Comment on "Initial Stages of Pattern Formation in Rayleigh-Bénard Convection"

In recent experiments on the initial stages of pattern formation in a Rayleigh-Bénard convection cell with a large aspect ratio,[1] the fluid was subjected to a heat current varying either linearly (ramping experiment) or periodically (oscillating experiment) in time from below to above the onset of the first instability. The observed cellular patterns appear to be similar to those one might expect in a laterally infinite system. Their random and irreproducible nature suggested the stochastic nature of their formation. However, whether the noise strength needed to explain the experiments could be attributed entirely to internal thermal fluctuations remained unresolved. Estimates of their strength have been made before with use of fluctuating hydrodynamics,[2] free-free boundary conditions, and a single-mode model. We present here results of a more complete calculation that uses stick boundary conditions and that, in particular, takes into account the fluctuations in *all* relevant modes near the critical Rayleigh number $R=R_c$, not just the single mode that survives in the final macroscopic state. For large systems, as in Ref. 1, inclusion of all modes is important in principle. It gives an increase in the strength of the fluctuations of about a factor of 30, but is still at least 4 orders of magnitude below the noise strength needed to explain the experiments.

Writing the local amplitudes of the fluctuating hydrodynamic fields as $\mathbf{a}(\mathbf{r},t)=\sum_\mu a_\mu(t)\mathbf{a}_\mu^R(\mathbf{r})$, where μ labels the relevant critical modes near R_c and $\mathbf{a}_\mu^R(\mathbf{r})$ is the right eigenvector associated with the hydrodynamic equations linearized around the laterally uniform stationary state, that is stable for $R<R_c$,[3] one obtains for the Nusselt number[4]

$$N-1\equiv J_{\text{conv}}/J_{\text{cond}}=(t_D/\nu R\tau_0)\sum_\mu\langle\,|a_\mu(t)|^2\rangle, \qquad (1)$$

where ν is the kinematic viscosity, τ_0 is a characteristic time[2] in units of the vertical thermal diffusion time t_D, and $\langle\,|a_\mu(t)|^2\rangle\sim V^{-1}$, with V the volume of the cell. To compare, for the ramping experiment, Eq. (1) with the corresponding expression for the single-mode model,[2] we introduce an effective single-mode noise strength F_{eff}. For a large system, when the sum in (1) can be replaced by an integral, one obtains[4]

$$F_{\text{eff}}=(t_D/2\pi\tau_0 t)^{1/2}k_B T t_D a_c^2(\Lambda\rho d^3\nu R)^{-1}, \qquad (2)$$

where for stick boundary conditions $a_c\approx3.119$, $R=1707$, and $\Lambda\approx1.20$. Thus F_{eff} is the value that should be used for the noise strength F of the noise f in Eq. (1) of Ref. 1, in order to obtain agreement with our Eq. (1). The summation over μ leads to a factor V, making N independent of V, and to an extra time dependence $\sim t^{-1/2}$ of N and F_{eff}. When our F_{eff} is compared with the corresponding expression for the thermal noise strength $\bar{F}_{\text{th}}$ in Appendix D of Ref. 2, one finds $F_{\text{eff}}\approx32\bar{F}_{\text{th}}$ for the aspect ratio of 10 considered in Ref. 1 and $t/t_D=1.2$. This shows the effect of the different boundary conditions and of the consideration of all critical modes rather than a single one. It appears to rule out internal thermal noise as the source of the stochastic behavior observed in the ramping experiment, since the noise strength is far below that observed experimentally.

For the oscillating experiment, the consideration of all relevant modes rather than a single one leads to a more complicated stochastic behavior than implied in Ref. 2; the fluctuations can now drive the system into many different convection patterns, instead of into just two. For large systems one expects, as in the ramping experiment, that the experimental results will be independent of V, because of the compensation between the strength of the fluctuations per mode $\sim V^{-1}$ and the number of modes $\sim V$.

The authors are most indebted to G. Ahlers, D. S. Cannell, C. W. Mayer, and R. Schmitz for helpful discussions and to National Science Foundation Grant No. PHY82-17853, supplemented by funds from NASA at the University of California at Santa Barbara. One of us (E.G.D.C.) was also partly supported by the U.S. Department of Energy Contract No. DE-AC02-81ER10807.

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Received 5 October 1987
PACS numbers: 47.20.Bp, 47.25.Qv

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