# Reflected Phase-Conjugate Wave in a Plasma

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We demonstrate that a plasma can be used as an effective nonlinear medium in a phase-conjugate reflector for subcentimeter electromagnetic waves. The method of production of the amplified phase-conjugate waves is almost-degenerate four-wave mixing. By excitation of collective modes in resonance with the incoming waves the amplification is enhanced significantly.

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There has been much interest recently in nonlinear interactions of electromagnetic (EM) waves with matter, which lead to the phenomenon of optical phase conjugation.[1,2] In this process a material body (e.g., solid or liquid) is stimulated by EM waves in such a way that another EM "signal" wave, which is sent into the system, is reflected from the body, with its wave-vector reversed and its phase conjugated. The signal beam is reflected by a phase-conjugate "mirror," and retraces its original path. Nonlinear optical effects in plasmas have attracted attention in recent years[3] including, in particular, some wave-mixing phenomena.[4,5] Recently, degenerate four-wave mixing in plasma was studied with use of fluid equations.[6] Also, small-signal (no amplification) phase-conjugate waves were produced by stimulated backward scattering in laser-driven plasmas.[7]

The purpose of the present paper is to demonstrate from first principles that a plasma can be used as a phase-conjugate reflector for EM waves in the subcentimeter region of wavelengths. It is found that for plasma of easily attained properties, the reflected radiation may be enhanced with respect to the incoming signal, and the plasma plays a dual role of a phase-conjugate reflector and amplifier. The method used here to generate the phase-conjugate wave is known in nonlinear optics as almost-degenerate mixing, with the plasma playing a role of the nonlinear medium. Collective modes of the plasma are set into resonance by mixing of the incoming waves and the reflected wave is then enhanced by a few orders of magnitude. The system is so arranged that two counter-moving pump waves, 1 and 3, run in the plasma continuously, with equal frequencies $\omega_0$ and antiparallel wave vectors $\mathbf{k}_0$ and $-\mathbf{k}_0$. These waves are "pure" transverse EM waves, with $\omega = ck$, outside the plasma, and "dressed" EM waves inside, with $\omega^2 = c^2k^2 + \omega_p^2$, where $\omega_p$ is the electron plasma frequency. Hereafter we will refer to both as EM waves. A third EM wave, the signal wave, 2, enters the system with $\omega_s$ and $\mathbf{k}_s$ ($\omega_s \sim \omega_0$). The signal 2 and the pump 1 form an interference grating (density mixing), and the Bragg-scattered second pump 3 generates a wave 4 which is phase conjugate to the signal 2. The generated-wave equation is

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}_4(\mathbf{r},t) = s(\mathbf{r},t) = 4\pi\left[\frac{1}{c^2}\frac{\partial}{\partial t}\mathbf{j}(\mathbf{r},t) + \nabla\rho(\mathbf{r},t)\right], \quad (1)$$

where $\mathbf{j}(\mathbf{r},t)$ and $\rho(\mathbf{r},t)$ are the current and charge-density response of the plasma to the incoming waves. We write for the external field

$$\mathbf{E}^e(\mathbf{r},t) = \sum_{i=1}^{3} \hat{\mathbf{e}}_i E_i \cos(\mathbf{k}_i \cdot \mathbf{r} - \omega_i t + \phi_i),$$

where $\hat{\mathbf{e}}_i$ is the polarization vector of the $i$th field, $\mathbf{k}_i$ and $\omega_i$ are the wave vector and frequency, and $\phi_i$ is the phase. We take $\omega_3 = \omega_1 = \omega_0$, $\mathbf{k}_3 = -\mathbf{k}_1$, $\mathbf{k}_1 = \mathbf{k}_0$, $\omega_2 = \omega_s$, $\omega_s \sim \omega_0$, and $|\mathbf{k}_0 - \mathbf{k}_s| \sim k_0$. The Fourier transform of $\mathbf{s}(\mathbf{r},t)$ in volume $V$ is

$$\mathbf{s}(\mathbf{k},\omega) = -(4\pi\omega/c^2)\{\mathbf{j}(\mathbf{k},\omega) - (c/\omega)^2\mathbf{k}[\mathbf{k}\cdot\mathbf{j}(\mathbf{k},\omega)]\},$$

where $\omega$ and $\mathbf{k}$ in $\mathbf{s}(\mathbf{k},\omega)$ are anticipated to be the frequency and the wave vector of the generated ("dressed") EM waves.

The plasma is described in terms of the Vlasov equation for the distribution function $f_\alpha(\mathbf{r},t,\mathbf{v})$ of the $\alpha$th species in space $\mathbf{r}$, time $t$, and velocity $\mathbf{v}$, i.e.,

$$\frac{\partial}{\partial t}f_\alpha + \mathbf{v}\cdot\nabla f_\alpha + \frac{q_\alpha}{m_\alpha}\left(\mathbf{E}_{sc} + \frac{1}{c}\mathbf{v}\times\mathbf{B}_{sc}\right)\cdot\frac{\partial}{\partial\mathbf{v}}f_\alpha = -\frac{q_\alpha}{m_\alpha}\left[\mathbf{E}^e(\mathbf{r},t) + \frac{1}{c}\mathbf{v}\times\mathbf{B}^e(\mathbf{r},t)\right]\cdot\frac{\partial}{\partial\mathbf{v}}f_\alpha. \quad (2)$$

Here $q_a$ and $m_a$ are the charge and mass, $\mathbf{E}^e(\mathbf{r},t)$ is the external electric field, and $\mathbf{B}^e(\mathbf{r},t)$ is the accompanying magnetic field. The $\mathbf{E}_{sc}$ and $\mathbf{B}_{sc}$ are the self-consistent fields in the plasma, i.e., they are the solution of Maxwell's equations with $\rho(\mathbf{r},t)=\sum_a q_a n_a(\mathbf{r},t)$, $n_a(\mathbf{r},t)=\int d^3v\, f_a(\mathbf{r},t,\mathbf{v})$, and $\mathbf{j}(\mathbf{r},t)=\sum_a q_a \int d^3v\, \mathbf{v} f_a(\mathbf{r},t;\mathbf{v})$. We assume the following: (a) The external EM waves (and the generated EM waves) must be able to penetrate the plasma. With $e$ and $m$ the electron charge and mass, $n_0$ the density, and $\omega_p^2=4\pi e^2 n_0/m$ the square of the electron plasma frequency, we take $(\omega_i/\omega_p)^2$ to be at least of order 10. (b) The thermal velocity of the electrons, $v_{\text{th}}$, is much smaller than the speed of light, i.e., $k_i v_{\text{th}} \ll \omega_i$. (c) The electron-to-ion mass ratio $m/M$ is very small. (d) The incoming waves 1 and 2, the mixing waves, can be tuned to resonate with some internal collective excitations of the plasma of frequency $\Omega$ and wave vector $\mathbf{q}$. We can set the waves so that $\omega_s=\omega_0 \pm \Omega$, $\Omega \ll \omega_s, \omega_0$, but $\mathbf{q}=\mathbf{k}_0-\mathbf{k}_s$ is of the order of $k_0$ or $k_s$. In our case, good candidates for excitations are the ion acoustic waves when the electron temperature $T_e$ is larger than $T_I$ (ions).

The mixing of the waves 1 and 2, and the Bragg scattering of the third wave, 3, generate a "third"-order current density,

$$\mathbf{j}(\mathbf{r},t)=\textstyle\sum_a q_a n_a^{(1,2)}(\mathbf{r},t)\mathbf{v}_a^{(3)}(\mathbf{r},t).$$

We denote by $n_a^{(1,2)}(\mathbf{r},t)$ the density response of the $\alpha$th species to the mixing waves $\mathbf{E}_1(\mathbf{r},t)$ and $\mathbf{E}_2(\mathbf{r},t)$, and by $\mathbf{v}_a^{(3)}(\mathbf{r},t)$ the velocity response to the scattered third wave, $\mathbf{E}_3(\mathbf{r},t)$. A similar term of $j$ is the product of $n^{(3,2)}$ with $v^{(1)}$.

We now introduce the condensed notations $k\equiv(\mathbf{k},\omega)$, $\sum_k \equiv (1/V)\sum_{\mathbf{k}}\int d\omega/2\pi$, $\delta(k-k_i)=\delta_{\mathbf{k},\mathbf{k}_i}\delta(\omega-\omega_i)$. Since $m/M \ll 1$, only electrons contribute to the current and

$$\mathbf{j}(k)=-e\textstyle\sum_{k'} n_e^{(1,2)}(k-k')\mathbf{v}_e^{(3)}(k'). \tag{3}$$

We solve for the electron velocity response to the transverse EM wave $\mathbf{E}_3(k)$, and find that

$$\mathbf{j}(k)=\hat{\mathbf{e}}_3\left(i\frac{e^2}{m}\right)\frac{E_3}{\omega_3}\tfrac{1}{2}\left[n_e^{(1,2)}(k-k_3)e^{i\phi_3}-n_e^{(1,2)}(k+k_3)e^{-i\phi_3}\right]. \tag{4}$$

The density response to the transverse EM waves $\mathbf{E}_1$ and $\mathbf{E}_2$ is then calculated with use of the Vlasov equation, Eq. (2), and Poisson's equation, $\nabla\cdot\mathbf{E}_{sc}(\mathbf{r},t)=4\pi\sum_a q_a n_a(\mathbf{r},t)$. Notice that for the density response we consider also the motion of the ions, since they do participate in the collective excitations in the plasma. The "second" order density due to the mixing (compare to Refs. 4 and 5) is

$$n_e^{(1,2)}(\mathbf{k},\omega)=[1-(m/M)(\omega_p/\omega)^2]g^{(1,2)}(\mathbf{k},\omega)/\epsilon(\mathbf{k},\omega), \tag{5}$$

where $\epsilon(\mathbf{k},\omega)$ is the linear dielectric function of the plasma, and

$$g^{(1,2)}(k)=-\frac{e^2}{m^2}\int d^3v\frac{1}{\omega-\mathbf{k}\cdot\mathbf{v}}\sum_{k'}\left\{\left[\mathbf{E}_1(k-k')+\frac{1}{c}\mathbf{v}\times\mathbf{B}_1(k-k')\right]\cdot\frac{\partial}{\partial\mathbf{v}}\left[\frac{1}{\omega'-\mathbf{k}'\cdot\mathbf{v}}\mathbf{E}_2(k')\cdot\frac{\partial}{\partial\mathbf{v}}f_e^0(\mathbf{v})\right]+(1\rightleftarrows 2)\right\}. \tag{6}$$

Near the ion acoustic resonance, $\epsilon(\mathbf{k},\omega)$ is

$$\epsilon_a(\mathbf{k},\omega)=1-(m/M)(\omega_p/\omega)^2+(k_D/k)^2-i\Gamma_a,$$

with $k_D=\omega_p/v_{\text{th}}$. For a plasma with cold ions, $T_I<T_e$, the width is narrow, i.e., $\Gamma_a=(\pi/8)^{1/2}(m/M)^{1/2}$. For low $\omega$, Eq. (6) yields

$$g^{(1,2)}(k,\omega)=-VI(k)(E_I E_2/\omega_1\omega_2)e^2 n_0/m^2 v_{\text{th}}^2, \tag{7}$$

with $I(k)$ of the form $v^{-1}\delta(k\pm k_1\pm k_2)\exp[i(\pm\phi_1\pm\phi_2)]$. We now substitute Eqs. (4), (5), and (7) in Eq. (1) and find, with $2\omega_0-\omega_s\approx\omega_s$, that the equation for the conjugate wave $\mathbf{E}_4=\mathbf{E}_c=\hat{\mathbf{e}}_c E_c(\mathbf{r},t)$ is

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t}\right)E_c=-r_0\frac{E_0^2}{\omega_0^2}(\hat{\mathbf{e}}_s\cdot\hat{\mathbf{e}}_0)\left[1-\frac{m}{M}\left(\frac{\omega_p}{\Omega}\right)^2\right]\frac{\omega_p^2}{mv_{\text{th}}^2}\left[\frac{1}{\epsilon_a(q,\Omega)}E_s\exp(i\mathbf{k}_s\mathbf{r}-i\omega_s t-i\phi_s)+\text{c.c.}\right], \tag{8}$$

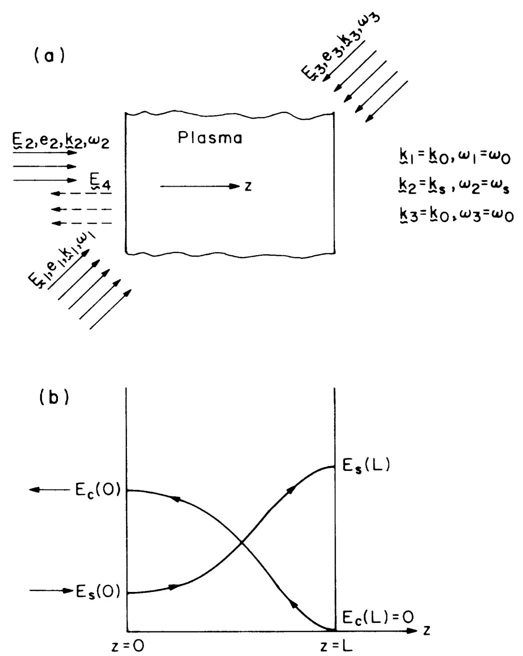


FIG. 1. (a) The geometry of phase conjugation in the plasma. The pump waves 1 and 3 are counterpropagating, and the reflected wave 4 is phase conjugate to the signal wave 2. (b) Schematic generation of the amplified conjugate wave, $E_c$, by the nonlinear interaction of the signal wave, $E_s$, with the pump waves.

where $q=\mathbf{k}_0-\mathbf{k}_s$, $\Omega=\omega_0-\omega_s$, and $r_0=e^2/mc^2$.

Following the procedure of Papper and Yariv (Ref. 1, p. 36) we assume that $E_c$ and $E_s$ vary slowly along the propagation direction $\boldsymbol{\kappa}_s=\hat{\mathbf{z}}\kappa_s$, and that the pump waves are not depleted, and rewrite Eq. (8) as

$$d\bar{E}_c(z)/dz=-i\kappa\bar{E}_s(z) \tag{9}$$

for the complex amplitudes of the waves. Here

$$\kappa=r_0\frac{1}{2k_s}\frac{E_0^2}{\omega_0^2}\frac{m}{M}\left(\frac{\omega_p}{\Omega}\right)^2\frac{\omega_p^2}{mv_{\text{th}}^2}\frac{1}{\epsilon_a(q,\Omega)} \tag{10}$$

plays the role of the gain coefficient of an active medium. If at the entrance to the plasma, $z=0$ [see Fig. 1(b)], the incoming signal $E_s$ is $E_s(0)$, and at $z=L$ no conjugate wave is present, $E_c(L)=0$, we find that the amplitude of the reflected wave is

$$\frac{E_c(0)}{E_s(0)}=i\frac{\kappa}{|\kappa|}\tan|\kappa|L. \tag{11}$$

We can now estimate the amplification parameter $\kappa$. Close to the acoustic resonance we take $|\epsilon_a|\sim\Gamma_a$ and rewrite Eq. (10) up to numerical factors of order 1 as

$$\kappa=r_0\frac{1}{8k_0}\frac{E_0^2}{mc^2}\left(\frac{mc^2}{T_e}\right)^2\left(\frac{\omega_p}{\omega_0}\right)^4\frac{1}{\Gamma_a}. \tag{12}$$

To evaluate $|\kappa|$ we consider the following parameters: $r_0=2.8\times10^{-13}$ cm, $n_0=1.0\times10^{14}$ cm$^{-3}$, $\lambda_0=0.1$ cm, $T_e=5.0$ eV, $E_0^2=1$ erg/cm$^3$, and $(m/M)^{1/2}=0.01$. This yields $k_0=67$ cm$^{-1}$, $E_0^2/mc^2=5\times10^7$ cm$^{-3}$, $mc^2/T_e=10^5$, and $(\omega_p/\omega_0)^2=0.08$. Even if we assume $\Gamma\simeq1$ we find $\kappa\simeq0.04$, and with $L=5$ cm we obtain $|\kappa|L\approx\pi/2$ and the $\tan|\kappa|L$ in Eq. (15) can be very large. We see that there is a wide range of plasma and radiation parameters where high gain can be easily achieved.

We wish to point out that the main enhancement factor comes from the self-consistent electric field associated with the ion acoustic wave, i.e., the term $(m/M)\times(\omega_p/\omega)^2\simeq10^4$ in the "mixed" density. The width of the resonance $\Gamma$ is much less important. This is, in fact, what makes the plasma at acoustic resonance such a strong nonlinear medium.

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