## Universality Classes for the $\Theta$ and $\Theta'$ Points

Duplantier and Saleur[1](DS) proposed exact values of the tricritical-point exponents for a 2D interacting self-avoiding walk which has both nearest-neighbor (nn) interactions and a special *subset* of the next-nearest-neighbor (nnn) interactions,[2] and describes exactly the hull of clusters at the percolation threshold. The tricritical point described by this set of interactions is termed the $\Theta'$ point, in contrast to the conventional $\Theta$ point which is described by a self-avoiding walk with only nn interactions. Reference 2 pointed out that the $\Theta$ and $\Theta'$ points may belong to the same universality class, and DS state their belief that this is indeed the case. Our purpose here is to point out that the relation of the $\Theta$ and $\Theta'$ points is an open scientific question.[3]

(i) First we note a rather subtle and surprising analogy between the present linear polymer problem and a general "branched polymer" problem (site-bond lattice animals with a nn interaction between adjacent sites[4]): The generating functions and the associated phase diagrams for both problems are of the same form. Specifically,

$$Z=\sum_{\text{config}} \mu_x^{N_x}\mu_b^{N_b}e^{\epsilon N_{\text{nn}}} ,$$

where $\mu_x$ and $\mu_b$ are chemical potentials, $N_b$ is the number of bonds, $N_{\text{nn}}$ the number of nn pairs, and $\epsilon$ is the nn interaction energy. For the branched polymer problem $N_x$ is the number of sites, while for the linear polymer problem $N_x$ is the subset of the nnn interactions defined in Ref. 2.

The corresponding phase diagram shows a line of $\Theta$ points for both the "linear" and "branched" problems. On this line there appears a special point, $\Theta'$. For the branched polymer case, the point $\Theta'$ corresponds to a *change* of universality class (from $\Theta$ point to percolation). For our linear polymer case, $\Theta'$ corresponds to the hull, and it is possible that the universality class also changes.

(ii) Second, to test this possibility we made extensive Monte Carlo simulations of both the $\Theta$ and $\Theta'$ points. Our results for the tricritical exponents $\nu$, $\gamma$, and $\phi$ at the $\Theta'$ point are in excellent agreement with the predictions of DS, thus providing a numerical confirmation of their arguments. For the $\Theta$ point, we find that $\nu$ and $\gamma$ are the same as for the $\Theta'$ point while $\phi=0.50\pm0.10$ is somewhat larger than the DS prediction $\phi=3/7$ (although the DS value is within our confidence limits). Thus we cannot conclude, from the numerical analysis alone, that the two points belong to *different* universality classes. However the near coincidence of the two sets of tricritical-point exponents cannot be used to support the opposite view that the two points belong to the *same* universality class. Indeed for the analogous branched polymer problem the numerical values for $d_f$, which were calculated[5] (for one point on the lattice animal "$\Theta$ line" of the phase diagram), are almost identical to the value of the percolation exponent at the special point $\Theta'$.

In conclusion, our purpose here is not to claim that the $\Theta$ and $\Theta'$ points are in different universality classes, but only to point out that this is still a genuinely open question.

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[3]The $\Theta$ and $\Theta'$ points would belong to the same universality class if all the nnn interactions were present. However a subtle feature is the presence here of only a *subset* of the nnn interactions (see Ref. 2).

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