Combined Effect of Zener and Quasiparticle Transitions on the Dynamics of Mesoscopic Josephson Junctions

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(Received 7 August 1987)

We study ultrasmall Josephson junctions using the coherent picture to describe the pair dynamics and the semiclassical approach for the quasiparticles. We find that the dynamics depends crucially on the ratio of the superconducting gap energy $\Delta(T)$ to the charging energy $e^2/2C$. The pair "Bloch oscillations" can be practically observed when $2\Delta(T) > e^2/2C$; for $2\Delta(T) < e^2/2C$ the I_{dc}/e oscillations due to quasiparticle tunneling can be observed. We discuss under what conditions the *I-V* characteristic will be S-shaped, exhibiting both oscillations at different values of the current.

PACS numbers: 74.50.+r, 72.15.Nj, 73.20.Dx

Among the many exciting new effects predicted in modern ultrasmall junctions[1] are the coherent Bloch oscillations in mesoscopic Josephson junctions driven by a current source.[2-8] Neglecting quasiparticle transitions, it is predicted that a Josephson junction of capacitance C driven by a current source I_{dc} will exhibit voltage oscillations of amplitude e/C at a frequency $I_{dc}/2e$. Similar oscillations, but with frequency I_{dc}/e and amplitude $e/2C$, are predicted in mesoscopic normal (nonsuperconducting) tunnel junctions.[5,8-14] Much subsequent efforts has been aimed at the inclusion of the stochastic tunneling of normal charges in addition to the coherent pair tunneling in the description of a mesoscopic Josephson junction.[4-9,11,13] In such a picture oscillations at frequencies I_{dc}/e or $I_{dc}/2e$ due to the normal electron tunneling and the pair tunneling, respectively, are expected resulting in an S-shaped *I-V* characteristic,[4,7,13] with the voltage first increasing the I_{dc} and then decreasing as we go from the I_{dc}/e to the $I_{dc}/2e$ oscillations, and finally increasing again as I_{dc} is increased even further.

In this Letter we include two new elements. First, we include the effect of the higher-energy bands by allowing Zener transitions[15] between energy bands.[8,9] Second, we treat the inelastic transitions as the tunneling of *quasiparticles* rather than normal charges. Thus the inelastic transition rate depends on the superconducting energy gap $\Delta(T)$. Our main new result is the discovery of the crucial role of the ratio of the superconducting gap energy $\Delta(T)$ to the charging energy of a single electron $e^2/2C$. We find that for $2\Delta(T) < e^2/2C$ only the I_{dc}/e oscillations are practically observable. In the opposite limit, at low currents only the $I_{dc}/2e$ oscillations should be present, while for higher currents the I_{dc}/e oscillations are also observable over a specific range of I_{dc}. We also show that the S-shaped *I-V* characteristic can be observed only over a very limited range of parameters. We emphasize that all of our predictions are based on the assumption that the Josephson tunnel junction is made from superconductors that are described by the Bardeen-Cooper-Schrieffer (BCS) theory,[16] so that the Josephson critical current I_J is related to the normal-state resistance R_N by[17]

$$I_J = [\pi\Delta(T)/2eR_N]\tanh[\Delta(T)/2k_BT].$$

We describe the dynamics of the pairs and the current source by a phenomenological Hamiltonian consisting of an electrostatic term and a pair tunneling term[5,8] $H = E_C(q - 2\hat{n}_P)^2 + E_J(1-\cos\hat{\theta})$, where $\hat{n}_P$ is the pair number operator, $\hat{\theta}$ is the phase operator conjugate to $\hat{n}_P$, and $\dot{q} \equiv I_{dc}/e$. Here $E_C = e^2/2C$, and E_J is the Josephson coupling energy $E_J = \hbar I_J/2e \lesssim E_C$. The energy spectrum as a function of q is a series of oscillating bands, analogous to those of the nearly-free-electron model in the extended zone scheme.[4,5]

In the adiabatic limit as q increases linearly in time we expect the system to follow the lowest-energy band. However, as the system passes through the gap regions some part of the wave function will "leak" into neighboring bands[8,18] by Zener tunneling.[15] Here we consider the simpler case in which the phase randomization time τ_ϕ due to the heat bath is longer than τ_Z, the Zener tunneling time, but shorter than the period of oscillation, $2e/I_{dc}$.[8,19] In this limit the system is always localized on a single band, but may display Zener tunneling when two bands draw together with probability[8,15,19]

$$P_{m,m+1} = P_{m+1,m} \approx \exp\{-\pi E_{gap}^2(m)/8\hbar E_C\dot{q}\},$$

where $E_{gap}(m)$ is the gap between bands m and $m+1$.

For the first band this can be written in a particularly simple form, namely

$$P_{1,2} = \exp\{-\pi I_J E_J / 16 I_{dc} E_C\} = \exp\{-I_Z/I_{dc}\},$$

where $I_Z \equiv \pi I_J E_J/16E_C$. Under these assumptions the system performs a random walk among the energy bands which can be described by discrete-time stochastic process for Q, the total charge across the junction[8]:

$$Q(t+\Delta t) = \begin{cases} Q(t) & \text{with probability } P_{m,m+\mathrm{sgn}(Q)}, \\ -Q(t) & \text{with probability } 1-P_{m,m+\mathrm{sgn}(Q)}. \end{cases} \tag{1}$$

In addition to Zener transitions the junction may also make inelastic transitions to lower band numbers by quasiparticle tunneling. Each quasiparticle that tunnels lowers the net charge across the junction by e, transferring an energy eQ/C to the heat bath. These inelastic transitions can be described by a continuous-time stochastic process:

$$Q(t+\Delta t) = \begin{cases} Q(t) + I_{dc}\Delta t - e, & \text{with probability } r(Q)\Delta t, \\ Q(t) + I_{dc}\Delta t + e, & \text{with probability } l(Q)\Delta t, \\ Q(t) + I_{dc}\Delta t, & \text{with probability } 1-[1(Q)+r(Q)]\Delta t, \end{cases} \tag{2}$$

where $r(Q)$ and $l(Q)$ are the quasiparticle transition rates in the forward and reverse directions, respectively. Such a semiclassical model implicitly assumes that the quasiparticles are well localized on either side of the tunnel junctions.

Considering the effect of both the superconducting energy gap and the electrostatic charging energy, the quasiparticle transition rate in the forward direction, say right to left, is given by[20]

$$r(Q) = \frac{1}{R_N C} \int_{-\infty}^{\infty} \tilde{D}_r(E) \tilde{D}_l(E-E_\mu) f(E)[1-f(E-E_\mu)]dE, \tag{3}$$

where $\tilde{D}(E)$ is the ratio of the quasiparticle excitation spectrum to the normal-state density of states at the Fermi level, f is the equilibrium Fermi distribution function, and energy is measured in units of E_C relative to the Fermi energy on the right-hand side of the junction. The transition rate in the opposite direction, $l(Q)$, is the reflection of $r(Q)$ about the $Q=0$ axis. The normal-state resistance R_N is the resistance when the system is driven by a voltage source above the critical temperature, T_C. The quantity E_μ is the change in the chemical potential of an electron that tunnels across the junction, $E_\mu = eQ/C - e^2/2C$.

We assume the BCS picture[21] so that $\tilde{D}(E) = |E|/[E^2 - \Delta(T)^2]^{1/2}$ for $|E| > \Delta(T)$ and is zero for $|E| < \Delta(T)$. At low temperatures ($T \ll T_C$, and $k_B T \ll E_C$) the rate $r(Q)$ is exponentially small up to a charge $Q_0 \equiv [2\Delta(T)C/e] + e/2$ and rises sharply to the line $(Q-e/2)/(eR_N C)$ thereafter. Increasing the temperature decreases $\Delta(T)$ and therefore Q_0 also, as well as increasing the transition rate for quasiparticles for $Q < Q_0$.

We first discuss the dynamics of the junction considering just the quasiparticle transitions of Eq. (2) (setting $P_{m,m\pm1} = 1$) in the limit $T=0$. We define Q_T as the charge at which a quasiparticle tunnels and $P(Q_T)$ to be the stationary probability density of such transitions for fixed I_{dc}. The average charge at which a transition occurs, $\bar{Q}_T$, is therefore $\bar{Q}_T \equiv \int P(Q_T) Q_T dQ_T$. For very large currents $\bar{Q}_T \gg Q_0$, and the response of the junction is independent of $\Delta(T)$ so that in this limit the I-V characteristic is the same as that of a normal junction, $I_{dc} \approx (Q - e/2)/R_N C$.[8,11] From the form of $r(Q)$ given above we can deduce that for small currents such that $I_{dc} \ll 2\Delta(T)/eR_N \equiv I_0$, we have that $\bar{Q}_T \approx Q_0$, and $P(Q_T)$ is peaked around this value and has a width smaller than e. The junction will repeatedly charge up to a value slightly greater than Q_0 and then a quasiparticle will tunnel, producing a sawtoothlike voltage oscillation at a frequency I_{dc}/e.[8,9] However, the average voltage is given by $\langle V \rangle = (\bar{Q}_T - e/2)/C \sim 2\Delta(T)/e$ for an arbitrarily small current. This is quite different from the case where the inelastic transitions come from the tunneling of normal electrons $[\Delta(T)=0]$, in which case

$$\lim_{I_{dc}\to 0} \langle V \rangle = 0.$$

At nonzero temperatures, although $r(Q)$ is not zero for $Q < Q_0$, for $T \ll T_C$ it is exponentially small, so that the sharp jump is rounded but still visible. In Figs. 1(a) $[2\Delta(T) < e^2/2C]$ and 1(b) $[2\Delta(T) > e^2/2C]$ we show typical I-V characteristics for the case where pair transitions are neglected, showing their different approaches to their asymptotes, in qualitative agreement with experiment.[22]

We now turn to the case where both Zener and quasiparticle transitions are included. For $2\Delta(T) < e^2/2C$ we have $Q_0 < e$, and hence for $I_{dc} < I_0$ the distribution $P(Q_T)$ has a width less than e, and is peaked near Q_0. This leads to oscillations at a frequency I_{dc}/e about a voltage $\langle V \rangle = 2\Delta(T)/e$. In order to see an effect from the pair current, $P(Q_T)$ must extend above e so that a pair transition can occur. Since the width of $P(Q_T)$ is approximately eI_{dc}/I_0 this implies that the pair current contributes at $I_{dc} \approx (1 - Q_0/e)I_0$. In Fig. 1(a) we see that at this point there is a knee in the I-V characteristic

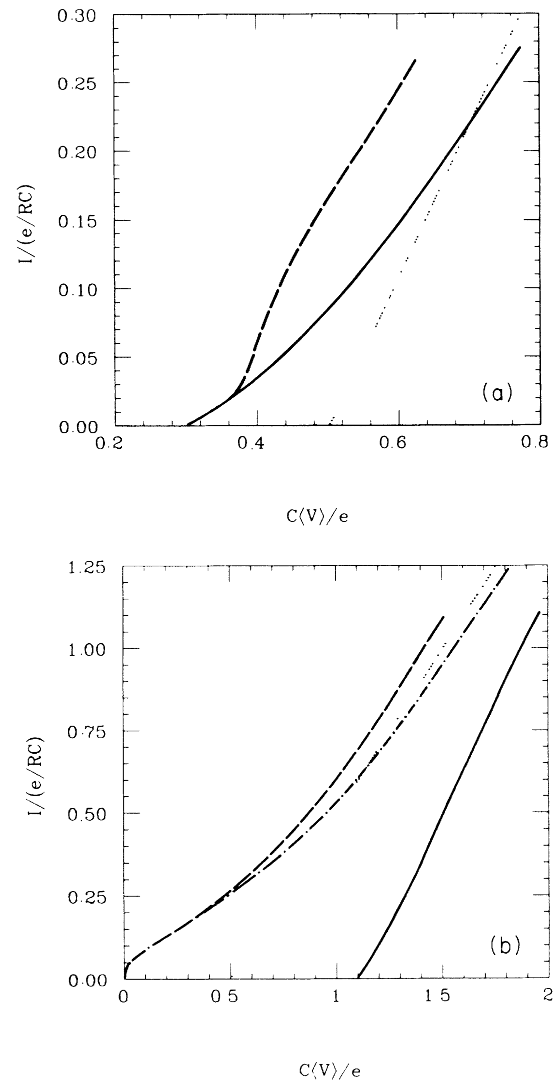

FIG. 1. The theoretically predicted *I-V* characteristic with the stochastic processes of Eqs. (1)–(3). (a) The *I-V* characteristic for $2\Delta(T) < e^2/2C$. The parameters are $T_C = 3.0$ K, $T = 0.25$ K, $C = 0.05$ fF, and $E_J/E_C = 2.0$. From these parameters it can be determined that $2\Delta(T)C/e^2 = 0.27$, $e/R_NC = 760$ nA. For the solid line the pair current was set equal to zero ($P_{m,m\pm1} = 1$); the dashed line includes the pair current. The two separate when a reasonable fraction of $P(Q_T)$ extends above $Q = e$. The straight dotted line gives the asymptotic *I-V* characteristic for large I_{dc}. (b) The *I-V* characteristic for $2\Delta(T) > e^2/2C$. The parameters are $T_C = 12.0$ K, $T = 1.0$ K, $C = 0.05$ fF, and $E_J/E_C = 2.0$. From these parameters it can be determined that $2\Delta(T)C/e^2 = 1.1$, $e/R_NC = 3.04$ μA. For the solid line the pair current was set to zero. The middle curve (dot-dash) includes the pair current in the case where $E_{gap}(m)$ shrinks geometrically in m. In this case $P_{m,m\pm1}$ rapidly approaches 1 with increasing band number, so that the *I-V* characteristic rapidly approaches the solid line. The upper, dashed curve includes the pair current when the gaps are independent of m. The straight dotted line is the asymptotic *I-V* characteristic for large I_{dc}.

reflecting the fact that the pairs start to carry some of the current and thus lower the overall resistance of the junction. However, a power spectrum of the voltage does not reveal a peak at a frequency $I_{dc}/2e$; the quasiparticle transitions still dominate the stochastic process. In order to observe the $I_{dc}/2e$ oscillations in the power spectrum it is necessary that $\bar{Q}_T > e$, which requires that $I_{dc} > I_0 = 4I_J/\pi$. From the form of $P_{1\to2}$ we see that this means that the Zener tunneling probability is large and the $I_{dc}/2e$ oscillations will be washed out. These results assume the Zener tunneling probability given above; we expect that the effect of the quasiparticles will be to increase $P_{m,m\pm1}$ above this value so that the knee in Fig. 1(a) will be smaller.

If we relax the constraint between I_J and R_N given by the BCS theory, then in the limit $\Delta(T) \to 0$, to increase $\bar{Q}_T$ above e means that[8] $I_{dc} > e/R_NC$. On the other hand, to observe the $I_{dc}/2e$ oscillations, I_{dc} should be smaller than I_Z. From these two conditions we obtain that in order to see the $I_{dc}/2e$ oscillations we require that $R_N \gtrsim (8/\pi)(E_C/E_J)^2\hbar/e^2$.

For $2\Delta(T) > e^2/2C$ and at low temperatures we have $Q_0 > e$, so that the probability for a quasiparticle transition before the system reaches the first band gap at $Q = e$ is exponentially small. The $I_{dc}/2e$ oscillations can be observed so long as $P_{1,2} \ll 1$, which means that $I_{dc} \ll I_Z$. As I_{dc} increases above this value, $P_{1,2} \to 1$ and the $I_{dc}/2e$ oscillations will be washed out.[8] In Fig. 1(b) we show typical *I-V* characteristics for this limit. The voltage for the combined case can be approximated by the voltage calculated with neglect of Zener transitions ($P_{m,m\pm1} = 1$) multiplied by $P_{1,2}$. We note that in this limit the I_{dc}/e oscillations are only observable for $I_Z < I_{dc} < I_0$. Both types of oscillations can be observed in a single junction under certain circumstances. When the temperature is increased the transition rate $r(Q)$ becomes appreciable for $Q < Q_0$. In effect, the junction has a high but finite resistance R_{eff} at low values of the current. We can choose a $\Delta(T)$ and T such that $Q_0 > e$, and that $e/R_{eff}C \ll I_Z$. For $I_{dc} < e/R_{eff}C$ the quasiparticle transition rate is large enough so that $\bar{Q}_T < e$ and the junction will exhibit I_{dc}/e oscillations. When the current is increased so that $e/R_{eff}C < I_{dc} < I_Z$ the junction reaches the first gap with a small probability of Zener tunneling so that the $I_{dc}/2e$ oscillations can be observed. At higher values of the current $P_{1,2}$ is large and the oscillations are washed out. Under such conditions the *I-V* characteristic will be S-shaped. We emphasize yet again that this prediction assumes the Zener tunneling probability given above, which may be modified when the effect of the quasiparticles on the Zener probability is considered.

We benefitted from many productive conversations with Y. Gefen. We also wish to thank M. Fisher, R. C. Jaklevic, and R. Wilkins for helpful discussions. This research was partially supported by National Science Foundation Grant No. DMR 8608305, the Army

Research Office under Grant No. DAAL03-87-K-0007, the Israel Academy of Science, and a Tel-Aviv University grant for basic research. One of us (K.M.) was supported by the Center For High Frequency Microelectronics at the University of Michigan. Numerical calculations were made possible by a National Science Foundation Grant on the San Diego Supercomputer.

Note added.— Subsequent to submission we learned of other work done on quasiparticle effects.[23]

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