Electron-Beam Radiation in a Strongly Turbulent Plasma

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A solution for the radiation power and frequency spectrum of a relativistic electron beam passing through a strongly turbulent plasma is presented. Regions of intense, localized electrostatic fields in the plasma are characterized as dipolar "solitons." A new regime of the beam-plasma interaction is identified with the range of beam parameters for which radiation from the beam dominates over the plasma radiation.

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Radiation generated by electron-beam discharges in plasmas appears to derive from conversion of nonlinear plasma-wave turbulence into electromagnetic waves, and generally occurs in narrow frequency bands at the plasma frequency and its second harmonic. This process occurs on a grand scale on the sun and other astrophysical objects,[1] and has been studied in the laboratory in plasma physics experiments,[2] and as a radiation source.[3]

The radiation produced by the acceleration of beam electrons is overlooked, mainly because it is regarded as incoherent, and is intrinsically small in low-density beams. Nonetheless, the radiation can provide useful information about the structure of the turbulence, and, in the case of dense relativistic beams, can become substantial relative to the plasma radiation. In this Letter, the beam radiation is calculated from a nonlinear turbulent state also assumed in models for the plasma radiation. The beam radiation is unique in being forward beamed (in the relativistic case) and having frequency components higher than the plasma frequency. Some coherence derives from the small-scale structure in the turbulence. Higher frequencies are produced when the beam is bunched.

Large amounts of beam radiation would seem possible when the electrostatic Langmuir wave produced in the beam-plasma instability acts as a "wiggler" field on the beam. However, stimulated emission by the beam has been found to have a small growth rate and inconsistent spectral features.[4] Also, any coherence in beam-generated waves is likely to be rapidly detuned by the tendency of large-amplitude electrostatic waves to clump (or collapse) into increasingly localized wave packets.[5] Such a plasma is said to be strongly turbulent.

The localized electrostatic structures in the strongly turbulent state are characterized as "solitons" or "cavitons." Caviton formation occurs through the expulsion of ions from local regions of high field into regions of less field by quasineutral coupling to electrons feeling the ponderomotive force. During this process, ions absorb the momentum of the beam-generated plasma waves, and, except for a residual velocity which is not greater than the ion-sound speed, the caviton is at rest in the plasma. As a result, the trapped-wave oscillation may be regarded as totally decoupled from the beam, and beam electron trajectories are only slightly perturbed in passage through a soliton. Radiation produced by the scattering of the beam electrons in the turbulence may be likened to a "bremsstrahlung" on the localized solitons.

Knowledge of the detailed structure and spectrum of strong turbulence is needed to model the radiation. Stationary but spherically symmetric three-dimensional solutions to the Zakharov nonlinear wave equations have been derived,[6] but their realization is questioned because their symmetry is inconsistent with the development of the density cavity by ponderomotive pressure.[7] Rather, strong turbulence appears to be a dynamic process wherein a nonlinear instability drives the cavitons to spatial sizes as small as five Debye lengths (λ_e), at which point dissipation processes intervene. (In the theory of Landau damping, this is also the smallest wavelength for which damping is negligible.)

A plausible model for a fully collapsed soliton which is not spherically symmetric is based on the following simple, localized, dipolar charge-density oscillation[8]:

$$\rho = \rho_0(\hat{\mathbf{p}}\cdot\mathbf{r}/a)\exp(-r^2/a^2)\exp(-i\omega_e t). \tag{1}$$

The distribution oscillates at the plasma frequency ω_e, has a dipole moment in the direction of the unit vector $\hat{\mathbf{p}}$, and is localized by a Gaussian envelope function with scale length a. When the light travel time across the caviton is small compared with the period of oscillation, the phase of the electric field over the acceleration region may be assumed to be the same. This condition is readily satisfied for all frequencies above the plasma frequency as long as $a \ll 2\pi c/\omega_e$. The envelope function is also assumed to be constant over the radiation time scale, which is justified if the supersonic collapse time is much greater than the light transit time. This condition is satisfied if

$$\frac{E_0^2}{8\pi n_0 k_B T_e} \ll \frac{1}{3(2\pi)^2}\frac{M}{m}, \tag{2}$$

where E_0 is the central electric field, n_0 and T_e are the

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plasma electron density and temperature, and M/m is the ratio of ion to electron mass. The Zakharov theory of strong turbulence is regarded as valid when $E_0^2/8\pi n_0 k_B T_e \ll 1$.

The electrostatic field associated with the dipolar charge distribution is calculated with Poisson's equation. In spherical polar coordinates with θ measured relative to the direction of the dipole moment, the electrostatic field is

$$E_r(r,\theta) = -\cos\theta(\pi\rho_0 a^4/r^3)\exp(-i\omega_e t)[(1+r^2/a^2)2(r/a)\exp(-r^2/a^2)-\sqrt{\pi}\,\mathrm{erf}(r/a)], \tag{3a}$$

$$E_\theta(r,\theta) = \sin\theta(\pi\rho_0 a^4/r^3)\exp(-i\omega_e t)[-(r/a)\exp(-r^2/a^2)+\tfrac{1}{2}\sqrt{\pi}\,\mathrm{erf}(r/a)], \tag{3b}$$

$$E_\phi(r,\theta) = 0. \tag{3c}$$

Near the center of the soliton, the electric field is simply

$$\mathbf{E}_0 = -\hat{\mathbf{p}}\frac{2\pi a^3}{3}\frac{\rho_0}{a^2}\exp(-i\omega_e t). \tag{4}$$

A spatial contour plot of constant electric field magnitude is presented in Fig. 1. The validity of this formulation is supported by comparison with numerical solutions of the Zakharov equations. Over a range of initial conditions, modulationally unstable Langmuir waves, in fact, appear to evolve into dipolelike collapsing solitons.[8,9]

The calculation of radiation from beam electrons passing through the soliton follows from the instantaneous acceleration of the electrons in the electrostatic field. The classical radiation for a moving charge is[10]

$$\mathcal{E}(\mathbf{x},t) = \frac{e}{c}\left[\frac{\hat{\mathbf{n}}}{(1-\hat{\mathbf{n}}\cdot\boldsymbol{\beta})^3 R}\times[(\hat{\mathbf{n}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]\right]_{\mathrm{ret}}, \tag{5}$$

where "ret" means evaluated at the retarded time and $\hat{\mathbf{n}}$ is a unit vector in the direction of the observation point, which is a distance R from the source.

The radiation field is a function of the angle χ between $\hat{\mathbf{n}}$ and $\boldsymbol{\beta}$, and the orientation of the soliton's dipole moment. In the following derivation, two coordinate systems are used. The coordinate system centered on the soliton is indicated by primes, and has $\hat{\mathbf{e}}'_z$ in the direction of the beam velocity. The plane x'-z' is coincident with the x-z plane of the observer's coordinates (unprimed), which has its $\hat{\mathbf{e}}_z$ in the direction of $\hat{\mathbf{n}}$.

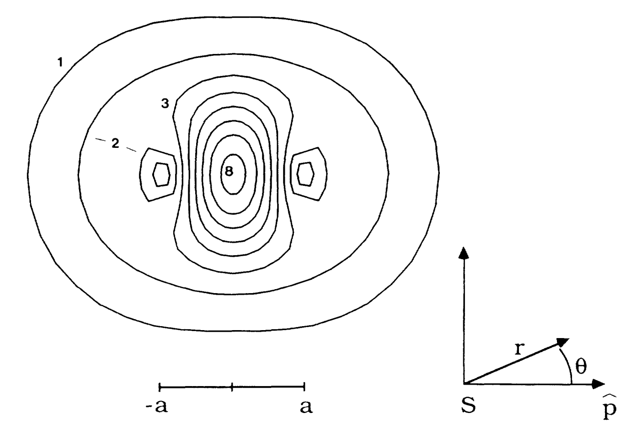

FIG. 1. Contour plot of electric field amplitude for soliton with scale length a and dipole moment in direction of $\hat{\mathbf{p}}$. In evenly spaced arbitrary units, 1 is the lowest and 8 the largest amplitude.

The total radiation field is a coherent sum over the radiating electrons in the soliton. The contribution from an individual electron is calculated from Eq. (5), with the substitution of the complete relativistic expression for the acceleration caused by the force of the soliton electric field [Eq. (3)], and with the use of the unperturbed velocity vector. The phase of the accelerating field at the retarded time in different parts of the soliton can be assumed to be the same. Multiplying the result by the number density in the beam provides the radiation per unit volume as a function of position within the source region. The total radiation field is found by integrating over source angle and radius, with the result

$$\mathcal{E}_x = \epsilon\frac{\hat{\mathbf{p}}\cdot\hat{\mathbf{e}}'_x(\beta-\cos\chi)/\gamma+\hat{\mathbf{p}}\cdot\hat{\mathbf{e}}'_z\sin\chi/\gamma^3}{(1-\beta\cos\chi)^3}, \tag{6a}$$

$$\mathcal{E}_y = \epsilon[\hat{\mathbf{p}}\cdot\hat{\mathbf{e}}'_y(\beta\cos\chi-1)/\gamma]/(1-\beta\cos\chi)^3, \tag{6b}$$

$$\epsilon = \tfrac{2}{3}\pi^{5/2}\rho_0 a^4 n_b e^2/mc^2 R\exp(-i\omega_e t). \tag{6c}$$

The special case of $\hat{\mathbf{p}} = \hat{\mathbf{e}}'_z$ is interesting because this is the natural orientation of fields produced in the plasma by the beam. The rms power radiated per unit area derived from the radiation field is

$$\frac{dP}{d\Omega} = cR^2\frac{\epsilon^2}{8\pi}\frac{\sin^2\chi}{\gamma^6}(1-\beta\cos\chi)^{-5}. \tag{7}$$

For large values of β, the angle for maximum emission is $\chi \sim 1/2\gamma$. It is not surprising that the radiation pattern has considerable relativistic beaming.[10]

Equation (7) is the power radiated coherently by the beam at the frequency of oscillation of the dipole. A single electron encounter actually produces a range of frequencies characteristic of collisional bremsstrahlung, where the role of the minimum impact parameter is played by the scale length a. This can be illustrated by the study, in detail, of the radiation from an electron passing through the center of the soliton, once again for the special case for which $\hat{\mathbf{p}} = \hat{\mathbf{e}}'_z$. The electric field is polarized along the observer's x direction. The energy radiated per unit solid angle per unit frequency interval is expressed in terms of the Fourier integral of this field component[10]:

$$\frac{dI(\omega)}{d\Omega}=\frac{e^2}{2\pi c^2}\left(\frac{e}{m\gamma^3 c}\right)^2 \sin^2\chi \left| \int_{-\infty}^{\infty} \frac{E_r(|\beta ct/(1-\beta\cos\chi)|,0)}{(1-\beta\cos\chi)^3} e^{i\omega t}\,dt \right|^2, \tag{8}$$

where the argument of the function E_r [given in Eq. 3(a)] is the electron's retarded radial position at the time t, assuming an unperturbed orbit (Born approximation). With the fact that most of the radiation is beamed at the angle $1/2\gamma$, the frequency spectrum for this orientation of the dipole moment can be written

$$\frac{dI(\omega)}{d\Omega}=\frac{E_0^2}{8\pi}\sigma_{\mathrm{T}} c\frac{a^2}{c^2}\left(\frac{27}{4\pi}\right)|A_1(\omega-\omega_e)|^2, \tag{9}$$

where

$$A_1(\omega)=\left(\frac{a\omega}{2\gamma^2 c}\right)^2 \int_0^{\infty}\frac{\mathrm{erf}(t)}{t}\cos\left(\frac{a\omega}{2\gamma^2 c}t\right)dt. \tag{10}$$

σ_{T} is the Thomson cross section. The function $A_1(\omega)$ is plotted in Fig. 2. The broad-band spectrum is observed to peak near the frequency $2\gamma^2 c/a$.

The frequency spectrum may be solved in a similar fashion for the case when the dipole moment of the electrostatic charge is oriented perpendicular to the velocity vector of the electron, i.e., $\hat{\mathbf{p}}=\hat{\mathbf{e}}_x'$, as might occur in fully developed turbulence:

$$\frac{dI(\omega)}{d\Omega}=\frac{E_0^2}{8\pi}\sigma_{\mathrm{T}} c\frac{a^2}{c^2}\left(\frac{27}{4\pi}\right)\gamma^2|A_2(\omega-\omega_e)+A_1(\omega-\omega_e)/2|^2. \tag{11}$$

The function $A_1(\omega)$ is same as in Eq. (10), and $A_2(\omega)$ is

$$A_2(\omega)=\exp[-(a\omega/2\gamma^2 c)^2/4]. \tag{12}$$

The spectrum extends to $2\gamma^2 c/a$, but now exhibits a low-frequency tail (see Fig. 2).

In the summation of such single-particle spectra over radiating beam electrons, phase averaging over electrons with different arrival times causes the radiation to be coherent only near the plasma frequency (namely, the dipole oscillation frequency). Thus, without an ancillary process, such as beam bunching, energy in the broad-band part of the spectrum will be less by many orders of magnitude, roughly $\gamma^2/\pi n_b a^3$, relative to the fundamental frequency. With beam bunching, the power spectrum will extend out in frequency according to the inverse

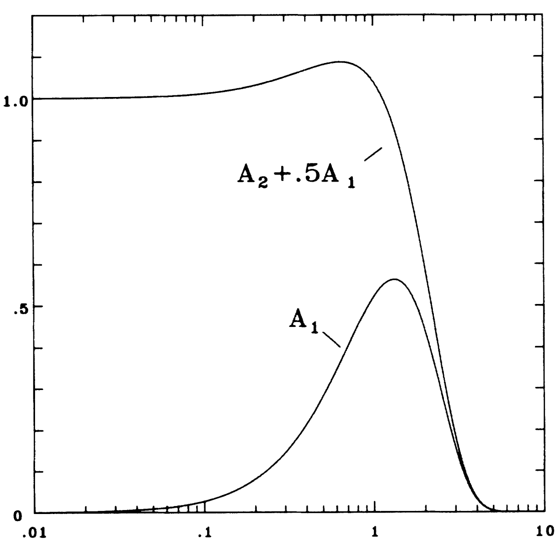

FIG. 2. Spectral functions $A_1(\omega)$ and $A_2(\omega)+A_1(\omega)/2$ corresponding to solitons with parallel and perpendicular orientation, respectively, relative to the beam velocity vector.

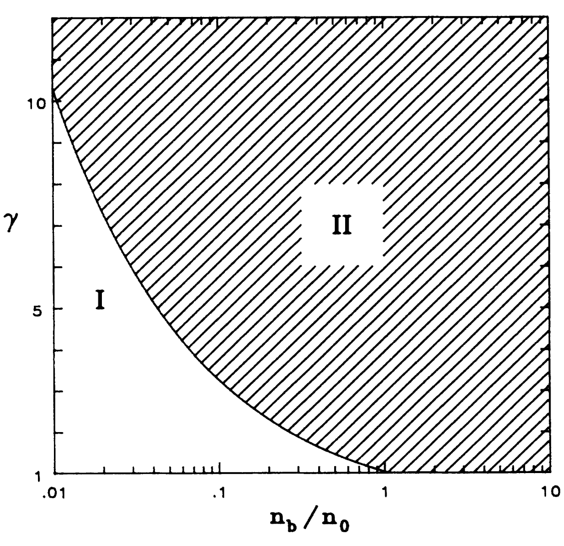

FIG. 3. Beam parameter space differentiated according to the dominant radiation component for a transverse dipole orientation: In region I, the maximum of $dP/d\Omega$ is larger for plasma emission than for beam emission; in II, the beam emission dominates.

transit time of the bunch across the soliton. Actual power spectra may be computed from knowledge or assumption of the soliton amplitude, scale length, number density, and statistics of the beam fluctuations.

Emission from the plasma will now be computed for comparison with Eq. (7). The explicit formulation of the high-frequency electron density in Eq. (1) makes the calculation of the plasma emission remarkably simple, since it follows directly from the classical formula for power radiated by an oscillating dipole. The dipole moment of the assumed charge distribution is $(\pi^{3/2}\rho_0 a^4/2)\hat{\mathbf{p}}$, so that the resulting time-averaged power per unit solid angle is

$$\frac{dP}{d\Omega}=\frac{27}{8}\pi^2\sin^2\theta\frac{E_0^2}{8\pi}\sigma_{\mathrm{T}}c(n_0a^3)^2. \tag{13}$$

Other soliton radiation models[11] derive their radiation integrals for the fundamental emission from an expansion in a purely longitudinal field, which makes the nonlinear current third order. This excludes the current producing the radiation given in Eq. (13), by implicit assumption that the soliton scale length is large relative to the electromagnetic wavelength, and scattering into the electromagnetic wave is incoherent. To argue that the opposite scaling adopted here is more appropriate for evolved solitons only a few Debye lengths in size, it suffices to use the fact that $c \gg (k_{\mathrm{B}}T_e/m)^{1/2}$.

Comparing the emission from the beam with conversion of the plasma oscillations, it is found that the maximum $dP/d\Omega$ is larger for beam emission of moderately relativistic beams ($\gamma=3$) when the beam number density is greater than 10% of the plasma density. This suggests a new regime of beam-plasma radiation where the beam radiation dominates (see Fig. 3). The beam emission is forward directed if the beam velocity is relativistic. An additional mechanism, such as beam bunching, can provide the coherence necessary for substantial emission above the plasma frequency.

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