Detection of Nucleon Correlations via Pion Double-Charge-Exchange Reactions

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Double-charge-exchange reactions induced by pions furnish a direct means of detecting nucleon-nucleon correlations in nuclei. For the particular case of double isobaric analog transitions the effect of correlations is easily isolated and quite strong. We show that the magnitudes of the double-charge-exchange cross sections for the isotopes ^{42}Ca, ^{44}Ca, and ^{48}Ca, recently measured at low energies, can be explained well by our taking account of the position correlations of the valence neutrons in the respective ground states.

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The existence of position correlations between nuclear particles is intrinsic to most models of the nucleus. Experimental efforts to measure those correlations, however, have a long and quite unproductive history. Their effects prove to be masked, in most experimental contexts, by other effects that are larger in magnitude and often quite uncertain as well. Double-scattering amplitudes, for example, are sensitive to correlations, but cannot easily be separated from the primary and dominant amplitudes for single scattering. Double-charge-exchange (DCX) processes of pions in nuclei, however, are uniquely exempt from this drawback; they are described by a double-collision amplitude that is not masked by any interfering single-collision amplitude. In particular, those double exchanges that take place between isobaric analog states are quite similar to elastic double-scattering processes, and are thus quite amenable to detailed theoretical analysis. Indeed we have shown, in a recent discussion of the DCX process in ^{14}C at 50 MeV,[1] that the magnitude and the shape of the differential cross section are quite responsive to the position correlations of the two valence neutrons that the reaction transforms into protons. Measurements have recently been reported[2-4] on DCX transitions in the calcium isotopes ^{42}Ca, ^{44}Ca, and ^{48}Ca. These now permit us to give a much more detailed and explicit demonstration of the role played by nucleon-nucleon correlations; it is, in fact, a dominant one.

The DCX processes we study are induced by positive pions incident on the nucleus (N,Z) and lead to the isobaric analog state in the nucleus $(N-2,Z+2)$. Such analog-state transitions are essentially elastic. The nucleon isospins are simply rotated, while the configuration space and spin wave functions of the nucleons undergo no change. The nucleus (N,Z) will be pictured as consisting of a closed-shell core, plus $N-Z$ valence neutrons. At the low energy of 35 MeV the nucleus is fairly transparent to pions. We have shown in our study[1] of DCX in ^{14}C at the somewhat higher energy of 50 MeV that no appreciable error is introduced by using the second-order impulse approximation to describe this DCX process. At 35 MeV that approximation should be comparably good and the integrated background effects of nuclear elastic scattering and absorption small.[1] Within this approximation DCX is simply a sequence of two single exchanges. In the double-analog transition neither of these exchanges can take place on nucleons in the nuclear core since such processes lead to final states outside the isobaric multiplet. The nuclear core therefore plays a passive role, and the matrix element of the $\pi^+ \to \pi^-$ process can be expressed simply as a sum of contributions of pairs (m,n) of valence nucleons. By introducing spin- and isospin-dependent transition operators

$$F_n(\mathbf{p}',\mathbf{p}) = e^{i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{r}_n}(\boldsymbol{\tau}_n\cdot\mathbf{T}_\pi)M_n(\mathbf{p}',\mathbf{p})$$

describing the charge-exchange amplitude on the nth nucleon, we can write the DCX transition amplitude[1] as

$$\mathcal{T}_{\mathrm{DCX}}(\mathbf{Q}) = \langle \pi^-(N-2,Z+2)\,|\,-\frac{1}{2\pi^2}\sum_{m\neq n}^{N-Z}\int F_m(\mathbf{k}_f,\mathbf{p})G(p)F_n(\mathbf{p},\mathbf{k}_i)d^3p\,|\,\pi^+(N,Z)\rangle, \tag{1}$$

where $\mathbf{Q}=\mathbf{k}_i-\mathbf{k}_f$ is the momentum transferred by the pions, and $\mathbf{T}_\pi$ and $\boldsymbol{\tau}_n$ are the isospin operators for the pion and the nth nucleon, respectively. The integration variable $\mathbf{p}$ is the momentum of the neutral pion that travels from one collision to the other, and $G(p)=(k^2-p^2+i\epsilon)^{-1}$ is the complex pion propagator. The transition amplitudes in Eq. (1) have been summed over all possible intermediate nuclear states by means of the closure approximation. Had we re-

stricted the summation to the analog intermediate state, we would, in fact, have excluded the possibility of treating correlations.[1]

When the isospin matrix elements are evaluated and the sum carried out over the contributing pairs of nucleons, Eq. (1) becomes

$$\mathcal{F}_{\mathrm{DCX}}(\mathbf{Q}) = [(N-Z)(N-Z-1)/2]^{1/2}[A^{(21)}_{N-Z}(\mathbf{Q}) + A^{(12)}_{N-Z}(\mathbf{Q})], \tag{2}$$

where $A^{(21)}_{N-Z}(\mathbf{Q})$ is the amplitude for an ordered DCX process on one arbitrarily chosen pair of neutrons, 1 and 2,

$$A^{(21)}_{N-Z}(\mathbf{Q}) = -\frac{1}{\pi^2}\langle 1,\ldots,N-Z \,|\, \int e^{i\mathbf{q}_1\cdot\mathbf{r}_1 + i\mathbf{q}_2\cdot\mathbf{r}_2} M_2(\mathbf{k}_f,\mathbf{p}) G(p) M_1(\mathbf{p},\mathbf{k}_i) d^3p \,|\, 1,\ldots,N-Z\rangle, \tag{3}$$

and $\mathbf{q}_1 = \mathbf{k}_i - \mathbf{p}$ and $\mathbf{q}_2 = \mathbf{p} - \mathbf{k}_f$.

The square of the matrix element (2) obviously contains as a factor the total number of valence neutron pairs. While that number increases, for example, from 1 to 28 in the sequence of isotopes from ^{42}Ca to ^{48}Ca, it would be quite erroneous to assume that the cross section for DCX increases in any comparable way. In fact, the effect of nucleon-nucleon correlations, as we shall show, is to make the sum of matrix elements $A^{(21)}_{N-Z}(\mathbf{Q}) + A^{(12)}_{N-Z}(\mathbf{Q})$ decrease in modulus, so that the DCX cross section undergoes much smaller changes in magnitude as $N-Z$ increases within a given nuclear shell.

It is clear from the structure of Eq. (3) that the dependence of $\mathcal{F}_{\mathrm{DCX}}(\mathbf{Q})$ on the nuclear wave functions enters the calculation through the two-particle form factor

$$S_{N-Z}(\mathbf{q}_1,\mathbf{q}_2) = \langle 1,\ldots,N-Z \,|\, e^{i\mathbf{q}_1\cdot\mathbf{r}_1 + i\mathbf{q}_2\cdot\mathbf{r}_2} \,|\, 1,\ldots,N-Z\rangle = \int e^{i\mathbf{q}_1\cdot\mathbf{r}_1 + i\mathbf{q}_2\cdot\mathbf{r}_2} \rho^{(2)}_{N-Z}(\mathbf{r}_1,\mathbf{r}_2) d^3r_1 d^3r_2, \tag{4}$$

where $\rho^{(2)}_{N-Z}(\mathbf{r}_1,\mathbf{r}_2)$ is the two-particle density of the valence neutrons in the initial ground state. The wave function also enters Eq. (3) through an analogous spin-dependent form factor that describes double-spin-flip processes. The role of these spin-flip transitions in the DCX process, although quantitatively significant, seems not to be as crucial as the effect of spatial correlation in explaining the general behavior of the observed cross sections. We shall therefore not present the spin-dependent terms explicitly in the present discussion and shall concentrate instead on the effects of spatial correlation, but we do this with the understanding that the results of our detailed calculations nonetheless include all effects of spin dependence.

If there were no correlations among the valence neutrons, the two-particle density $\rho^{(2)}_{N-Z}(\mathbf{r}_1,\mathbf{r}_2)$ would factorize into the product $\rho(r_1)\rho(r_2)$ of the two single-particle densities, and in that case, as we have shown in Ref. 1, the only intermediate nuclear state that would then contribute to the DCX process would be the isobaric analog state. We are assuming, on the contrary, that correlations are present and have therefore summed over all accessible intermediate states, both analog and nonanalog. To express the effect of correlation more explicitly we introduce the correlation function

$$C_{N-Z}(\mathbf{r}_1,\mathbf{r}_2) = \rho^{(2)}_{N-Z}(\mathbf{r}_1,\mathbf{r}_2) - \rho(r_1)\rho(r_2), \tag{5}$$

which permits us thus to write the form factor (4) as the sum of two terms,

$$S_{N-Z}(\mathbf{q}_1,\mathbf{q}_2) = S_0(q_1)S_0(q_2) + \int e^{i\mathbf{q}_1\cdot\mathbf{r}_1 + i\mathbf{q}_2\cdot\mathbf{r}_2} C_{N-Z}(\mathbf{r}_1,\mathbf{r}_2) d^3r_1 d^3r_2, \tag{6}$$

with $S_0(q) = \int e^{i\mathbf{q}\cdot\mathbf{r}}\rho(r)d^3r$. The first of these terms is then associated with transitions that take place through the analog state; the two single-particle form factors it contains tend to be strongly peaked in the forward direction. The second term, which expresses the effect of correlations, is associated with transitions that take place only through the nonanalog states, and vanishes for $\mathbf{q}_1 = 0$ or $\mathbf{q}_2 = 0$. These two terms then will lead, in general, to transition amplitudes having different angular dependences.

The primary sources of the correlations of positions and spins of the valence neutrons are the constraints implicit in nuclear shell structure. The angular momenta of the valence nucleons must sum to zero for an even-even nucleus, a constraint that leads to particularly strong spatial correlation for a single pair, $N-Z=2$. The attractive short-range nuclear force causes additional neutron pairs added to the valence shell to have angular momenta coupled to zero as well. We have applied these constraints, together with the requirement of antisymmetry, to the wave functions for the shell of $j = l + \frac{1}{2}$ orbitals. The single-paricle density $\rho(r)$ that results is just the squared-radial wave function for the shell and is spherically symmetric. The correlation function, on the other hand, for $N-Z$ even, is

$$C_{N-Z}(\mathbf{r},\mathbf{r}') = \kappa_{N-Z}\rho(r)\rho(r')\{(l+1)[P_l(\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}')]^2 + (l+1)^{-1}[(P_l^1(\hat{\mathbf{r}}\cdot\hat{\mathbf{r}}')^2 - 1\}, \tag{7}$$

where the constant κ_{N-Z} is given by

$$\kappa_{N-Z} = [2j+3-2(N-Z)]/[(2j-1)(N-Z-1)], \tag{8}$$

and P_l and P_l^1 are the normal and associated Legendre functions, respectively. We note that C_{N-Z} has the same shape for all values of $N-Z$ within the $j=l+\frac{1}{2}$ shell. The coefficient κ_{N-Z} evidently governs the magnitude and sign of the correlation effect. In particular, for a filled shell with $N-Z\to\infty$, it leads to $C_{N-Z}(\mathbf{r},\mathbf{r})\to -\frac{1}{2}\rho^2(r)$, which expresses the correlation characteristic of a Fermi gas.

The complete expression for the spin-independent form factor (4) that corresponds to the correlation function (7) is

$$S_{N-Z}(\mathbf{q}_1,\mathbf{q}_2)=S_0(q_1)S_0(q_2)+\kappa_{N-Z}\sum_{L\neq 0}a_L S_L(q_1)S_L(q_2)P_L(\hat{\mathbf{q}}_1\cdot\hat{\mathbf{q}}_2), \tag{9}$$

where $S_L(q)=\int dr\, r^2 j_L(qr)R_{nl}^2(r)$ is the form factor associated with the intermediate nuclear state with angular momentum L, j_L is a spherical Bessel function and R_{nl} the radial wave function[5] corresponding to principal quantum number n and orbital momentum l, and

$$a_L=(2j+1)(2l+1)^2\left[\begin{Bmatrix} l & l & L \\ j & j & \frac{1}{2}\end{Bmatrix}C^{L0}_{l0l0}\right]^2.$$

We have foregone presenting the lengthier expression for the spin-dependent form factor. We may note, however, that it contains no contribution from the uncorrelated part of the two-particle density. In effect, the double-spin-flip transitions take place only on correlated pairs.

In the course of the DCX process a neutral pion must propagate from one of a pair of participating neutrons to the other. Its propagator provides a weighting in the integrand of Eq. (1) that is naturally greatest when the two neutrons have small separation. It is that tendency of the process to favor configurations in which the valence neutrons are close together that gives particular emphasis to the effects of spatial correlations.[1]

Another measure of the importance of correlations can be gained by defining the separation density function

$$\rho_{\rm sep}(r)=\int \rho^{(2)}_{N-Z}\left(\mathbf{R}+\frac{\mathbf{r}}{2},\mathbf{R}-\frac{\mathbf{r}}{2}\right)d^3R$$
$$=\frac{1}{(2\pi)^3}\int e^{-i\mathbf{q}\cdot\mathbf{r}}S_{N-Z}(\mathbf{q},-\mathbf{q})d^3q. \tag{10}$$

It is this function, weighted with the spatial pion propagator and its derivatives, and integrated over $\mathbf{r}$, that governs the magnitude of the forward values ($\mathbf{Q}=0$) of the DCX cross section. We have used a set of harmonic-oscillator-based wave functions for the $f_{7/2}$ shell[5] to evaluate the separation densities for the valence neutrons of the calcium isotopes of even A. The results, which are shown in Fig. 1, illustrate a strong tendency for the two valence neutrons of ^{42}Ca to lie close together and a progressive weakening of the tendency as further neutron pairs are added to the same shell. The dotted curve in Fig. 1 represents the separation density with the correlation term omitted, and is the same for all four isotopes. The central values $\rho_{\rm sep}(0)$ exceed the uncorrelated value for ^{42}Ca and ^{44}Ca and are smaller than that value for ^{46}Ca and ^{48}Ca. Indeed, since $\rho_{\rm sep}(0)=(1+l\kappa_{N-Z})\times\int\rho^2(\mathbf{R})d^3R$, the central values of $\rho_{\rm sep}$ are in the ratios

$$\rho_{\rm sep}^{^{42}{\rm Ca}}(0):\rho_{\rm sep}^{^{44}{\rm Ca}}(0):\rho_{\rm sep}^{^{46}{\rm Ca}}(0):\rho_{\rm sep}^{^{48}{\rm Ca}}(0)=1:\tfrac{1}{3}:\tfrac{1}{5}:\tfrac{1}{7},$$

and the uncorrelated value is $\frac{1}{4}$, on the same scale.

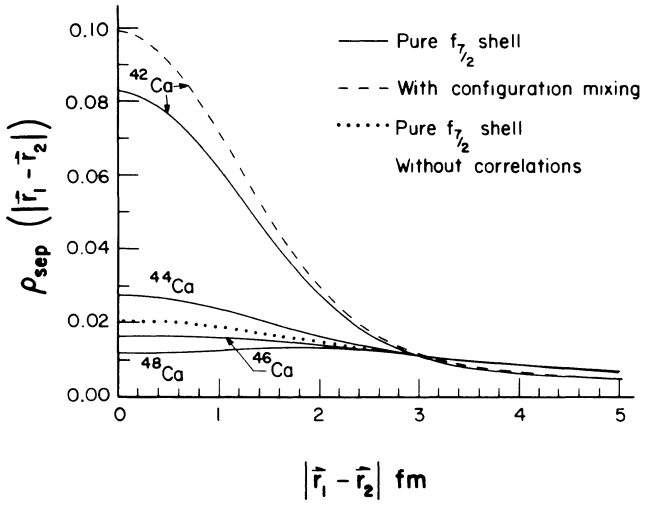

FIG. 1. The separation densities for ^{42}Ca, ^{44}Ca, ^{46}Ca, and ^{48}Ca (solid lines). The dotted curve is the separation density with the correlation function C_{N-Z} set equal to zero. The dashed curve corresponds to a configuration-mixed representation of ^{42}Ca noted in the text.

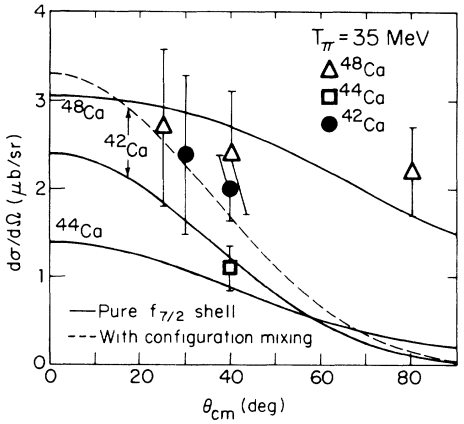

FIG. 2. Comparison between the theoretically predicted differential cross sections and the experimental data for the reactions $\pi^+ {}^A{\rm Ca}(0^+)\to\pi^- {}^A{\rm Ti}(0^+)$, for $A=42$, 44, and 48, at 35 MeV.

The fact that the DCX cross sections are proportional, roughly speaking, to the squares of the values of the separation density, shows how dramatically the correlations affect the cross sections. If we take these ratios as a guide, we find the DCX cross section for the single valence pair in ^{42}Ca to be some 49 times larger than that for any given pair in ^{48}Ca. The correlation effect in ^{42}Ca is thus so strong that its single pair yields a DCX cross section significantly larger than the 28 pairs present in ^{48}Ca.

In more accurate terms, the cross sections are determined by the shapes as well as the central values of the separation-density curves. The differential cross sections we have calculated using the momentum- and spin-dependent πN charge-exchange amplitudes[6] are shown for the Ca isotopes at 35 MeV in Fig. 2.

The DCX cross sections differ not only in their magnitudes, but in their angular dependences as well. The cross section for ^{42}Ca is strongly forward peaked and adding neutron pairs to the valence shell flattens it appreciably. The reason for this behavior is that the angular distribution contributed by any neutron pair (m,n) is, roughly speaking, the form factor for the distribution of its centroid[1] $\mathbf{R} = \frac{1}{2}(\mathbf{r}_m + \mathbf{r}_n)$. When the members of the pair are closely spaced, as in ^{42}Ca, the distribution of the centroid is similar to the distribution of the valence neutrons themselves. It extends to large radii and consequently has a sharply peaked form factor. For ^{48}Ca, on the other hand, the members of the valence pairs are generally much further apart. The distribution of their centroids tends therefore to be concentrated at smaller radii, and its form factor to be much less strongly peaked.

In the foregoing calculations we have used pure $f_{7/2}$ wave functions to describe the ground state of the Ca isotopes. Any configuration mixing present in these states may also affect the DCX cross sections, however, by changing both the correlated and uncorrelated parts of the two-particle density. We have carried out a sample calculation for a mixed representation of the ground state of ^{42}Ca [with 9% of $(p_{3/2})^2$ and 91% of $(f_{7/2})^2$ for the two neutrons[7]], and find, as is evident in Fig. 1, that it increases the separation density significantly, and leads to a corresponding increase of the differential cross section. We note that the difference between the differential cross section for ^{42}Ca calculated with and without the configuration mixing is large and comparable to the differences between the cross sections for Ca isotopes that follow from a model based on pure $f_{7/2}$ configurations. (A simplified analysis of DCX reactions on $f_{7/2}$ nuclei, based on spin-independent πN interactions has been reported, e.g., by Auerbach, Gibbs, and Piasetzky.[8])

The six recently measured[2-4] values of the DCX cross sections for the isotopes ^{42}Ca, ^{44}Ca, and ^{48}Ca are all in fair agreement with the shell-model predictions shown in Fig. 2 (including configuration mixing for ^{42}Ca). The fact that the forward DCX cross sections of the Ca isotopes are all of comparable magnitude is direct testimony to the importance of shell-model correlation effects; they are, as we have noted, strong enough to offset large combinatorial factors in Eq. (2). The measurements thus furnish clear evidence for the detection of correlation effects in the valence shell.

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