Spin Content of the Proton

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Data on polarized lepton-proton scattering are shown to give no clear conclusions on the spin content of the proton. We argue that errors have been underestimated, specifically those arising from the uncertainty of the extrapolation to infinite energy ($x \to 0$). Estimates of the spin content are sensitive to such uncertainty and possible ways of surmounting this problem are discussed.

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Precise measurements of the asymmetry $A^p(x)$ in polarized muon-proton scattering have recently been reported by the European Muon Collaboration.[1] These data are consistent with previous experiments at SLAC[2] but extend to smaller values of x where they are significantly lower than had been anticipated.[2] From these data, the polarized structure function $g_1^p(x)$ is computed with use of data on the unpolarized structure function,

$$xg_1^p(x)=A_1^p(x)xF_1^p(x)=A_1^p(x)\frac{F_2^p(x)}{2[1+R(x)]}, \tag{1}$$

each quantity also depending on Q^2. As a result of the unexpectedly low values of the asymmetry, it is further claimed[1] that the estimate of the integral of g_1, $I_p=\int_0^1 dx\, g_1^p(x)$, is much lower than that predicted by Ellis and Jaffe[3]—so much so that the resulting inferred fraction of proton spin carried by the quarks, κ, is consistent with zero. Such a conclusion, if true, will require a reexamination of our understanding of nucleon structure and has dire implications for some proposed searches for dark matter.[4]

With this in mind, we examine the validity of this estimate of the spin content and question the assumptions on which it depends. We find that while the surprisingly low value of $\kappa \approx 0$ is indeed possible, much larger values are equally likely since the procedure for the extrapolation to $x=0$ is far from unique. In addition we suggest a more efficient procedure for extracting information on the proton's spin constitution. The procedure of Ref. 1, in terms of F,D ratios, is not the most efficient and leads to inconsistencies in the analysis.

The aim is to extract information about the helicities of the partons within a polarized target and throughout we emphasize helicities rather than secondary quantities—such as the F,D values for the octet of β decays. This approach immediately shows where the small value of κ comes from and highlights the sensitivity of the polarized structure function g_1 to the quark polarizations.

We define

$$\Delta q_+(x)\equiv[q^{\uparrow}(x)+\bar{q}^{\uparrow}(x)]-[q^{\downarrow}(x)+\bar{q}^{\downarrow}(x)] \tag{2}$$

as the distribution of quarks or antiquarks with positive ($\uparrow$) or negative ($\downarrow$) helicity within an infinite-momentum proton of positive helicity. The quark model then gives

$$\int_0^1 dx\, g_1^p(x)=\tfrac{1}{2}\int_0^1 dx[\tfrac{4}{9}\Delta u_+(x)+\tfrac{1}{9}\Delta d_+(x)+\tfrac{1}{9}\Delta s_+(x)], \tag{3}$$

(for g_1^n, interchange $\Delta u \leftrightarrow \Delta d$). We now decompose Eq. (3) into the **3** and **8** components (nonsinglet) and singlet component of flavor,

$$I_{p,n}=\int_0^1 dx\, g_1^{p,n}(x)=\pm I_3+I_8+I_0, \tag{4}$$

with

$$I_3=\tfrac{1}{12}\int_0^1 dx[\Delta u_+(x)-\Delta d_+(x)], \tag{5}$$

$$I_8=\tfrac{1}{36}\int_0^1 dx[\Delta u_+(x)+\Delta d_+(x)-2\Delta s_+(x)], \tag{6}$$

$$I_0=\tfrac{1}{9}\int_0^1 dx[\Delta u_+(x)+\Delta d_+(x)+\Delta s_+(x)]. \tag{7}$$

The QCD correction factors are different for the nonsinglet and singlet components,[5]

$$\left[1-\frac{\alpha_s}{\pi}\right]I_{3,8} \text{ and } \left[1-\left[1-\frac{2N_f}{\beta_0}\right]\frac{\alpha_s}{\pi}\right]I_0, \tag{8}$$

where N_f=number of flavors. Thus the net helicity of quarks and antiquarks is $9I_0$ [Eq. (7)] and is given by the difference between the integrated structure function, I_p, and the sum of I_3 and I_8 (corrected by QCD). This nonsinglet combination can be related to known values of g_A/g_V for baryons and turns out to be large while the *difference* δ between I_p and I_3+I_8 turns out to be small. As a consequence, the inferred net quark helicity ($=9\delta$) is rather sensitive to small percentage errors in the estimate of I_p and/or in the values of I_3 and I_8. Therefore we critically examine some possible sources of uncertainty in these quantities and adopt a procedure which will efficiently minimize those errors.

I_3, I_8, g_A/g_V, and F,D values.—Integrals of the (differences of) distributions of polarized quarks and antiquarks are given by the measured (g_A/g_V) of the baryons.[6,7] Thus writing $2S_z[q]=\int dx\,\Delta q_+(x)$ we have the following constraints:

$$(g_A/g_V)_{np}=2S_z[3(u-d)] \equiv F+D=1.258\pm0.004 \quad (9a)$$

(Aguilar-Benitez *et al.*[8] and Green[9]),

$$(g_A/g_V)_{\Lambda p}=2S_z[(u-d)+(u-s)] \equiv F+\tfrac{1}{3}D=0.694\pm0.025 \quad (9b)$$

(Ref. 8 and Bourquin *et al.*[10]),

$$(g_A/g_V)_{\Xi\Lambda}=2S_z[(u-s)+(d-s)] \equiv F-\tfrac{1}{3}D=0.25\pm0.05 \quad (9c)$$

(Refs. 8 and 10), and

$$(g_A/g_V)_{\Sigma n}=2S_z[3(d-s)] \equiv F-D=|0.362\pm0.043| \quad (9d)$$

(Ref. 8 and Bourquin *et al.*[11]). The triplet contribution can be expressed, with use of Eq. (4), as $I_3=(g_A/g_V)_{np}$ and immediately gives the Bjorken sum rule[12]

$$\int_0^1 dx[g_1^p(x)-g_1^n(x)] = \tfrac{1}{6}\,|g_A/g_V|_{np}(1-\alpha_s/\pi). \quad (10)$$

The nonsinglet combination I_3+I_8 can be expressed in several combinations of the (g_A/g_V):

$$(1-\alpha_s/\pi)^{-1}(I_3+I_8)=\begin{cases}\frac{1}{6}(g_A/g_V)_{\Lambda p}=0.116\pm0.004, & (11a)\\ \frac{1}{18}[(g_A/g_V)_{\Sigma n}+2(g_A/g_V)_{np}]=0.119\pm0.002, & (11b)\\ \frac{1}{12}[(g_A/g_V)_{\Xi\Lambda}+(g_A/g_V)_{np}]=0.125\pm0.004. & (11c)\end{cases}$$

Expression (11b) has the least error as it most strongly emphasizes the precisely known $(g_A/g_V)_{np}$. Using the weighted average, with $\alpha_s=0.25\pm0.02$ we obtain

$$I_p=(0.110\pm0.002)\pm(0.108\pm0.002)\kappa, \quad (12)$$

where $\kappa=2S_z[u+d+s]$, the net fraction of spin of the proton carried by the quarks. The sensitivity of κ to the value of I_p is illustrated in Fig. 1.

It is instructive to see how this relates to the Ellis-Jaffe sum rule.[4] From Eq. (7) we write

$$I_0=4I_8+2S_z[\tfrac{1}{3}s], \quad (13)$$

and if one assumes that the strange sea is unpolarized, the last term vanishes and Eqs. (4), (9), and (13) give

$$I_p=I_3+5I_8\equiv\frac{1}{12}\left|\frac{g_A}{g_V}\right|_{np}\left[1+\frac{5}{3}\frac{3F-D}{F+D}\right]. \quad (14)$$

Note the redundancy here: Equation (14) uses $g_A/g_V=F+D$ but this constraint is not actually satisfied in the analyses of Sloan[1] and Jaffe.[13] This can be traced back to the analysis of Bourquin *et al.*[14] carried out at a time when there were two conflicting estimates for $(g_A/g_V)_{np}$. Reference 14 chose to "omit from the fit the neutron decay correlation (which yields) $g_A/g_V=1.258\pm0.009$ and which differs significantly from the result 1.239 ± 0.009 required by the neutron-lifetime measurements." The $F+D$ used in Refs. 1 and 13 fitted the latter value. Today we know that Ref. 14 made the wrong choice; the modern value for the neutron lifetime yields a value for g_A/g_V that is consistent with that from the angular-

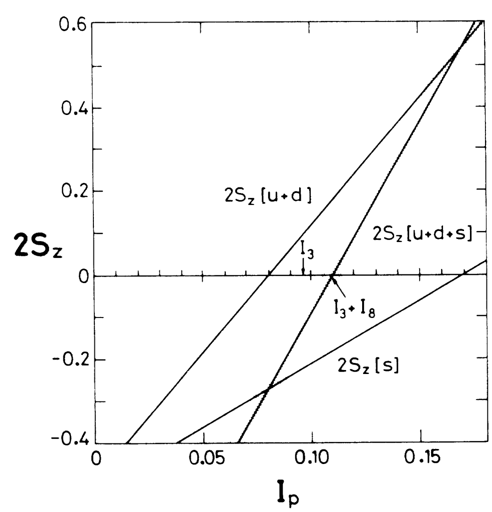

FIG. 1. Plot of $\kappa=2S_z[u+d+s]$ vs $I_p=\int_0^1 dx\,g_1^p(x)$ given by Eq. (12), illustrating the sensitivity to small changes in the value of I_p. The separate estimates for $2S_z[u+d]$ and $2S_z[s]$ come from our using the extra constraint $2S_z[u+d-2s]=I_8=0.015\pm0.002$ where I_8 is expressed in different combinations of the (g_A/g_V).

asymmetry result. The best value of g_A/g_V averaging modern results for τ_n and decay asymmetries gives[9] 1.258 ± 0.004 and this is what we have used in Eq. (12). Attention to such apparent detail is necessary because of the sensitivity of the resulting value of κ to such considerations.

Uncertainty in estimate of I_p.— A major source of uncertainty arises from the assumed behavior of $xg_1^p(x)$ as $x \to 0$. Even for unpolarized structure functions the small-x behavior is a matter of debate. In this region, $F_2(x)$ appearing in Eq. (1) almost certainly increases rapidly since it is intimately related to the behavior of the gluon distribution $xG(x)$, as $x \to 0$. This may go like $\exp[(\ln 1/x)^{1/2}]$ or perhaps, as emphasized by Collins,[15] as $x^{-\delta}$ with $\delta \approx \frac{1}{2}$. Consequently the simplest expectation from Regge behavior, $F_2(x) \sim$const, would be wrong and lead to an *underestimate* for the extrapolation of $F_2(x)$ as $x \to 0$. For $xg_1(x)$ we have even less faith in any extrapolation since there is disagreement over the Regge prediction. The relevant amplitudes are the t-channel helicity amplitudes $f^{+}_{1/2-1/2;10}$ for the virtual Compton scattering. The asymptotic expansion obtained from the Sommerfeld-Watson transformation is subject to constraints by the so-called conspiracy equations. Known Regge poles with intercept $0 \le \alpha(0) \le 1$ do not contribute to the leading behavior of $xg_1(x)$ and the decoupling of the Pomeron trajectory itself is a well-known consequence of theorems on the spin dependence of high-energy scattering amplitudes. The Pomeron-Pomeron cut does contribute, however (the negative parity piece), and represents the leading term in the asymptotic expansion. The next-to-leading terms are given by the A_1 trajectory and we have[16]

$$xg_1(x) = \beta_{PP} \nu^{\alpha_c(0)-1}/\ln^2(\nu/M) + \beta_{A_1} \nu^{\alpha_{A_1}(0)-1}, \quad (15)$$

so that as $x \to 0$, $xg_1(x) \sim 1/\ln^2 x$. The detailed analysis carried out in Ref. 16 seems to have been overlooked by later workers in the field; for example, Heimann[17] claims that the cut behavior is simply $\nu^{\alpha_c(0)-2}$, i.e., the cut behavior is essentially the same as the A_1 pole, $xg_1(x) \sim x$. Given this theoretical uncertainty in the small-x behavior we believe that this uncertainty must be reflected in the phenomenological estimates of the integral I_p. Therefore we take these two estimates for the $x \to 0$ extrapolation as a measure of the theoretical uncertainty.

Firstly we take $A_1^p(x)$ as measured in Ref. 1 but since the values of unpolarized structure functions $F_2(x)$ measured by the European Muon Collaboration fall below another muon deep-inelastic experiment, we choose to compute $xg_1(x)$, according to Eq. (1), using the new measurements of $F_2^p(x)$ from the Bologna-CERN-Dubna-Munich-Saclay collaboration[18] to reflect the uncertainty in the unpolarized measurements. In any case the difference between the European Muon Collaboration and Bologna-CERN-Dubna-Munich-Saclay measurements differ most in the region where $A^p(x)$ is smallest, thus deemphasizing the disagreement.

In Fig. 2 we show the resulting values of $xg_1(x)$ vs $\ln x$ computed at $Q^2 = 20$ GeV^2. To study the effect of varying the $x \to 0$ extrapolation, we simply draw two curves as follows: (i) a hand-drawn curve through the data for $x > 0.05$ together with an extrapolation $xg_1(x) = 0.38x$ for $x < 0.05$—giving $I_p = 0.1205 + 0.0190 = 0.1395$; (ii) a hand-drawn curve through the data for $x > 0.05$ together with an extrapolation $xg_1(x) = 0.135/\ln^2 x$—giving $I_p = 0.1175 + 0.0450 = 0.1625$.

Looking at Fig. 1 we see that this uncertainty implies that values of κ as large as 50% are as probable as the vanishingly small values claimed in Ref. 1.

Conclusions.— For a given value of I_p, we get a value for $\kappa = 2S_z[u+d+s]$ and also, using Eq. (13) with I_8 expressed in terms of the measured g_A/g_V, a value for $2S_z[s]$. Thus $\kappa \approx 0$ implies that the strange sea is polarized and opposite to the direction of proton polarization; see Fig. 1. Our discussion on the uncertainty of I_p suggests that while one cannot be definite as to whether the sea is polarized or not, some residual negative polarization for $s\bar{s}$ is likely. Close and Sivers[19] showed, within the framework of tree-level QCD, that the $q\bar{q}$ sea can be polarized, positively at large x, negatively at small x. However, the *integrated* polarization vanished because of the vanishing of the relevant anomalous dimension. In fact, perturbative QCD alone seems unable to derive a negative polarization. Jaffe[13] concludes that there must be a rapid Q^2 dependence at low Q^2 [his $\lambda(Q^2)$ is proportional to our κ, but $\lambda(Q^2) \ge 0$ is *assumed*] associated with nonperturbative effects.

Such effects must clearly be nontrivial since there must be a "matching" of the integrals of $g_1^{p,n}(x)$ at finite Q^2 to the $Q^2 = 0$ limit of the Drell-Hearn-Gerasimov sum rule.[20] While the Bjorken sum rule [Eq. (10)]

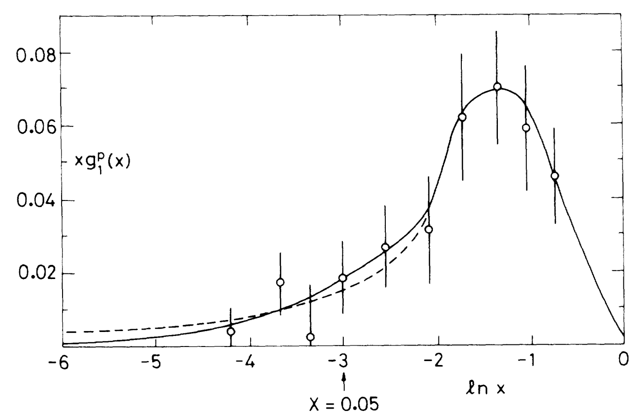

FIG. 2. Values of $xg_1^p(x, Q^2 = 20)$ computed, via Eq. (1), with $A^p(x)$ from Ref. 1 and $F_1^p(x, Q^2 = 20)$ from Ref. 18. The solid curve corresponds to a hand-drawn curve for $x > 0.05$ with an extrapolation $xg_1 = 0.38x$ for $x \le 0.05$. The dashed curve corresponds to a hand-drawn curve for $x > 0.05$ with an extrapolation $xg_1 = 0.135/\ln^2 x$ for $x \le 0.05$.

would appear to match "smoothly" to the Drell-Hearn-Gerasimov result for the difference $g_1^p - g_1^n$, there must be a change of sign for each $g_1^{p,n}$ individually.

To conclude, we have highlighted the difficulty of estimating $I_p = \int_0^1 dx\, g_1^p(x)$ to good accuracy—because of the large error associated with the $x \to 0$ extrapolation. In this context it would be helpful to measure the very small-x behavior of the *unpolarized* structure functions, $F_2^p(x)$ and $F_1^p(x)$—this may well be one of the most important results which will come from the DESY *ep* collider HERA. Also we have emphasized that estimation of I_0 (and hence κ) using a measurement of I_p in Eq. (4) is hampered by the dominance of the right-hand side by I_3. Clearly a future experiment on a polarized isoscalar target (though very difficult) would measure I_{p+n} and the right-hand side would then be dominated by I_0 allowing a precise estimate of κ.

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