preceded and followed by one at a reference power $\tilde{Q} = 0.490$ μW/cm^2, and $\delta T_\lambda = T_\lambda(\tilde{Q}) - T_\lambda(Q)$ (which is relatively unaffected by drifts in the thermometry) was fitted by the equation

$$\delta T_\lambda / T_\lambda = (Q/Q_0)^x - (\tilde{Q}/Q_0)^x. \quad (3)$$

For $Q < 10$ μW/cm^2, the fit yielded $x = 0.813 \pm 0.012$ and $Q_0 = 568 \pm 200$ W/cm^2. The root mean square deviation from the fit was 13×10^{-9}, corresponding to 28 nK, and is consistent with our thermometer resolution and stability. In Fig. 4 we show the data for $\Delta T_\lambda / T_\lambda = \delta T_\lambda / T_\lambda + (\tilde{Q}/Q_0)^x$ vs Q on logarithmic scales. The straight line through the data corresponds to the fit for $Q \lesssim 10$ μW/cm^2. Up to $Q \simeq 15$ μW/cm^2, it fits the data well. Beyond that power, the data points lie slightly above the line and corrections for the temperature difference $\Delta T_{\rm II}$ across the superfluid are required.

A detailed prediction of the λ-point shift was made by Onuki.[6] He obtains[8]

$$\Delta T_\lambda / T_\lambda = A (Q/Q_0)^x,$$

with A a constant of order unity and $x = 1/(1 + \nu - x_\lambda)$. Here $\nu = 0.672$ is the correlation-length exponent[7] and x_λ is the effective exponent of the thermal conductivity λ of He I when this variable is approximated by $\lambda = \lambda_0 t^{-x_\lambda}$. From measurements[16] of λ, we estimate $x_\lambda = 0.423$ at $t \cong 3 \times 10^{-7}$, and thus obtain $x = 0.80$ from the theory, in excellent agreement with the experiment. The theory also gives $Q_0 = \lambda_0 T_\lambda / \xi_0$, where ξ_0 is the correlation-length amplitude. From this we estimate Q_0 to be equal to 2300 W/cm^2. When the data are fitted with x fixed at 0.80, we find $Q_0 = 754$ W/cm^2. This differs from the theoretical value only by a factor of 3, and suggests that $A \cong 2.4$, i.e., indeed of order 1.

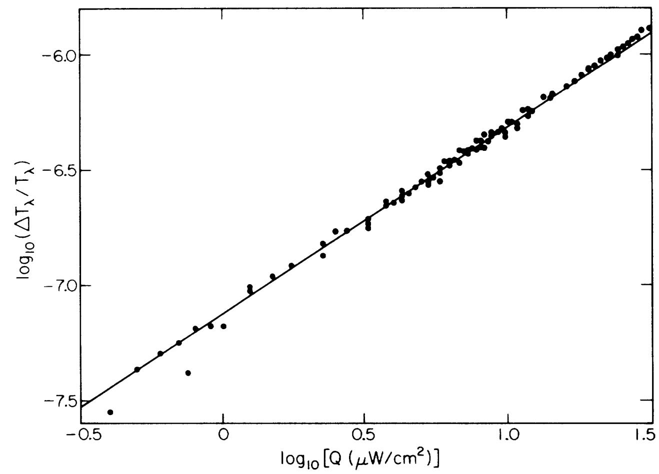

FIG. 4. The λ-point shift $\Delta T_\lambda(Q)/T_\lambda$ vs the heat-current density Q on logarithmic scales. The solid line is the best fit of a power law [Eq. (3)] to the data, and corresponds to $x = 0.813$.

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