Observation of a Topological Phase by Means of a Nonplanar Mach-Zehnder Interferometer

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We report the direct observation of a topological phase, i.e., an Aharonov-Bohm-type phase, as a fringe shift in an optical interferometer, which consisted of a modified Mach-Zehnder interferometer, in which the light traveled along nonplanar paths in its two arms. These arms were arranged symmetrically so as to have nearly equal path lengths, but opposite senses of handedness. The relationship between the phase acquired by a circularly polarized light beam and the solid angle subtended by the circuit of the spin vector of a photon in this beam was found to be a linear one with a slope of unity. The sign of the fringe shift also agreed with theory.

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Berry's phase,[1,2] and its generalizations,[3-5] have been the subject of much recent interest. Berry discovered that in the quantum adiabatic theorem, there exists an extra phase, in addition to the usual dynamical phase, which is picked up by a system after it has gone through a closed circuit in the parameter space of the Hamiltonian. The essential assumptions are that the evolution of the *Hamiltonian* be *cyclic* and *adiabatic*. Because of its generality, it can manifest itself in all fields of physics. In particular, in optics Chiao and Wu[6] showed that a spinning photon whose direction is gently turned through a closed circuit C in momentum space acquires a phase γ given by

$$\gamma(C) = -\sigma\Omega(C), \qquad (1)$$

where σ is the helicity, and $\Omega(C)$ is the solid angle subtended by C at the origin of momentum space, of the photon. They predicted that this phase would manifest itself as a topological optical activity, which was subsequently observed by Tomita and Chiao in a helical optical fiber.[7] The phase γ is formally identical to the Aharonov-Bohm phase which a unit electric charge picks up after it has traveled around a closed circuit in the presence of a Dirac monopole of topological charge $-\sigma$. Since the Aharonov-Bohm phase is topological, we shall also call γ a topological phase.

Chiao and Wu[6] also suggested an interference experiment in which two helical fibers of opposite handedness served as the two arms of an interferometer. In this Letter, we report on an experiment in which we replaced, for practical reasons, these two fibers with two nonplanar mirror configurations, which are similar to the one proposed by Kitano, Yabuzaki, and Ogawa.[8] These two mirror configurations have opposite handedness. The topological activity of a handed mirror configuration, and its analog in NMR, have recently been observed.[9]

However, the adiabatic assumption used by Berry[1] and by Chiao and Wu[6] is violated by the sudden reversals of helicity when a spinning photon bounces off a mirror. Berry[10] viewed this as "antiadiabatic" evolution. Aharonov and Anandan[5] generalized Berry's phase to nonadiabatic situations. Their assumption is that the evolution of the *state* of the system be *cyclic,* i.e., that it returns to its starting point, adiabatically or not. We shall interpret our experiment in terms of the Aharonov-Anandan phase, and replace parameter space, here momentum space, by projective Hilbert space,[5] which in the present case is the sphere of *spin* directions of the photon. Hence C in Eq. (1) is to be reinterpreted as a closed circuit on the sphere of spin directions, and σ is the sign, or handedness, of the initial helicity. Otherwise, the calculation of the solid angle Ω in momentum space yields incorrect results.

A schematic of the apparatus is shown in Fig. 1. Let the x axis coincide with the initial direction of the laser beam, the y axis point towards the mirror M6, and the z axis point out of the plane of the paper. The unpolarized output of a He-Ne laser (Spectra Physics model 105-2) is incident upon a polarization-preserving beam splitter B1 (Ealing model 24-3949), which consists of a half-silvered mirror embedded along a diagonal of a glass cube. (We used an unpolarized laser and cube beam splitters in order to avoid systematic errors arising from s-p phase shifts.) This beam of light is composed of photons.[11] Consider what happens to a positive-helicity photon in this beam, i.e., one in a helicity state $|+\rangle$, so that $\langle \mathbf{s}_0 \cdot \mathbf{k}_0\rangle = +1$, where $\langle \mathbf{s}_0\rangle = \mathbf{k}_0 = (1,0,0)$ is its initial spin and propagation direction. Upon reflection from an infinitely conducting mirror or beam splitter, its helicity flips sign. Its helicity does not flip sign upon transmission through a beam splitter. The probability amplitude for its transmission is nearly equal to that for its

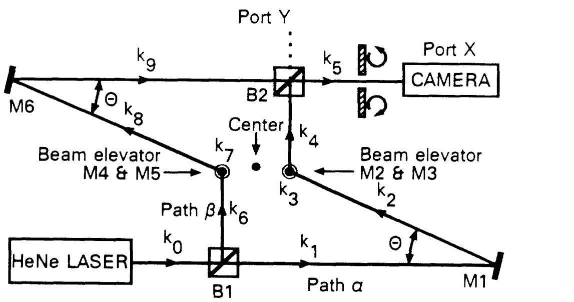

FIG. 1. Top view of nonplanar Mach-Zehnder interferometer. The beams in the upper half of the diagram are at a greater height than in the lower half of the diagram by 41 mm. B2 and M3 are 95 mm apart.

reflection at B1. Upon transmission through B1, it then travels along what we shall call path α. Its direction $\mathbf{k}_1=(1,0,0)$ immediately after transmission, and its spin $\langle\mathbf{s}_1\rangle=\mathbf{k}_1$, are unchanged. After reflection from mirror M1 (all mirrors are aluminized front-surface mirrors), it travels along direction $\mathbf{k}_2=(-\cos\theta,\sin\theta,0)$, and its spin is $\langle\mathbf{s}_2\rangle=-\mathbf{k}_2$. Then it ascends by means of a "beam elevator" (Newport Research model 670) along direction $\mathbf{k}_3=(0,0,1)$, and its spin is $\langle\mathbf{s}_3\rangle=\mathbf{k}_3$. The beam elevator consists of a pair of rotatable 45° mirrors M2 and M3 in a vertical configuration, in order to deflect the beam upwards to a greater height. At the output of the beam elevator, the photon emerges along direction $\mathbf{k}_4=(0,1,0)$, and its spin is $\langle\mathbf{s}_4\rangle=-\mathbf{k}_4$. It then strikes the second beam splitter B2, which is identical to B1, whereupon it is either transmitted through to port Y, where it is lost, or reflected to port X, where it is detected. Upon reflection at B2, it travels along $\mathbf{k}_5=(1,0,0)$, and its spin is $\langle\mathbf{s}_5\rangle=\mathbf{k}_5$. Note that $\mathbf{k}_5=\mathbf{k}_0$ and $\langle\mathbf{s}_5\rangle=\langle\mathbf{s}_0\rangle$. Thus its evolution is cyclic.

We summarize the history of the photon along path α in Fig. 2, where we construct a unit sphere to represent all possible directions of the spin of the photon. Since its evolution is cyclic, the tip of the spin vector of the photon traces out on this sphere a *closed* curve. Point A corresponds to the spin direction $\langle\mathbf{s}_1\rangle=(1,0,0)$, etc. Let us connect points on the sphere by geodesics, i.e., arcs of great circles. The action of mirror M1 corresponds to the geodesic AB, etc. This construction needs further theoretical justification, but suffice it to say here that it yields results which agree with experiment. The closed curve consists of a spherical triangle BCD, with a doubled-back path AB which encloses no area. By the Gauss-Bonnet theorem, i.e., that the sum of the interior angles of a spherical triangle is 180° plus the solid angle it subtends at the center of the sphere, one can show that the solid angle enclosed by the closed curve $ABCDA$ is

$$\Omega=\pi/2-\theta. \tag{2}$$

The solid angle Ω was varied by adjustment of mirrors M1 and M6, etc., so as to vary θ.

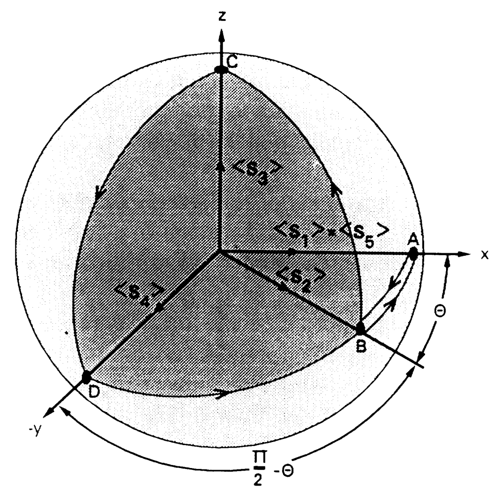

FIG. 2. Sphere of spin directions of the photon. (This is *not* the Poincaré sphere.) The heavily shaded area is the solid angle Ω.

Next, returning to Fig. 1, let us follow the photon along its other path β, which it traverses upon reflection from beam splitter B1. After this reflection, it moves along $\mathbf{k}_6=(0,1,0)$, and its spin is $\langle\mathbf{s}_6\rangle=-\mathbf{k}_6$, etc. Finally, after beam splitter B2, it recombines with its *alter ego* from path α, either upon its transmission at beam splitter B2, emerging in the same direction and spin to be detected at port X, or upon its reflection at B2, to be lost out port Y. (Recall that in quantum mechanics, a *single* photon interferes with itself.[11])

Figure 2 again summarizes the history of the photon along path β, except that now the spherical triangle is traversed in the opposite sense. The phase shift seen in interference is determined by the difference between the histories of paths α and β, i.e., $ABCDA''$ minus $AD'C'B'A''$, where the unprimed letters refer to points on the sphere associated with path α, and the single-primed quantities (not shown in Fig. 2) refer to those associated with path β. The final state A'' (not shown in Fig. 2) is, by arrangement, very close to the initial state A. For the case where the two arms of the interferometer are exactly symmetric, the dynamical phases, which are directly related to the optical path lengths of the two arms, exactly cancel, and the resulting phase difference, which is entirely geometrical (or topological), is *twice*[6] the solid angle subtended by the spherical triangle BCD. Moreover, one obtains upon reversing the sense of circular polarization of the light incident on B1, which does not affect the dynamical phases, a phase shift *4 times* this solid angle. Since the two arms are never exactly symmetric, we chose this quadrupling procedure.

Care was taken, nevertheless, to make the interferometer as nearly symmetrical as possible in its two arms. In particular, its two optical path lengths were nearly equal.

However, they had the opposite senses of handednes. [This can be seen from Fig. 1: Beam splitters B1 and B2, etc., are images of each other under inversion ($\mathbf{r} \to -\mathbf{r}$) through the center of the interferometer.] Thus the topological phases of the two arms add rather than cancel.

All the components of the interferometer were carefully mounted and bolted down onto a small optical table (Newport Research Model XS-24), to ensure the mechanical rigidity of the entire system. Fringes were produced by a slight deliberate misalignment of B2. The optical table rested via shock absorbers on an ordinary laboratory table, whose feet in turn rested via further shock absorbers on the floor of the laboratory. Thus the fringes were stabilized for detection.

The detection apparatus placed at port X consisted of a $10\times$ beam expander (not shown in Fig. 1), followed by two circular polarization filters[12] of opposite senses (Polaroid Model HNCP37) placed side by side, and a camera. Any fringe shift which would have resulted from reversal of the sense of circular polarization of the light incident on B1 would instead show up in a direct comparison at the single output port X, by use of the side-by-side circular polarization filters after B2.

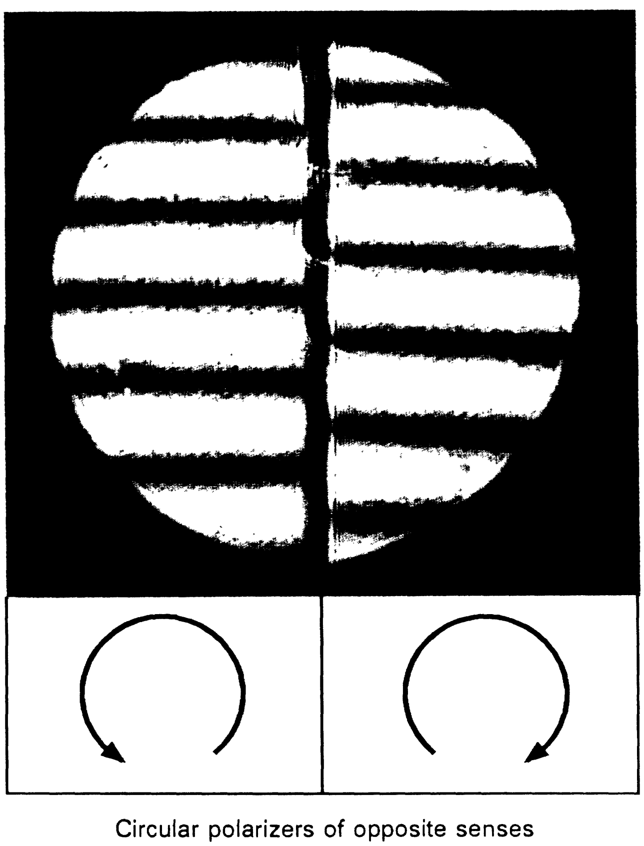

FIG. 3. Interferogram for $\theta = 45°$.

In Fig. 3, we show a photograph of the fringes for $\theta = 45°$. When the beam in either arm of the interferometer was blocked, the fringes disappeared. Thus the interference did not originate from spurious reflections. When both arms were unblocked, but the circular polarization filters were removed, they again disappeared from view, as if no interference had occurred. This only happened for $\theta = 45°$. Such a disappearance of fringes would not have happened if light were scalar waves like sound, and demonstrates the spinor nature of light. In Fig. 3, the maxima of one half of the interferogram coincide with the minima of the other half. This shows that a π phase shift has occurred between the two opposite senses of circular polarization. In other words, a left-handed photon and its right-handed twin have "aged" differently, although they have traveled together over the same total distance.

In Fig. 4, the fringe shifts for various θ are converted into a topological phase γ for one arm of the interferometer, and plotted against the solid angle Ω. The typical vertical error bar represents one standard deviation in the measurement of the fringe shift. The typical horizontal error bar represents the uncertainty in the measurement of θ. The solid line represents theory. Thus Eq. (1) is verified, up to a sign. This sign was checked in an auxiliary experiment by comparison of the sign of the fringe shift for small γ with that obtained from the insertion of a cell filled with a sugar solution of known optical activity into one arm of the interferometer, when both arms were set to zero γ.[13]

We have analyzed the interferometer classically using the Jones calculus. The method used was similar to that for nonplanar laser ring gyros.[14] Details will be given elsewhere.

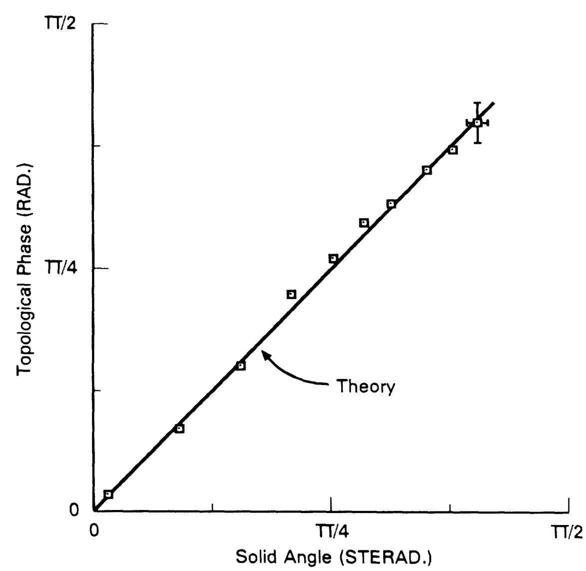

FIG. 4. Topological phase γ vs solid angle Ω.

In conclusion, the fringe shift presented in Fig. 3 represents a direct observation of a topological phase in an optical interferometer. The fringe shift does not originate from the difference in optical path lengths of the two arms of the interferometer, but rather from the difference in their handedness. The phase–solid-angle relation, Eq. (1), was verified. This phase is closely related to Berry's phase, and is perhaps the Aharonov-Anandan phase. We leave open the question of why the use of geodesic constructions on the sphere of spin directions is theoretically justified.

In the following Letter, Pines and co-workers report the observation of a topological phase in NMR interference.

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[1]M. V. Berry, Proc. Roy. Soc. London A **392**, 45 (1984).

[2]B. Simon, Phys. Rev. Lett. **51**, 2167 (1983).

[3]J. H. Hannay, J. Phys. A **18**, 221 (1985); M. V. Berry, J. Phys. A **18**, 15 (1985).

[4]F. Wilczek and A. Zee, Phys. Rev. Lett. **52**, 2111 (1984).

[5]Y. Aharonov and J. Anandan, Phys. Rev. Lett. **58**, 1593 (1987).

[6]R. Y. Chiao and Y.-S. Wu, Phys. Rev. Lett. **57**, 933 (1986).

[7]A. Tomita and R. Y. Chiao, Phys. Rev. Lett. **57**, 937 (1986).

[8]M. Kitano, T. Yabuzaki, and T. Ogawa, Phys. Rev. Lett. **58**, 523 (1987).

[9]J. Anandan, R. Y. Chiao, G. Chingas, K. M. Ganga, R. A. Harris, H. Jiao, A. S. Landsberg, H. Nathel, A. Pines, and D. Suter, to be published; D. Suter, G. Chingas, R. A. Harris, and A. Pines, Mol. Phys. **61**, 1327 (1987).

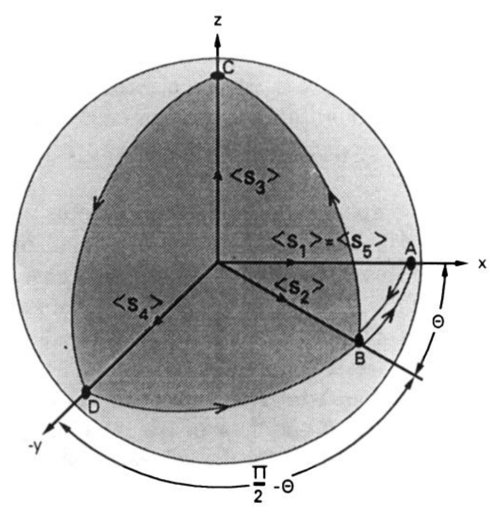
[10]M. V. Berry, Nature (London) **326**, 277 (1987).

[11]R. Y. Chiao and A. Tomita, Phys. Rev. Lett. **59**, 1789 (1987).

[12]These filters, composed of a $\lambda/4$ plate and a linear polarizer glued together, were placed so that the beam emerging from port X impinged on the $\lambda/4$ plate first. They were not true circular polarizers, since the light emerging from them towards the film was linearly polarized. Nevertheless, they did block out light of different senses of circular polarization, and thus served as circular polarization *filters*.

[13]A possible ambiguity of the phase, i.e., either $\pi/2-\theta$ or $+\theta$, was resolved in favor of the first of these two possibilities by the determination of the sign of the fringe shift. An increment of $\delta\theta$ causes the phase to change by $-\delta\theta$ and $+\delta\theta$, respectively, for these two possibilities, leading to opposite signs of the fringe shift.

[14]W. W. Chow, J. Gea-Banacloche, L. M. Pedrotti, V. E. Sanders, W. Schleich, and M. O. Scully, Rev. Mod. Phys. **57**, 61 (1985).



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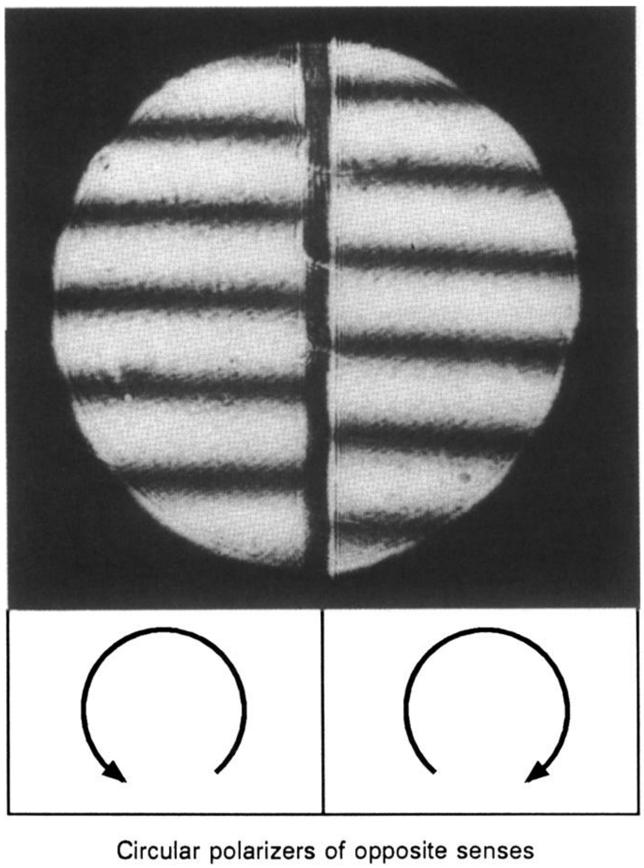

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