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Observation of Topological Phase by Use of a Laser Interferometer

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We report a direct experimental observation of Pancharatnam's phase, which is closely related to Berry's topological phase. The experiment involves measurement of the phase change in one beam of a laser interferometer as the polarization state of light is taken along a closed circuit on the Poincaré sphere. The phase change is found to be equal to half the solid angle subtended by the circuit at the center of the Poincaré sphere. Apart from providing a striking demonstration of the topological phase, the experiment demonstrates that unitary time evolution of a system is not essential for the appearance of the topological phase.

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Recently, Berry[1] and Ramaseshan and Nityananda[2] have pointed out the close connection between the topological phase, commonly referred to as Berry's phase, and the phase discovered in the fifties by Pancharatnam[3] in his study of interference of polarized light. Berry's phase has been the subject of several experimental studies.[4-7] The only experiment reported so far with light[4] measures the rotation of the plane of polarization of light when it passes through a coiled optical fiber. In such experiments, the parameter space corresponds to the surface of the sphere of directions of wave vector **k**. In contrast to this, as pointed out by Berry,[1] Pancharatnam's phase corresponds to a circuit on the sphere representing states of polarization of light, namely the Poincaré sphere, and has a different dependence on solid angle Ω. For the sphere of directions, the topological phase is given by Ω, whereas for the Poincaré sphere, it is given by $\Omega/2$ as in the case of Berry's phase for spin-$\frac{1}{2}$ rotations. In this Letter, we report an experimental study of the latter topological phase, namely the phase on the Poincaré sphere, by use of laser interferometry.

Consider the circuit *APBA* in Fig. 1. *AP* represents a beam of light, initially linearly polarized along the x direction, passing through a quarter-wave plate (QWP1) whose axis (say the fast axis) is oriented at an angle of 45° to the x axis. This takes the beam to state P, which is right-hand circularly polarized. *PB* represents the beam passing through a second QWP (QWP2) with axis oriented at an angle $(\alpha-90°)/2$ to the x axis. This takes the beam to state B, which is linearly polarized at

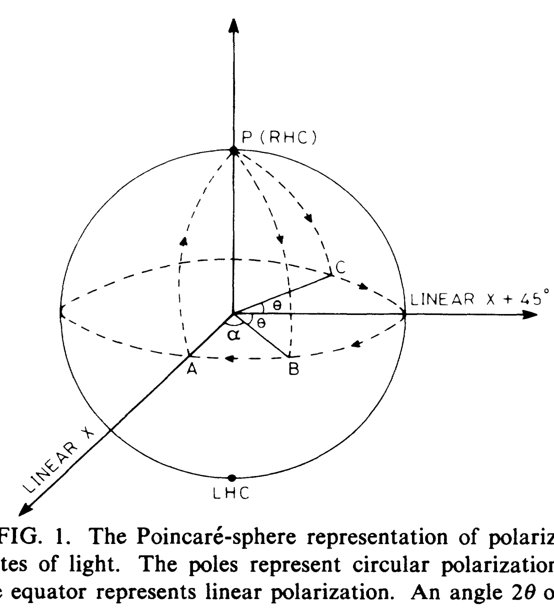

FIG. 1. The Poincaré-sphere representation of polarization states of light. The poles represent circular polarization and the equator represents linear polarization. An angle 2θ on the equator corresponds to a rotation through an angle θ of the polarization plane in real space. All other points represent elliptical polarization.

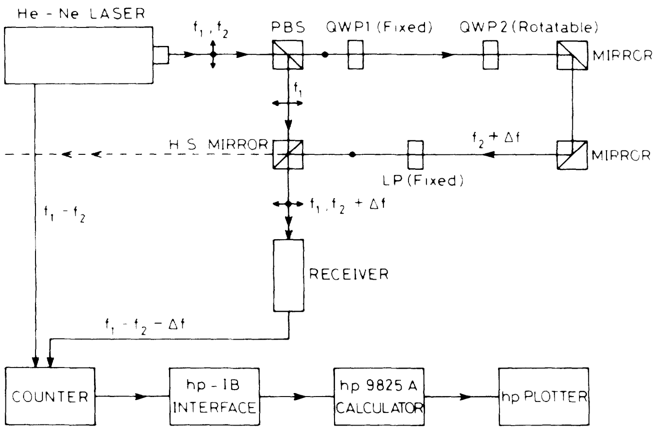

FIG. 2. Schematic diagram of experimental setup. A two-frequency stabilized, single-mode He-Ne Zeeman laser (6328 Å) is used. When the optical path length in one of the two interfering beams is changed by some means, the phase change produced can be looked upon as the time integral of an instantaneous frequency shift Δf of the frequency of that beam. The counter integrates Δf to yield the total phase change. PBS is a polarizing beam splitter; H.S. MIRROR is half silvered mirror; and LP is a linear polarizer (Polaroid).

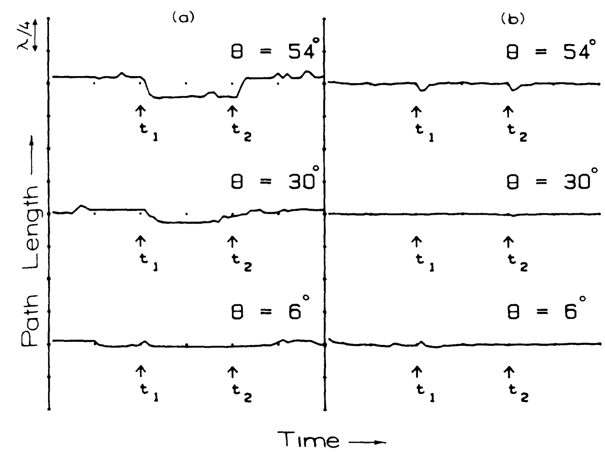

FIG. 3. Experimental traces of the path length for a few angles. One vertical division equals a quarter wavelength ($\lambda/4$). The sampling interval in time equals $\frac{1}{10}$ of a horizontal division. Each horizontal division is approximately 20 sec. (a) Main experiment, where at times marked t_1, QWP2 was rotated by an angle θ and at t_2 by an angle $-\theta$. (b) Auxiliary experiment, where at times marked t_1 and t_2, QWP2 was rotated by the same angle as in the main experiment.

an angle $\alpha/2$ to the x direction. From B, the beam can be brought back to A by passage through an optically active medium of the right thickness. Alternatively, the beam can be sent through a Polaroid (linear polarizer) with its axis aligned along x. The two alternatives are qualitatively different, in that in the second alternative, the beam undergoes a "nonunitary" evolution since the intensity is not preserved.[8] This is the alternative chosen in this experiment. In either case, the beam comes back with an added topological phase which is equal to half the solid angle subtended by the area APB at the center, i.e., $\alpha/2$. This is in addition to a large dynamical phase which is introduced at every stage. As pointed out by Bitter and Dubbers[6] and Berry,[9] the presence of this large dynamical phase makes the topological phase very difficult to measure. The quantity that can be measured sensitively with a laser interferometer, however, is the change in topological phase as the circuit $APBA$ is changed slowly to the circuit $APCA$. This corresponds to our rotating QWP2 through an angle θ, while the optical path length is being continuously monitored by the interferometer. It must be ensured, however, that the change in dynamical phase is zero when QWP2 is rotated. This is true under the following conditions: (i) The circuits on the Poincaré sphere are of the form shown in the figure, i.e., the state of light goes to P and comes back; (ii) the plate being rotated is optically uniform and has parallel faces; and (iii) the axis of rotation coincides with the laser beam.

To ensure that conditions (ii) and (iii) are met, one can do an auxiliary experiment in which QWP1 is absent and QWP2 is rotated through the same range of angles as in the main experiment. Analysis shows that in this experiment if QWP2 satisfies (ii) and (iii), the total change in phase ϕ on rotation of its axis by an angle β is given by $\tan\phi = \tan^2\beta$, where $\beta = 0°$ corresponds to the fast axis of QWP2 being aligned along the incident linear polarization. The curve of ϕ vs β given by the above equation, has minima at $\beta = 0°$ and 180°, has maxima at $\beta = 90°$ and 270°, and is symmetric about all these four points. Let β_m denote one of these maxima or minima. If, therefore, one selects the rotation range of QWP2 such that β goes from $\beta_m - \theta/2$ to $\beta_m + \theta/2$, one expects to see no change of phase on rotation. A null result in this auxiliary experiment is achieved by our ensuring that (ii) and (iii) are met. In the main experiment, then, one chooses the same rotation range, resulting typically in a change of circuit from $APBA$ to $APCA$ (Fig. 1).

Figure 2 shows a schematic diagram of the experimental setup. A custom-built laser interferometer system designed for machine-shop applications was slightly modified and adapted for the purpose. The least count of the instrument is $\frac{1}{40}$ of a wavelength and the path length is continuously monitored and plotted by an on-line computer. At a certain time t_1, QWP2 (vibration isolated from the main setup) is gently manually rotated through angle θ. At a later time t_2, QWP2 is rotated back to the original position. The resulting steps in the optical path length for a few angles are shown in Fig. 3(a). The results for the auxiliary experiment (with QWP1 removed) for the same angles are shown in Fig. 3(b).

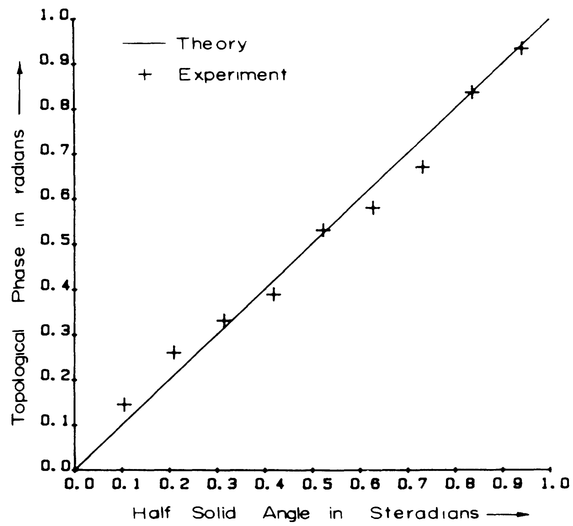

FIG. 4. Observed topological phase vs solid angle.

Figure 4 shows the extent of quantitative agreement between the expected values of the topological phase (solid line) and the measured values (plusses). It was observed that noise in the experiment is a function of thermal conditions in the laboratory. The scatter of the measured points about a straight line decreases as thermal conditions become more uniform and stable. The data presented here represent a run made under the best conditions.

Apart from the fact that this experiment provides the most direct demonstration of a topological phase so far, (rotate QWP2 and you see a step on the chart), the experiment raises an interesting question.

Since the discovery of Berry's phase,[9] it has gradually come to be realized that the phenomenon is more general than envisaged in Ref. 9. Aharanov and Anandan,[10] for instance, have shown that the assumption of adiabaticity is not essential for the appearance of Berry's phase. This experiment suggests yet another generalization, namely, the assumption of unitarity in the time evolution of the system is not essential for the appearance of the phase.

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