Quark-Lepton Mass Ratio in Technicolor

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(Received 21 December 1987)

QCD corrections may substantially enhance a techniquark condensate relative to a technilepton condensate when the technicolor β function is small. This enhancement factor appears in quark-lepton mass ratios. There is much less enhancement of the techniquark-technilepton mass ratio.

PACS numbers: 12.15.Ff, 12.50.Lr

In a technicolor (TC) theory, quark and lepton masses are proportional vacuum expectation values of local technifermion bilinears calculated in the presence of an effective ultraviolet cutoff. This cutoff Λ_s is the mass scale of "sideways" interactions which couple quarks and leptons to technifermions. In this paper we shall assume that a quark mass is proportional to a techniquark condensate $\langle\bar{Q}Q\rangle_{\Lambda_s}$ and that a (charged-) lepton mass is proportional to a technilepton condensate $\langle\bar{L}L\rangle_{\Lambda_s}$ (in the following we shall omit the Λ_s subscript). Both techniquarks and technileptons carry TC, whereas only techniquarks carry color. We will explore the possibility that the ratio $\langle\bar{Q}Q\rangle/\langle\bar{L}L\rangle$ is an important factor in determining the quark-lepton mass ratio.

I shall argue that ordinary color interactions may suitably enhance $\langle\bar{Q}Q\rangle$ compared to $\langle\bar{L}L\rangle$ in the case that the TC running coupling is slowly varying as a function of momentum. A slowly varying TC coupling has been well studied in the context of enhancing $\langle\bar{Q}Q\rangle$ and $\langle\bar{L}L\rangle$ and thereby suppressing flavor changing neutral currents.[1-5] In such a theory we will find that the value of $\langle\bar{Q}Q\rangle$ is very sensitive to a QCD radiative correction, even though the value of the color coupling at and above the TC scale is small, $\alpha_{\rm QCD}\approx\frac{1}{10}$. This is, the fractional shift in $\langle\bar{Q}Q\rangle$ due to the correction is much greater than the naive estimate of order $\alpha_{\rm QCD}$.

The full ladder Schwinger-Dyson (SD) equation may be linearized by introduction of an infrared cutoff self-consistently.[6,7] This linearization is an appropriate approximation when the coupling is slowly varying, since then the integral is dominated by large momentum.[5-7] The result takes the form

$$\Sigma(k^2)=\frac{\lambda(k^2)}{k^2}\int_{\kappa^2}^{k^2}dp^2\Sigma(p^2)+\int_{k^2}^{\infty}dp^2\frac{\lambda(p^2)\Sigma(p^2)}{p^2},\quad \Sigma(\kappa^2)=\kappa. \tag{1}$$

These equations must determine the infrared cutoff κ as well as $\Sigma(k^2)$.

The self-energy of the technilepton, $\Sigma_L(k^2)$, is determined by (1) with $\lambda(k^2)$ replaced by

$$\lambda_L(k^2)=\tfrac{1}{4}\alpha(k^2)/\alpha_c,\quad\text{with } \alpha_c\equiv\pi/3C_2, \tag{2}$$

where $\alpha(k^2)$ is the TC running coupling and C_2 the Casimir for the technifermion representation of TC. $\Sigma_Q(k)$ is determined by (1) with

$$\lambda_Q(k^2)=\frac{1}{4}\left[\frac{\alpha(k^2)}{\alpha_c}+\frac{\alpha_{\rm QCD}(k^2)}{\alpha_{{\rm QCD}c}}\right], \tag{3}$$

where $\alpha_{\rm QCD}(k^2)$ is the color running coupling. We shall assume that the techniquarks are in the triplet representation of color, in which case $\alpha_{{\rm QCD}c}\equiv\pi/3C_{2{\rm QCD}}=\pi/4$. The technilepton and techniquark masses are respectively of order κ_L and κ_Q.

We write

$$\alpha(k^2)/\alpha_c=2A/\ln(k^2/\Lambda^2), \tag{4}$$

corresponding to asymptotically free TC with $k\partial_k\alpha(k^2)=-b\alpha^2(k^2)$ and $A\equiv1/b\alpha_c$. Since chiral-symmetry breaking occurs when $\alpha(k)/\alpha_c\approx1$ and since we may write

$$k\partial_k[\alpha(k^2)/\alpha_c]=-(1/A)[\alpha(k^2)/\alpha_c]^2,$$

large A is synonymous with a slowly varying coupling. And note[5] that (4) only holds $k>\kappa_{L,Q}$; large A may not be applicable well below the technifermion mass scale since technifermions no longer contribute to the TC β function. Thus the TC confinement scale may in fact be much closer to $\kappa_{L,Q}$ than the $\Lambda\ll\kappa_{L,Q}$ appearing in (4).

Since the color coupling $\alpha_{\rm QCD}(k^2)$ is small at the TC scale it will vary slowly over the momentum range of interest above the TC scale. And the presence of techniquarks will also tend to make the color β function smaller. With this in mind, we find it convenient to study the case of a constant color coupling, $\alpha_{\rm QCD}(k^2)=\alpha_{\rm QCD}$.

The reader may appreciate the following oversimplified explanation for the main result of this paper. Assume that the dynamical mass is generated when the effective coupling has reached a certain critical value, for example, when $4\lambda_L(\kappa_L^2)=4\lambda_Q(\kappa_Q^2)=1$. This naive[2,7] assumption along with (2), (3), and (4) gives $\kappa_Q/\kappa_L\approx\exp(4A\alpha_{\rm QCD}/\pi)$. Then as long as $A\alpha_{\rm QCD}\approx1$ we have κ_Q/κ_L deviating significantly from unity. This in turn yields a large enhancement for $\langle \bar{Q}Q\rangle/\langle \bar{L}L\rangle$, since on dimensional grounds one might expect this ratio to be κ_Q^3/κ_L^3. But this last statement is also naive because, as well shall see, the condensates have a nontrivial dependence on Λ_s as well. A more careful treatment is needed.

I briefly describe how explicit solutions to (1) are obtained when A is large; more details are given elsewhere.[7-9] We convert (1) to a differential equation by changing variables from k^2 to $x = \ln(k^2/\Lambda^2)$ and by defining $G_{L,Q}(x) \propto x^{1/2} e^{x/2} \Sigma_{L,Q}(x)/\kappa$. $x_{0L,Q} \equiv \ln(\kappa_{L,Q}^2/\Lambda^2)$ is the smallest x we need consider and for large A we will find that $x_{0L,Q} \gtrsim A$. This allows us to drop $1/x^2$ terms in the resulting equation. In the technilepton case,

$$G_L''(x) + \left(\frac{-1}{4} + \frac{A-1}{2x}\right) G_L(x) = 0. \tag{5}$$

In the techniquark case,

$$G_Q''(x) + \left(\frac{-d^2}{4} + \frac{A-1}{2x}\right) G_Q(x) = 0, \tag{6}$$

where $d^2 = 1 - 4\alpha_{\rm QCD}/\pi$.

The first equation[8] is Whittaker's equation if $1/x^2$ terms are again ignored in the general form of Whittaker's equation. The appropriate Whittaker function solution is determined by the requirement of the correct asymptotic behavior for dynamical symmetry breaking. When written in terms of the confluent hypergeometric function we have

$$G_L(x) \propto x^{1/2} e^{-x/2} U(1-A/2, 1, x). \tag{7}$$

In the techniquark case we may substitute $x = y/d$ into (6) to obtain

$$\hat{G}_Q''(y) + \left(\frac{-1}{4} + \frac{B-1}{2y}\right) \hat{G}_Q(y) = 0, \tag{8}$$

where $\hat{G}_Q(y) \equiv G_Q(y/d)$ and $B - 1 \equiv (A-1)/d$. Thus the solution $G_Q(x)$ to (6) takes the form

$$G_Q(x) \propto (xd)^{1/2} e^{-xd/2} U(1-B/2, 1, xd). \tag{9}$$

The differential equations are supplemented by boundary conditions

$$\Sigma_L(x_{0L}) = \kappa_L, \quad \Sigma_Q(x_{0Q}) = \kappa_Q,$$
$$\Sigma_L'(x_{0L}) = 0, \quad \Sigma_Q'(x_{0Q}) = 0. \tag{10}$$

The first two correspond to the boundary condition already present in (1) and serve to normalize the solutions. The last two follow from the integral equation itself and serve to determine $x_{0L,Q}$ as the largest values of x for which the functions $\Sigma_{L,Q}(x) \propto x^{-1/2} e^{x/2} G_{L,Q}(x)$ have local maxima, since we require $\Sigma_{L,Q}(x)$ to be monotonically decreasing above $x_{0L,Q}$. We may now write the self-energies for the technilepton and techniquark, respectively.

$$\Sigma_L(x) = \kappa_L e^{(x_{0L}-x)} U(1-A/2, 1, x)/U(1-A/2, 1, x_{0L}),$$
$$\Sigma_Q(x) = \kappa_Q e^{(1+d)(x_{0Q}-x)/2} U(1-B/2, 1, xd)/U(1-B/2, 1, x_{0Q}d). \tag{11}$$

Quark and lepton masses are proportional to $\langle \bar{Q}Q\rangle$ and $\langle \bar{L}L\rangle$, respectively. These condensates are given by divergent integrals effectively cut off as Λ_s, since the nonrenormalizable operators which couple technifermions to quarks and leptons are characterized by Λ_s. With $\langle \bar{Q}Q\rangle \equiv \frac{1}{3}\langle \Sigma_a \bar{Q}_a Q_a\rangle$ (summing over three colors) we have

$$\langle \bar{Q}Q\rangle \propto \int^{\Lambda_s} k^2 dk^2 \frac{\Sigma_Q(k^2)}{k^2 + \Sigma_Q(k^2)^2}, \tag{12}$$

with a similar expression for $\langle \bar{L}L\rangle$. After linearizing as before we obtain

$$\langle \bar{Q}Q\rangle/\langle \bar{L}L\rangle = \exp[\tfrac{3}{2}(x_{0Q} - x_{0L})] I_Q/I_L = \kappa_Q^3 I_Q/\kappa_L^3 I_L, \tag{13}$$

where

$$I_Q = \int_{x_{0Q}}^{x_s} dx \exp[(1-d)(x - x_{0Q})/2] U(1-B/2, 1, xd)/U(1-B/2, 1, x_{0Q}d),$$

and

$$I_L = \int_{x_{0L}}^{x_s} dx\, U(1-A/2, 1, x)/U(1-A/2, 1 x_{0L}),$$

with $x_s \equiv \ln(\Lambda_s^2/\Lambda^2)$.

We may avoid the integrals by making use of an expression[8] obtained by differentiating (1). This yields

$$\langle \bar{Q}Q\rangle \propto \Sigma_Q'(\Lambda_s^2)/[\lambda(\Lambda_s^2)/\Lambda_s^2]', \tag{14}$$

where the prime denotes differentiation with respect to Λ_s^2. Thus for large Λ_s we obtain [ignoring the $\lambda'(\Lambda_s^2)$ term since it is suppressed by x_s^{-1}]

$$\frac{\langle\bar{Q}Q\rangle}{\langle\bar{L}L\rangle}=\exp[\tfrac{3}{2}(x_{0Q}-x_{0L})]\frac{\exp[(1-d)(x_s-x_{0Q})/2]}{1+(1-d^2)x_s/2A}R, \tag{15}$$

$$R\equiv\frac{\{[(1+d)/2]U(1-B/2,1,x_sd)-d(B/2-1)U(2-B/2,2,x_sd)]\}U(1-A/2,1,x_{0L})}{[U(1-A/2,1,x_s)-(A/2-1)U(2-A/2,2,x_s)]U(1-B/2,1,x_{0Q}d)}.$$

We find that (15) agrees with (13) to within $\approx 5\%$ $A=5$ and $\approx 1\%$ at $A=15$. In evaluating these expre sions we fix x_s relative to x_{0Q}; that is we fix the ratio Λ_s/κ_Q. This is because it is κ_Q and not κ_L which is largely determined by the W and Z masses.

I display $\langle\bar{Q}Q\rangle/\langle\bar{L}L\rangle$ from (15) in Fig. 1 for two values of the color coupling $\alpha_{\rm QCD}$ ($\frac{1}{11}$ and $\frac{1}{9}$) and two values of Λ_s/κ_Q (10^3 and 10^4). I also give some typical values for κ_Q/κ_L in the form $(\kappa_Q/\kappa_L)_A$: $(1.45)_6$, $(1.79)_8$, $(2.24)_{10}$, $(2.82)_{12}$, and $(3.56)_{14}$ for $\alpha_{\rm QCD}=\frac{1}{11}$ and $(1.60)_6$, $(2.09)_8$, $(2.77)_{10}$, $(3.70)_{12}$, and $(4.96)_{14}$ for $\alpha_{\rm QCD}=\frac{1}{9}$. The results clearly display a large sensitivity to $\alpha_{\rm QCD}$.

We now discuss a few complications. The first is that the real color coupling will run (slowly), either decreasing or increasing above the TC scale depending on the sign of the color β function. If $\alpha_{\rm QCD}(k^2)$ happens to increase, then the enhancement of $\langle\bar{Q}Q\rangle$ will be even more pronounced. The same is true if techniquarks are in a different color representation with a larger Casimir $C_{2\rm QCD}$ (e.g., for sextet techniquarks, $C_{2\rm QCD}=10/3$); the smaller $d^2=1-3C_{2\rm QCD}\alpha_{\rm QCD}/\pi$ would produce a drastic enhancement of $\langle\bar{Q}Q\rangle$ for the same A. Interesting values of $\langle\bar{Q}Q\rangle/\langle\bar{L}L\rangle$ then occur for much smaller values of A, and the large A analysis of this paper would not be appropriate.

A second complication is that techniquarks do not fully contribute to the β function in the momentum range $\kappa_L<k<\kappa_Q$. If for this range we replace A by $\tilde{A}$ we have

$$k\partial_k[\alpha(k^2)/\alpha_c]=-(1/\tilde{A})[\alpha(k^2)/\alpha_c]^2;$$

this implies that $\alpha(k^2)/\alpha_c\to\infty$ at $k\approx\exp(-\tilde{A})\kappa_Q$. κ_L cannot be less than $\approx\exp(-\tilde{A})\kappa_Q$ and thus we expect smaller κ_Q/κ_L for smaller $\tilde{A}$. For the sake of obtaining analytical results we have neglected this effect and have set $\tilde{A}=A$. I may therefore have somewhat overestimated the quantity κ_Q/κ_L for a given A. On the other hand, we have found that phenomenologically interesting values of the quantity $\langle\bar{Q}Q\rangle/\langle\bar{L}L\rangle$ occur for κ_Q/κ_L as small as 2 or 3. It is clear that such values for κ_Q/κ_L may occur naturally even when a more realistic β function is used.

A third complication is that the sideways physics necessary for the generation of quark and lepton masses will alter the TC dynamics above the scale Λ_s. This suggests, for example, that an ultraviolet cutoff at Λ_s be imposed on the SD equation (1). But we have purposely constrained $A<15$ so that $\Lambda_s\Sigma(\Lambda_s)\ll\kappa\Sigma(\kappa)$. In this case $\Sigma(k)$ will not be significantly affected by the cutoff,[6,7] except possibly for $k\approx\Lambda_s$. On the other hand, $\langle\bar{Q}Q\rangle$ and $\langle\bar{L}L\rangle$ are given by divergent integrals involving $\Sigma(k)$, and thus it is not quite so clear that these quantities would be insensitive to the change in $\Sigma(k)$ due to a cutoff.

To check this we turn to a numerical treatment. Here we obtain $\Sigma(k)$ through an iterative procedure[2,5] applied directly to the nonlinear ladder SD equation in the presence of an ultraviolet cutoff (the nonlinear SD equation does not require an infrared cutoff). The numerical results for $\langle\bar{Q}Q\rangle/\langle\bar{L}L\rangle$ are displayed in Fig. 2. And numerical values for $[\Sigma_Q(0)/\Sigma_L(0)]_A$ are typically within $\approx 5\%$ of the above values for $(\kappa_Q/\kappa_L)/_A$, except for large values of A when some sensitivity to the cutoff develops.

We thus find qualitative agreement between the numerical and analytical results; the differences indicate the degree to which the initial linearization of the SD equation is to be trusted. Although further results will

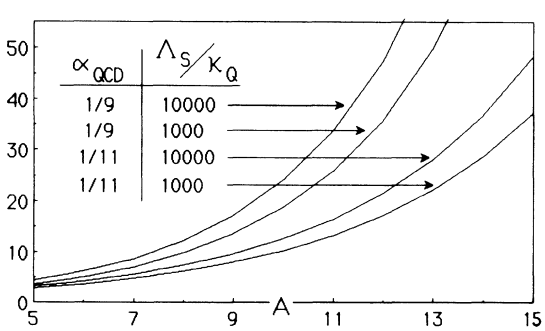

FIG. 1. $\langle\bar{Q}Q\rangle/\langle\bar{L}L\rangle$ (analytical).

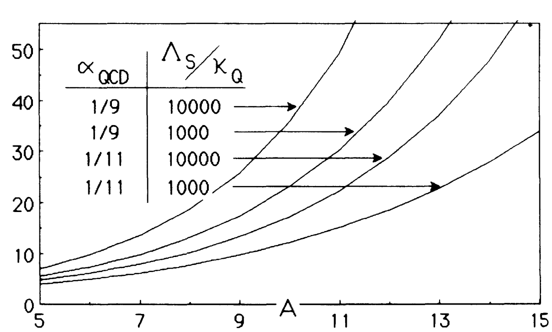

FIG. 2. $\langle\bar{Q}Q\rangle/\langle\bar{L}L\rangle$ (numerical).

not be presented here, the numerical method also allows one to check the effect of a changing TC β function below κ_Q and the running of the QCD coupling $\alpha_{\rm QCD}(k^2)$.

I conclude by mentioning another possibility for producing mass splittings among technifermion condensates. There may be corrections due to the exchange of gauge bosons much more massive than the TC scale. Such gauge bosons could distinguish between and cause splittings among different techniquarks for example. This in turn is of interest for causing a mass hierarchy between different quarks which couple to different techniquarks. This effect may be analyzed numerically, as was done in Ref. 2. But it is not difficult to argue that if the TC running coupling does not significantly decrease until a scale $\tilde{\Lambda} \gg \kappa$, then any new interaction with a mass scale less than or of order $\tilde{\Lambda}$ may have a large effect on the technifermion condensates.

Summarizing this paper, we have been concerned with corrections to technifermion condensates from a known source, in particular QCD interactions above the weak scale, and have indicated when these apparently weak corrections may be important in the determination of quark-lepton mass ratios.

This research was supported in part by the Natural Sciences and Engineering Research Council of Canada.

[1]B. Holdom, Phys. Rev. D **24**, 1441 (1981).

[2]B. Holdom, Phys. Lett. **150B**, 301 (1985).

[3]T. Akiba and T. Yanagika, Phys. Lett. **169B**, 432 (1986).

[4]K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986), and Phys. Lett. B **178**, 308 (1986).

[5]T. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986); T. Appelquist and L. C. R. Wijewardhana, Phys. Rev. D **35**, 774 (1987), and **36**, 568 (1987); T. Appelquist, D. Carrier, L. C. R. Wijewardhana, and W. Zheng, Phys. Rev. Lett. **60**, 1114 (1988).

[6]B. Holdom, University of Toronto Report No. UTPT-87-12, 1987 (to be published).

[7]B. Holdom, Phys. Lett. B **198**, 535 (1987).

[8]M. Bando, T. Morozumi, H. So, and K. Yamawaki, Phys. Rev. Lett. **59**, 389 (1987).

[9]B. Holdom and J. Terning, Phys. Lett. B **200**, 338 (1988).