Simulating SU(3) Gauge Theory with a Realistic Quark Spectrum: Strangeness Production in Heavy-Ion Collisions

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SU(3) gauge theory with a light isodoublet and a heavier strange quark is simulated on a 4×8^3 lattice. A first-order chiral-symmetry–restoring phase transition is found and metastable states are exhibited. The gluon, isodoublet, and strange-quark contributions to the energy density are in the ratio 8:4:1 and each internal energy jumps discontinuously at T_c.

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Stochastic differential equations for lattice gauge theory have yielded several algorithms which make first-principles simulations of SU(3) gauge theories with light dynamical fermions feasible on modern supercomputers. These methods have been used to study the thermodynamics of quantum chromodynamics and to show that the theories with two, three, or four light quarks have a first-order chiral-symmetry–restoring transition at $T_c \simeq 100$–200 MeV.[1-4] In this Letter we present data for a relatively realistic quark spectrum, an isodoublet of light quarks and a heavier strange quark, and we display metastable states at the transition.[5] We also measure the gluonic, isodoublet, and strange-quark contributions to the system's internal energy and find discontinuous jumps in each internal energy which are in the ratio 5:4:1. This suggests that the strangeness content in a quark-gluon plasma made in a heavy-ion collision should be considerable and could provide a signal for the creation of the plasma itself.

Before discussion of the results, consider some issues of lattice technology and algorithms. We have simulated small lattices, 4×8^3, where the temperature T is related to the lattice spacing a and the temporal extent $N_\tau=4$ by $aT=1/N_\tau$. Quark masses were chosen to be $am_{u,d}=0.0125$ for the isodoublet and $am_s=0.25$ for the strange quark. If $T_c\simeq155$ MeV, as favored by phenomenology[6] as well as crude lattice spectrum calculations,[7] then the bare mass of the isodoublet is 7.75 MeV and the bare mass of the strange quark is 155 MeV. These masses are near the quark-mass estimates based on current algebra.[8] Therefore, aside from serious questions of assessing finite-lattice-spacing and finite-volume effects, these simulations are quite realistic. There is no need to consider mass extrapolations here as done in most applications. We use staggered-lattice-fermion methods, as in most studies of this subject, in order to control mass renormalization in the interacting theory.

Our numerical results were obtained with a version of the hybrid algorithm which propagates the gluon fields in computer time t while accounting for the effects of quarks through additional stochastic terms in the gluon equation of motion.[9] The gluon equation of motion is discretized with basic time steps $dt=0.01$ and systematic errors of $O((dt)^2)$ have been observed in the algorithm's time averages. To accommodate an isodoublet of quarks a random complex field $\eta_{u,d}$ is placed on the sites of the lattice and an independent complex field η_s is also placed on the lattice to simulate the effects of a strange quark. Two terms, each bilinear in $\eta_{u,d}$ and η_s, then appear in the gluon equation of motion, and the strength of each term can be adjusted continuously so that an isodoublet and a strange quark are simulated. The precise algorithm for one random pseudofermion field η was introduced in Ref. 9. Those authors invented a finite-differencing scheme which only required a single inversion of the lattice Dirac operator for each sweep through the entire lattice and which had $O((dt)^2)$ systematic errors in long-time averages of observables. This algorithm is better controlled and more efficient than its predecessors which could also simulate an arbitrary number of quarks.[10] We have generalized the algorithm of Ref. 9 to a realistic quark spectrum by introducing two pseudofermion stochastic fields $\eta_{u,d}$ and η_s discussed above and by generalizing its time-differencing scheme to again keep systematic errors of time averages to $O((dt)^2)$. The details of this procedure will be presented elsewhere.

Now consider the results of our simulations. Our first task was to find the chiral-symmetry–restoring transition which separates the hadronic matter from the quark-gluon–plasma phase. Our previous simulations with a light isodoublet of quarks placed the critical coupling at $\beta_c=6/g_c^2\simeq5.27$–$5.29$, where metastable states were displayed.[11] We expected to find a first-order transition in the theory with a realistic quark spectrum at a β_c shifted to a smaller value as a result of the additional color screening caused by the strange quark. This effect was observed. In Fig. 1 we show the time evolution of the observables $\langle\bar{\psi}\psi\rangle$, the chiral-order parameter of the light isodoublet, the Wilson line (WL) and S_0, the average plaquette of Wilson's lattice gauge action, at $\beta=5.25$. The two curves correspond to a hot and cold start, and we see both configurations evolve into a hot, quark-plasma state where $\langle\bar{\psi}\psi\rangle$ is almost zero and WL is relatively large. The total number of sweeps in Fig. 1 is 10000, and the observables were averaged into bins 100 sweeps wide. The time evolution of hot and cold starts at $\beta=5.20$ are shown in Fig. 2 for 30000 sweeps and a clear two-state signal was found. This is our best evidence for the first-order character of the transition and metastable states. It is hard to judge the degree of metastability observed here except to note that for β chosen away from β_c, the time-correlation length in the algorithm (the number of sweeps required to generate a statistically independent configuration) is typically 100–300 sweeps, so that a 30000-sweep run would generate from 100 to 300 new gauge-field configurations. Finally, in Fig. 3 we show a metastability search at $\beta=5.15$ and find that the states evolve into a unique state of ordinary hadronic matter where $\langle\bar{\psi}\psi\rangle$ is large and WL is small. Only 10000 sweeps were needed in Fig. 3.

In summary, SU(3) gauge theory with a realistic quark spectrum has a first-order chiral transition on a 4×8^3 lattice at a coupling $\beta=5.20\pm0.05$.

To understand the character of this transition we also measured other observables such as the pressure and energy densities. The internal-energy densities were extracted from the simulation with standard methods extensively discussed and analyzed in Ref. 4 and by Engels *et al.*,[12] and Heller and Karsch.[13] Measurements of the pressure are quite delicate and will be deferred to another article. The internal-energy measurements are shown in Fig. 4 and in Table I which also includes the average values and statistical errors (including time correlations) for other observables at a wide range of β values. We see from the table that just below $\beta=5.20$ the Wilson line jumps discontinuously (from 0.127 to 0.489) as does $\langle\bar{\psi}\psi\rangle$ for the isodoublet (from 0.718 to 0.217). We note from the figure that the three internal energies also jump at $\beta=5.20$ as expected of a strong first-order transition. Unfortunately the extraction of physically interesting numbers from Fig. 4 requires some analysis and assumptions because of the presence of large finite-size effects on small lattices. First we note that the dimensionless energy densities ϵ/T^4 are substantial fractions of the Stefan-Boltzmann free-field internal energies on lattices

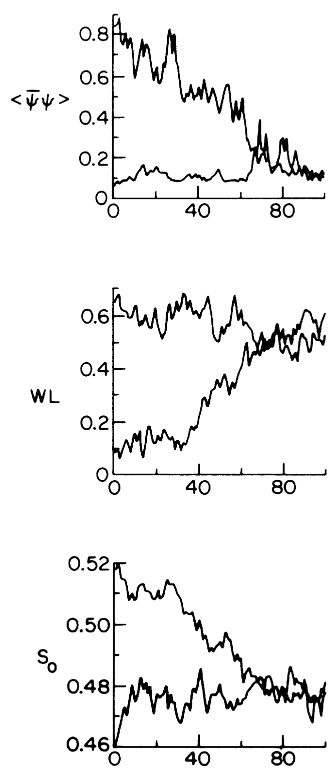

FIG. 1. Time evolution of $\langle\bar{\psi}\psi\rangle$, the Wilson line WL, and the average plaquette S_0 from hot and cold starts at $\beta=5.25$. 10000 sweeps.

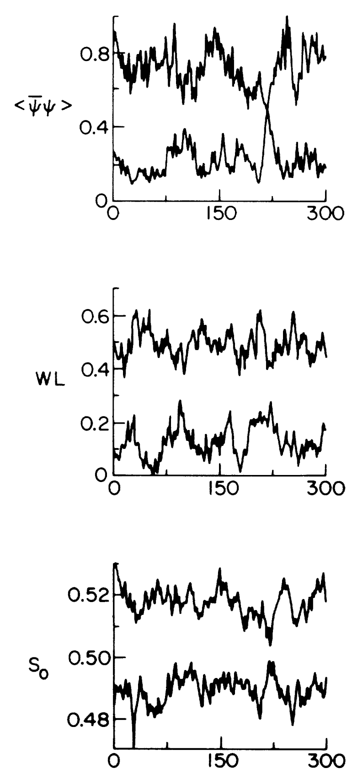

FIG. 2. Same as Fig. 1 except $\beta=5.20$, and 30000 sweeps.

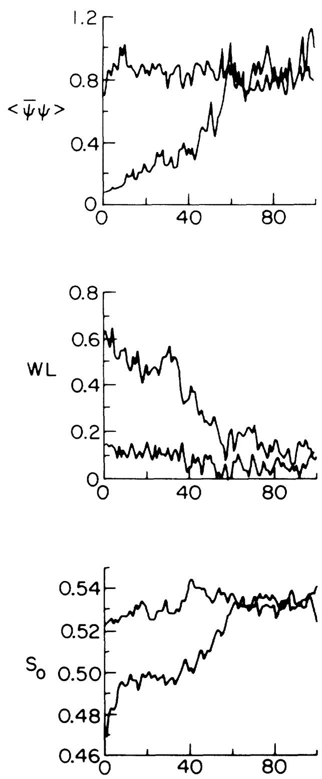

FIG. 3. Same as Fig. 1 except $\beta = 5.15$.

of this size.[14] For the gluon internal energy, the ratio of the internal energy shown in Fig. 4 at β slightly above 5.20 to the Stefan-Boltzmann value for a free lattice Bose gas of sixteen degrees of freedom (eight for color, times 2 for spin) is 2.20 ± 0.20. For the isodoublet the ratio is 0.85 ± 0.15, and for the heavier strange quark it is 0.48 ± 0.06. Presumably this ratio is smaller for the heavy quark because of its considerable mass, $m_s \simeq T_c$. Next we note that the free Bose gas on a 4×8^3 lattice has an internal energy which exceeds the space-time continuum Stefan-Boltzmann value ($\epsilon_b/T^4 = \pi^2/30$) by a

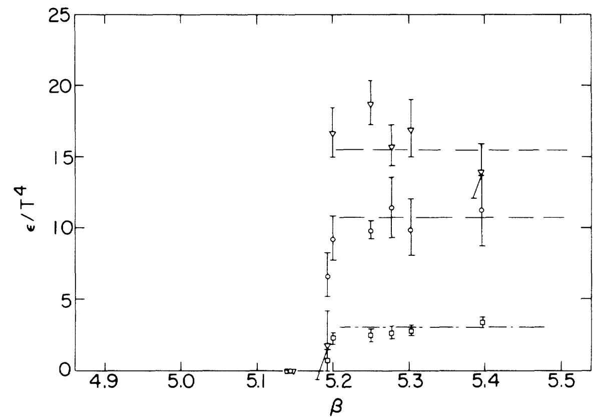

FIG. 4. Internal energies, ϵ/T^4, for gluons (inverted triangles), light quarks (circles), and heavy quark (squares).

factor of 1.40 as a result of finite-size effects, and a free-quark gas exceeds its Stefan-Boltzman value ($\epsilon_f/T^4 = 7\pi^2/60$) by a factor of 1.86 for $am = 0.0125$ and by a factor of 1.69 for $am = 0.25$.[14] If we correct for these finite-size effects by dividing our data by these numerical factors, we have

$$\Delta\epsilon_g = (12.0 \pm 2.00) T^4,$$
$$\Delta\epsilon_{u,d} = (6.30 \pm 1.00) T^4, \qquad (1)$$
$$\Delta\epsilon_s = (1.70 \pm 0.25) T^4.$$

This procedure for handling finite-size effects has proven quantitively successful in pure-gauge-field simulations where small-lattice results have been compared to large-lattice results where finite-size corrections are quite small.[12] However, the huge gluon-energy density observed near T_c (almost twice the Stefan-Boltzmann value) should be confirmed on large lattices before Eq. (1) is accepted.[13] Simulations of the four-flavor theory on a 6×10^3 lattice also gave a similarly large gluon-energy density near the transition.[2] It is interesting that the strange-quark energy density just above T_c is only

TABLE I. Average values of observables over long runs. S_0 is the average plaquette, WL is the Wilson line, $\bar\psi\psi_l$ is the chiral-order parameter $\langle\bar\psi\psi\rangle$ for the light isodoublet, $\bar\psi\psi_h$ is the same for the strange quark, ϵ_g/T^4 is the internal energy density for the gluons, $\epsilon_{u,d}/T^4$ is for the isodoublet, and ϵ_s/T^4 is for the strange quark.

β	S_0	WL	$\bar\psi\psi_l$	$\bar\psi\psi_h$	ϵ_g/T^4	$\epsilon_{u,d}/T^4$	ϵ_s/T^4
10.000	0.214 ± 0.001	1.846 ± 0.010	0.028 ± 0.001	0.525 ± 0.001	13.1 ± 2.6	11.5 ± 0.3	5.0 ± 0.1
6.000	0.387 ± 0.001	1.017 ± 0.015	0.045 ± 0.001	0.737 ± 0.001	13.6 ± 2.6	11.2 ± 2.1	3.7 ± 0.8
5.397	0.452 ± 0.002	0.665 ± 0.020	0.073 ± 0.003	0.845 ± 0.001	13.6 ± 0.26	11.3 ± 2.2	3.4 ± 0.9
5.301	0.468 ± 0.002	0.659 ± 0.020	0.112 ± 0.005	0.951 ± 0.005	16.9 ± 2.6	10.0 ± 2.1	2.9 ± 0.8
5.278	0.472 ± 0.003	0.602 ± 0.025	0.112 ± 0.008	0.995 ± 0.010	15.6 ± 2.8	11.4 ± 2.0	2.7 ± 1.0
5.248	0.478 ± 0.002	0.550 ± 0.015	0.131 ± 0.008	0.975 ± 0.008	18.7 ± 1.5	9.9 ± 1.3	2.5 ± 0.8
5.200	0.489 ± 0.001	0.489 ± 0.018	0.217 ± 0.025	1.016 ± 0.006	16.6 ± 3.1	9.2 ± 2.0	2.2 ± 0.3
5.193	0.517 ± 0.002	0.127 ± 0.022	0.718 ± 0.037	1.139 ± 0.010	1.8 ± 3.0	6.7 ± 2.0	0.8 ± 1.0
5.143	0.534 ± 0.002	0.064 ± 0.012	0.865 ± 0.025	1.209 ± 0.010	0.0 ± 1.3	0.0 ± 1.0	0.0 ± 0.2

half (roughly) the ideal-gas limit. This number is also subject to finite-size uncertainties. However, this kinematic suppression may be real because the $\beta=10.00$ data in Table I correspond to very high T and, at $\beta=10.00\epsilon_s/T^4$, is roughly twice its value at $\beta=5.20$. Ideally, one would like to hold the quark masses fixed in physical units while changing T. This will be possible on a large lattice where the renormalization-group trajectories relating bare and physical quantities have been well computed.

The major interest in Eq. (1) is that each internal energy is large and that they stand in the ratio 8:4:1 (approximately). Therefore, this lattice-gauge-theory simulation predicts considerable $s\bar{s}$ production in heavy-ion collisions. Models of the hadronization of the quark-gluon plasma suggest that a relatively large fraction of $s\bar{s}$ quarks should lead to considerable antihyperon production.[15] Model builders and experimentalists should find Eq. (1) provocative and useful in quark hadronization studies of the evolution of the system's final state.

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