Observation of the Meissner Effect in a Lattice Higgs Model

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The lattice-regularized U(1) Higgs model in an external electromagnetic field is studied by Monte Carlo techniques. In the Coulomb phase magnetic flux can flow through uniformly. The Higgs phase splits into a region where magnetic flux can penetrate only in the form of vortices and a region where the magnetic flux is completely expelled, the relativistic analog of the Meissner effect in superconductivity. We present evidence for symmetry restoration in strong external fields.

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One of the most striking manifestations of superconductivity, and now a phenomenon posing a critical test which all candidates for superconducting materials must pass, is the observed anomalous behavior of magnetic flux in the superconducting medium. Briefly summarized, two different situations can arise when a superconductor is subjected to an external magnetic field. In the first case, known as type I, the magnetic field decreases exponentially on the surface of the sample and disappears completely in the bulk. This is known as the Meissner effect. In the other case, type II, magnetic flux is totally expelled only up to a certain critical magnetic-field strength B_{c1}, beyond which the superconducting medium allows for a partial penetration of magnetic fields in the form of narrow vortices with quantized units of flux.[1] These different manifestations of superconductivity are all very well explained within the framework of the phenomenological Ginzburg-Landau theory.[2,3]

From what appears to be an entirely different direction, one considers in relativistic quantum field theory the action for the U(1) Higgs model in Euclidean space,

$$S=\int d^4x\left[\tfrac{1}{4}F_{\mu\nu}^2+(D_\mu\phi)^*(D_\mu\phi)+m^2|\phi|^2+\lambda|\phi|^4-\tfrac{1}{2}F_{\mu\nu}F_{\mu\nu}^{\rm ex}\right]. \tag{1}$$

Here $\lambda>0$, but m^2 is allowed to take negative values, in which case one has, at tree level, a "broken" or Higgs realization of the U(1) symmetry. In Eq. (1) we have introduced a coupling to an external electromagnetic field strength source $F_{\mu\nu}^{\rm ex}$: this source is kept fixed in the functional integral of the theory. Although this model is studied in elementary-particle physics [because it is one of the simplest models exhibiting the relativistic Higgs mechanism, and because of its similarity to the standard SU(2)⊗U(1) theory of electroweak interactions], it can also be viewed as the Lorentz-invariant generalization of the Ginzburg-Landau theory. Indeed, this model is known to possess topological excitations, Nielsen-Olesen vortices,[4] which are classical static solutions to the equations of motion, and which are the "relativistic" analogs of flux tubes in type-II superconductors.

In this Letter we shall present the results of a numerical study of the full quantum theory associated with the U(1) Higgs model. This allows us to test the importance of classical vortex solutions in the full dynamics of the theory, and gives us the first opportunity to assess in a completely nonperturbative manner earlier perturbative predictions[5,6] concerning the characteristics of this model in external electromagnetic fields.

In order to perform this numerical study, we latticize the action (1) in a standard way,[7] by making the following substitutions:

$$\phi_c\to a^{-1}\sqrt{\kappa}\phi_x,\quad (am_c)^2=(1-2\lambda-8\kappa)/\kappa,\quad \lambda_c=\lambda/\kappa^2,\quad A_\mu\to(e/a)\theta_\mu(x),\quad \beta=1/e^2, \tag{2}$$

where the subscript c denotes continuum quantities, a is the lattice spacing, and the notation otherwise is obvious. This corresponds to a lattice action of the form

$$S=\beta\sum_p(1-\cos\theta_p)-\kappa\sum_{x,\mu}(\phi_x^*e^{i\theta_\mu(x)}\phi_{x+\mu}+\text{H.c.})+\lambda\sum_x(|\phi_x|^2-1)^2+\sum_x|\phi_x|^2, \tag{3}$$

where the sums run over plaquettes and lattice points, as indicated. This lattice action will then be treated by standard Monte Carlo methods. The external electromagnetic field strength source has not been incorporated in Eq. (3). Its lattice version is not unique, and we shall here choose it by the replacement

$$\cos\theta_p\to\cos(\theta_p-\theta_p^{\rm ex}). \tag{4}$$

We work on finite lattices of size L^4 using periodic boundary conditions. We restrict ourselves to *constant* external electromagnetic fields $\theta_p^{ex}=\theta_{\mu\nu}^{ex}-\Delta_\mu\theta_\nu^{ex}-\Delta_\nu\theta_\mu^{ex}$, nonzero in, say, all 1-4 planes. This external field θ_p^{ex} is chosen such that there are lattice configurations $\theta_\mu(x)$ with $\theta_p=\theta_p^{ex}(\mathrm{mod}2\pi)$. One consequence of this is that $\theta_p^{ex}=(2\pi/L^2)n_{ex}$, corresponding to $2\pi n_{ex}$ total flux through the lattice. We can construct such configurations explicitly because of the compact nature of the group.[8] They contain Dirac sheets, the surfaces of plaquettes spanned by Dirac strings on the dual lattice,[9] one Dirac sheet for each 2π unit of flux. In a numerical simulation they can constitute very long-lived metastable states. This is so, because generally a monopole-antimonopole pair needs to be created, loop around the lattice, and then annihilate again, in order to "unwind" a Dirac sheet and hence change the flux by 2π.[10] Although a change to a configuration with different total flux can occur within the usual Monte Carlo updating, it is advantageous to propose from time to time global updates that change the total flux by one unit and then accepting or rejecting them with a Metropolis criterion.

To start, we show in Fig. 1(a) the behavior of $\langle\phi^* U_\mu\phi\rangle$

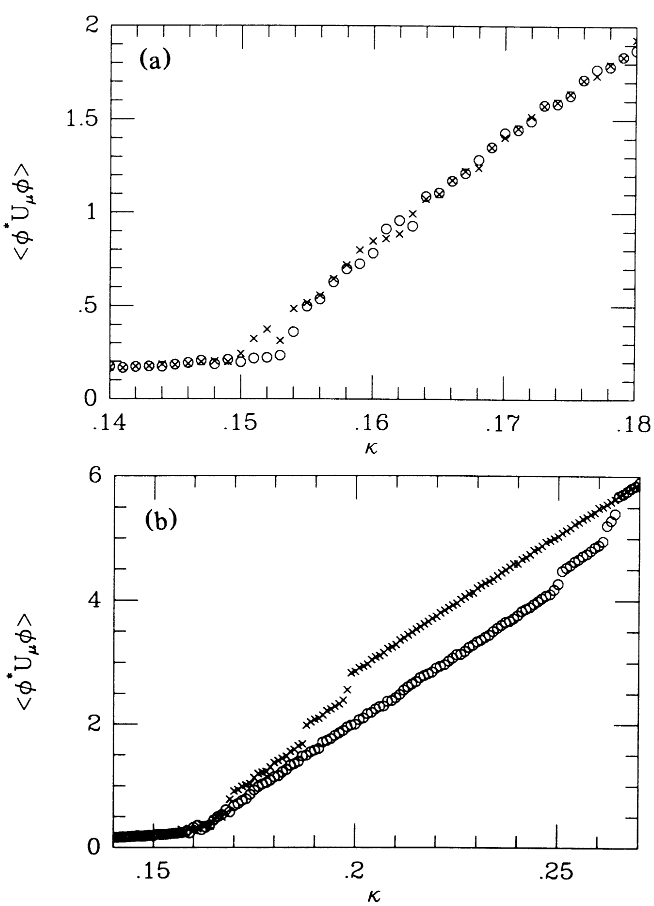

FIG. 1. Hysteresis runs on a 6^4 lattice at $\beta=2.50$ and $\lambda=0.10$. Plotted is the average link $\langle\phi^* U_\mu\phi\rangle$ for increasing κ (circles) and for decreasing κ (crosses). Each point is the average over 150 iterations after 50 iterations for "thermalization." (a) No external field, $n_{ex}=0$, and (b) $n_{ex}=3$.

in a "heating-cooling" exploration of the Coulomb-Higgs phases at fixed $\beta=2.50$, $\lambda=0.10$, and $n_{ex}=0$ (i.e., with no external field). A clear change of regime is seen to occur at $\kappa_c\simeq0.153$, in agreement with earlier localizations of the Coulomb-Higgs phase transition at these parameter values.[11] Next we turn on an external magnetic field by choosing, e.g., $n_{ex}=3$, and equilibrate well inside the Coulomb phase, alternating local Monte Carlo sweeps and global updates. After a short time an appropriate number of global changes get accepted, so as to make the total *measured* flux through 1-4 planes equal to 6π, the flux of the externally *applied* field. This conforms with our intuition of the electromagnetic properties of the Coulomb phase. At the classical level it corresponds to the solution

$$\theta_{\mu\nu}(x)=\theta_{\mu\nu}^{ex}(x),\quad |\phi_x|^2=0, \tag{5}$$

of the equations of motion associated with the action (3) and (4).

Performing now a heating-cooling run as for $n_{ex}=0$, we obtain, for $\langle\phi^* U_\mu\phi\rangle$, the behavior shown in Fig. 1(b). In Fig. 2 we plot the corresponding expectation value of plaquettes in the 1-4 planes. The three units of flux get annihilated successively as κ is increased and we go deeper into the Higgs phase. This causes small jumps in $\langle\phi^* U_\mu\phi\rangle$ and bigger ones in the 1-4 plaquettes. Beyond $\kappa\simeq0.265$ the total magnetic flux flowing through the lattice is *zero*. In this part of the phase diagram the applied magnetic field is thus totally expelled. *We identify the total expulsion of magnetic flux as the relativistic analog of the Meissner effect.* This phase corresponds at the classical level of this lattice model to the isotropic solution

$$\theta_{\mu\nu}(x)=0,\quad |\phi_x|^2=(8\kappa+2\lambda-1)/2\lambda. \tag{6}$$

Similarly, one would be tempted to take the stagewise disappearance of magnetic flux between $\kappa\simeq0.24$ and

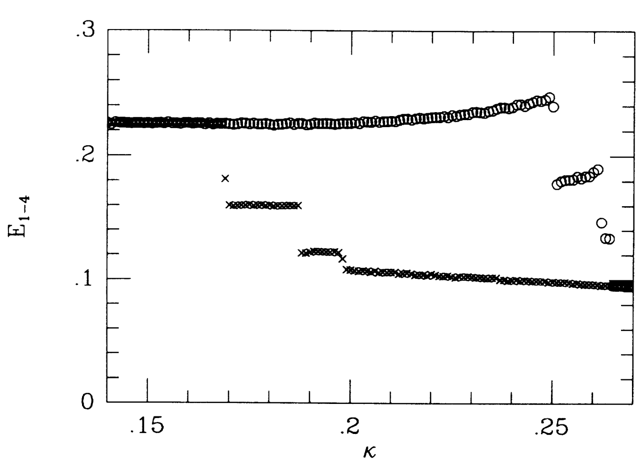

FIG. 2. The average plaquette $E_{1\text{-}4}=\langle1-\cos(\theta_{14})\rangle$ in the 1-4 planes, the planes with $\theta_p^{ex}\neq0$, for the same run as Fig. 1(b).

0.265 as evidence for a "mixed," type-II, phase of this Higgs model. Indeed, as we go backwards from the high-κ phase three units of flux reappear successively in roughly the same κ region, though showing a clear hysteresis, such that once we cross back into the Coulomb phase the total flux penetrating the lattice again equals 6π. A much more direct way of characterizing this type-II phase with localized topological excitations, vortices, will be presented below.

Note that the critical κ at which $\langle \phi^* U_\mu \phi \rangle$ changes from showing Coulomb to Higgs characteristics, κ_c, shifts slightly upward as an external electromagnetic field is applied. (In Fig. 1 we only present results for $n_{ex}=0$ and $n_{ex}=3$; a clear monotonic increase is found as n_{ex} is increased through the integers.[12]) There are thus parts of the Higgs phase which cease to show Higgs behavior as the external field strength increases, and instead switch to Coulomb behavior. From the superconducting analogy this is again as expected: It corresponds to the destruction of the superconducting state as the external magnetic field passes beyond a critical value B_{c2}.[13]

We can also study how, in the course of the simulation, the system relaxes to equilibrium. In Fig. 3 we show the "time evolution" obtained from starting with a configuration with two units of flux, taken from what was tentatively identified as the type-II region, as it equilibrates in the high-κ domain. The magnetic flux is seen to disappear in steps over rather few Monte Carlo iterations, and not surprisingly, the system equilibrates to configurations containing no net flux. It is interesting to note that these jumps in the flux occurred in the *local* Monte Carlo updatings, and not as a result of global changes being accepted. This brings further credence to the belief that what we have identified as the type-II phase indeed is populated with *localized* topological excitations in the form of vortices. In contrast, when equilibrating configurations with no flux in the Coulomb phase in an external field, we find a much higher probability of flux changing as the result of global updates being accepted. This indicates that in the Coulomb phase the magnetic flux is *uniformly* distributed.

To complete the above picture, we must make a clear identification of the role played by lattice vortices in this

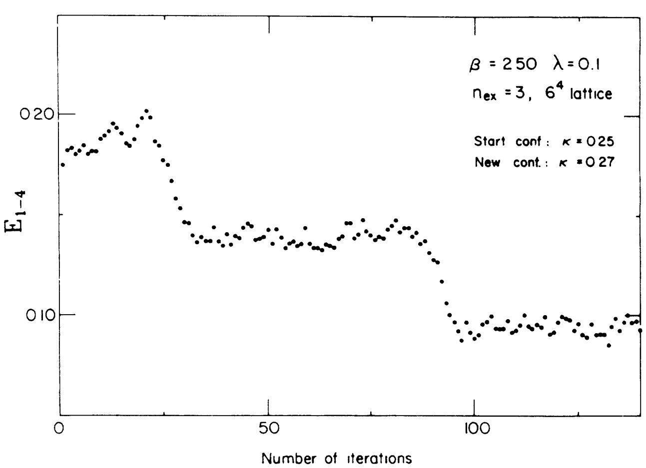

FIG. 3. A typical run history showing the relaxation of a configuration with two units of flux in the flux-expelling region. Same quantity as shown in Fig. 2. The lattice size is again 6^4, and the starting configuration ($\beta=2.50$, $\lambda=0.10$, and $\kappa=0.25$) is taken from the region we shall later establish to be of type II. The stepwise disappearance of magnetic flux occurs here in the local Monte Carlo updatings.

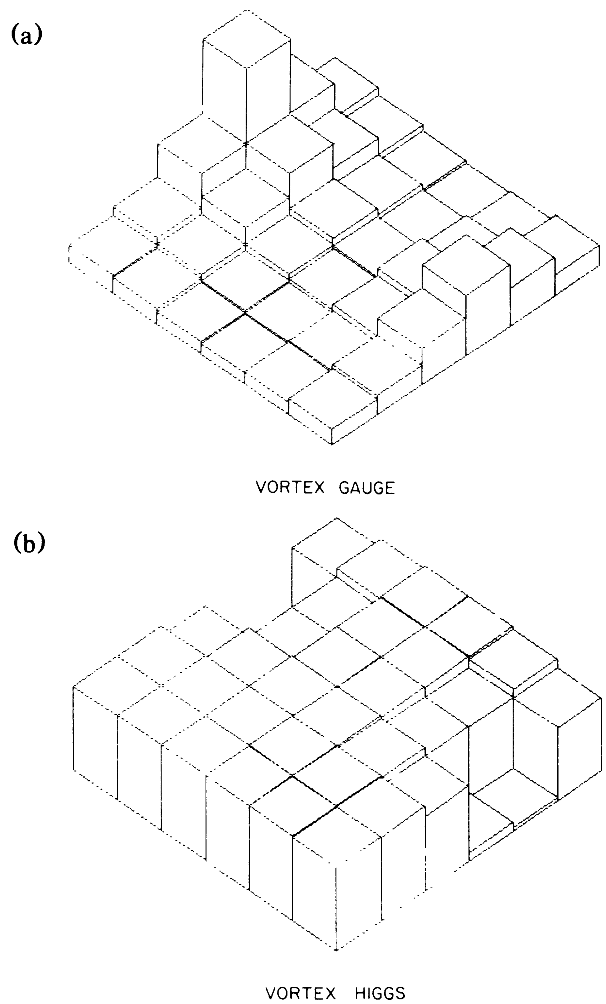

FIG. 4. An equilibrium configuration with one unit of flux in the type-II Higgs phase is "cooled" to its nearest saddle-point solution. The lattice is of size 6^4, and the parameters are $\beta=2.50$, $\lambda=0.10$, and $\kappa=0.20$. Even on this rather coarse-grained lattice, the field distributions show the clear characteristics of a Nielsen-Olesen vortex. (a) The local magnetic-flux density, and (b) the magnitude of the Higgs field $\rho=(\phi^*\phi)^{1/2}$ with its minimum value subtracted. Since ρ (arbitrarily) was chosen at one particular corner of each plaquette, the minimum of ρ is not found exactly on top of the magnetic-flux maximum.

model. By translational invariance we can concentrate on the two-dimensional subspace selected by the external magnetic field, i.e., in our case the 1-4 planes. In the continuum, Nielsen-Olesen vortices[4] have quantized magnetic flux that can be associated to the (integer) topological charge of two-dimensional U(1) gauge fields

$$\frac{e}{2\pi}\Phi = \frac{e}{4\pi}\int d^2x\, \epsilon_{\mu\nu}F_{\mu\nu}(x) = Q. \qquad (7)$$

In terms of our angular variables $\theta_\mu(x)$ this can be transcribed to the lattice as[14]

$$Q = \frac{1}{2\pi}\sum_{x,\mu,\nu}\bar{\theta}_{\mu\nu}(x). \qquad (8)$$

The sum runs over plaquettes in the 1-4 planes and the reduced plaquette variables $\bar{\theta}_{\mu\nu}(x)$ are forced to lie in the interval $(-\pi,\pi]$ through the identification

$$\bar{\theta}_{\mu\nu}(x) = \theta_{\mu\nu}(x) + 2\pi n_p, \qquad (9)$$

with $n_p \in \{-2,-1,0,1,2\}$. On a 4D lattice the "topological charge," Eq. (9), counts the number of units of magnetic flux through the plane considered.

If our interpretation of the intermediate-κ region as being a type-II phase is correct, we should expect one vortex configuration for each unit of magnetic flux passing through the lattice in that region. This can be tested directly by a relaxation procedure, a method of stripping quantum fluctuations off an equilibrium configuration by "cooling" it to the nearest classical minimum of the action.[12,15] The resulting gauge/Higgs-field distributions for a typical type-II candidate configuration with *one* unit of flux are shown in Fig. 4. A very clear example of a Nielsen-Olesen vortex can be seen. In contrast, if we cool a configuration with one unit of flux in the Coulomb phase of this model, we end up with completely *flat* field distributions. As expected, the flux distribution is there, apart from quantum fluctuations, uniform. Similarly, if we cool from what is clearly identified as part of the high-κ region in the Higgs phase, we find *zero* magnetic flux through all plaquettes and a constant $\phi^*\phi$ distribution—yet another way of seeing the Meissner effect.

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