

Forced Organization of Flute-Type Turbulence by Convective Cell Injection

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Nonlinear interactions between flute-type turbulence and an externally excited convective cell in a strongly magnetized plasma are investigated. During the interaction the azimuthal-mode-number spectrum of the turbulence is deformed and a broad spectrum evolves, indicating an inverse cascade. As a result of a modification in phase and amplitude of the fluctuations, an organized structure is created in the turbulence. The macroscopic behavior is well explained by a Van der Pol-type equation.

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The self-organization process in turbulent and linearly unstable systems is one of the most interesting and important phenomena in nonlinear plasma dynamics as well as in fluid dynamics.¹ It has been demonstrated for two-dimensional flows, for instance, that the situation consisting of very many vortices is deformed through a coalescence of these vortices to create ultimately a large-scale structure, indicating an inverse cascade in the turbulence.² In distinction from these spontaneous organizations, we here report a forced organization of turbulence in a plasma, accompanied by an inverse cascade. This process is the result of a nonlinear interaction between spontaneously generated turbulence and an externally excited convective cell.^{3,4}

The experiment is carried out in a linear machine,^{3,5} where a plasma is produced by surface ionization of cesium on a hot tantalum plate of 3 cm diam. The plasma column is confined by an axial magnetic field B_0 (≈ 0.35 T). Typical parameters are as follows: central plasma density $n_0 \approx 10^9$ cm⁻³, and temperatures $T_e \approx T_i \approx 0.2$ eV. Outside the central plasma column, a residual or scrapeoff plasma layer exists with the same temperature but a reduced plasma density.⁵ A convective cell is excited by our applying a positive square-wave pulse of 20- μ s duration to an 8-mm-diam disk which is placed in the residual plasma outside the main plasma column. The repetition rate of the pulses is 50 Hz. Between the pulses the bias of the disk is equal to that of the cold end plate (-15 V).

In the residual plasma, fluctuations always appear, which show many features of a flute-type instability.^{6,7} The driving mechanism for the instability which gives rise to these oscillations was identified as the azimuthal Kelvin-Helmholtz, or velocity shear, instability; see Ref. 6. Fluctuations in floating potential were measured by two probes with exposed spherical platinum tips of 1 mm diam.^{3,4} High-input impedance amplifiers (100 M Ω) with bandwidth 300 kHz were placed in the intermediate vicinity of the probes. Probes and detecting circuits were tested by measurements of grid-excited ion acoustic waves.³ Figure 1(a) shows a typical frequency spectrum of the potential fluctuations. We find that the fluctua-

tion around $f_0 = 5.2$ kHz is associated with an $m = 2$ azimuthal mode number and propagates approximately with the local $\mathbf{E}_0 \times \mathbf{B}_0$ velocity, where \mathbf{E}_0 is the radial electric field at the plasma edge. This result was explicitly confirmed by the measurement of \mathbf{E}_0 . By changing the potential gradient in the residual plasma we may also obtain a broadband fluctuation.⁷ Here we consider mainly the narrow-band case.

In order to investigate the interaction between the background turbulence and a convective cell, we inject a positively polarized cell into the turbulence. The injection takes place at a random phase relative to the spontaneous fluctuations. The potential fluctuations are measured by a probe at different azimuthal positions θ and are eventually averaged by a boxcar averager. Figure 1(b) shows a typical evolution of the averaged potential $\langle \phi \rangle$ with θ as a parameter. Here the radius of the cir-

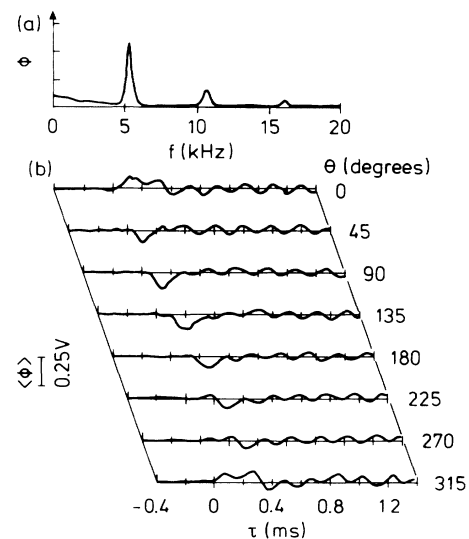


FIG. 1. (a) Frequency spectrum of the spontaneously excited fluctuation. (b) Temporal variation of the averaged potential $\langle \phi \rangle$ at different azimuthal positions θ with $r = 1.8$ cm. A convective cell is injected at $t = 0$. The exciter disk is centered at $\theta = 0$.

cumference for the measurement is $a=1.8$ cm and the exciter disk is placed at $\theta=0$. Before the cell injection, $t < 0$, no structure appears and the value of $\langle \phi \rangle$ is zero. After the cell injection, $t > 0$, however, there appears an organized structure which propagates azimuthally. This organized structure lasts longer than the cell lifetime of approximately $200 \mu\text{s}$ as described in Ref. 3. Its velocity is about 3.3×10^4 cm/s, which is almost the same as the velocity of the background fluctuations.

The effect of the cell on the turbulence was further examined by use of the spontaneous fluctuations as a trigger of the entire measuring system. For changing injection time of the cell within one wave period, the potential fluctuations are measured at the fixed position $\theta=90^\circ$. The results are shown in Fig. 2(a), where arrows indicate the times when the cell is injected. The top trace shows the fluctuation without cell injection. Soon after the injection, the amplitude and the phase of the inherent wave are deformed, but eventually the inherent perturbations reappear and continue again long after the cell decay, but with a phase locked by the injected cell. In order to estimate the phase shift of the fluctuation, the evolution of the peaks of the signals is plotted in Fig. 2(b) with circles. The top trace corresponds to the fluctuation without cell injection. Two parallel lines with narrow spacing show the time, phase and interval of the applied pulse for the cell excitation. Note that the results cannot be explained by a simple linear superposition of the background fluctuations and the injected cell. If all the signals in Fig. 2(a) are shifted in time to have the same cell injection time and added, there would appear a finite signal after the cell injection, $t > 0$, but nothing for $t \leq 0$. This would agree with the observations in Fig. 1(b).

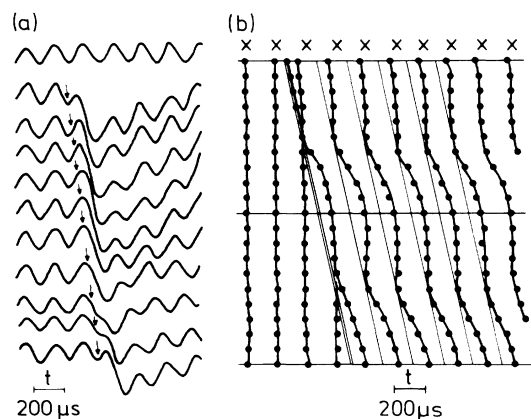


FIG. 2. (a) Temporal variations of potential fluctuations ϕ with cell injection. The arrows show the time when the cell is injected. (b) Variations of time phases of the signals shown in (a). Circles show the peaks of the signals. The size of the circles indicates the accuracy of the determination of the position. Top traces in (a) and (b) correspond to the signal without cell injection.

The spatial potential distribution in the θ direction is analyzed numerically to obtain a mode-number spectrum. Figure 3 shows the evolution of the intensities of each azimuthal component. The $m=2$ mode exists in the plasma before the cell injection at $t < 0$ as discussed previously. The mode grows at first after the injection, but soon afterward it decays, and the lower mode of $m=1$ becomes strong after a time $t \approx 150 \mu\text{s}$. Subsequently, this mode also decays and then the lowest mode of $m=0$ grows and dominates for $190 \mu\text{s} < t < 230 \mu\text{s}$. As the lower modes decay, the initial state with $m=2$ reappears for $t > 450 \mu\text{s}$. The results in Fig. 3 are explained in the following way: The inherent turbulent structure ($m=2$) coalesces with a convective cell to generate a larger spatial structure, i.e., an inverse cascade from the higher mode to the lower mode.^{2,4} This process continues during the cell lifetime, and during this time a very broad spectrum is realized, as shown in Fig. 3 ($t \approx 200 \mu\text{s}$). After the decay of the cell the initial mode reappears. It is important that because of the interaction, the phase and amplitude of the inherent fluctuation are altered and as a result an organized structure is formed.

To explain the phase locking, the following simple model is considered. The narrow-frequency-band oscillations in the plasma appear as a linearly growing mode, with a nonlinear saturation mechanism. A theoretical model based on the Van der Pol equation is well established for such plasma phenomena.^{8,9} In particular, it is proposed by Hioki and Okuda⁹ for conditions very similar to ours. The inherent fluctuation with an amplitude which is saturated before the cell injection in the weakly unstable plasma is nonlinearly coupled with a cell during its lifetime. The cell appears as an external force for the inherent fluctuation. Since the inherent fluctuation and the cell move with essentially the same azimuthal velocity $\mathbf{V}_d = \mathbf{E}_0 \times \mathbf{B}_0 / B_0^2$, this model enables us to use the Van

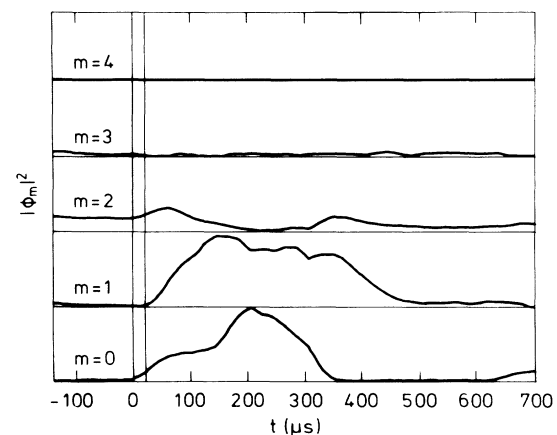


FIG. 3. Variations of intensities $|\phi_m|^2$ of the azimuthal modes with mode numbers m as functions of time t . The two vertical lines show the duration of the applied pulses.

der Pol-type equation with a pulselike forcing term where the time variable t is related to the azimuthal position through $t = a\theta/V_d$:

$$d^2\phi/dt^2 - \epsilon(1 - \beta\phi^2)d\phi/dt + \phi = \phi_{\text{ext}}(t), \quad (1)$$

where ϕ is the fluctuation amplitude normalized with the saturated amplitude ϕ_0 , t is the time normalized with $\Omega_0 = 2\pi f_0 = 2\pi/T_0$, ϵ is the linear growth rate normalized with Ω_0 , and β is the coefficient of the damping term which is assumed to be proportional to the wave power, i.e., ϕ^2 . In the case without external force $\phi_{\text{ext}}(t)$, the condition for a saturated amplitude of $\phi_0 = 1$ gives $\beta = 4$. To simulate the external forcing term we use

$$\phi_{\text{ext}}(t) = \phi_{\text{ext}0} \exp\{- (t - t_0)^2 / (\delta t)^2\}, \quad (2)$$

where $\phi_{\text{ext}0}$ is the amplitude normalized with ϕ_0 , t_0 is the normalized time when the cell grows up to the maximum amplitude, and δt is the effective growth and decay time of the cell.

Measurements⁷ of the correlation function $R = \langle \phi(\theta = 0)\phi(\theta) \rangle$ between two probes at $r = a$ as a function of θ show that the correlation length of the fluctuation is approximately $L \approx 2\pi a$. With the assumption of a convective

tively growing instability, the growth rate can be estimated as

$$1/\epsilon = \Omega_0(L/V_d) = \Omega_0 L / (\Omega_0 a / m) \approx 4\pi,$$

where $m = 2$. Then $\epsilon \approx 0.08$. All the other constants in Eq. (2) are obtained from Fig. 2(a), i.e., $\phi_{\text{ext}0} \approx 5$, and $\delta t = \Omega_0 t_1 \approx \pi$, since the growth time of the cell, t_1 , is about $100 \mu\text{s}$ and the wave period $T_0 \approx 200 \mu\text{s}$.

Figure 4(a) shows a wave form obtained numerically from Eqs. (1) and (2) with parameters $\epsilon = 0.1$, $\phi_{\text{ext}0} = 5$, while $1/(\delta t)^2 = 0.1$, and $t_0 = 20$. Here we obtain a signal similar to the result in Fig. 2(a); see, for instance, trace No. 4 or 5 from the bottom. The inherent saturated fluctuation is strongly modified in amplitude and in phase when the external force becomes dominant at around $t = 20$ and the inherent mode reappears after the disappearance of the external force. The important point is that the phases of the oscillations remain shifted also for times after the decay of the external pulse. In order to investigate the effect of varying t_0 on the phase shift of the rearranged fluctuation, we change t_0 within one period as $t_0 = 20 + j(T_0/5)$, where $j = 1, 2, \dots, 6$. The results for $\phi_{\text{ext}0} = 5$ are shown in Fig. 4(b) by filled circles which correspond to the times of peaks of the signals. The arrows show t_0 . In the top trace the peaks for the case without external force are plotted. Here we clearly find that the temporal position of the peaks follows t_0 . When $\phi_{\text{ext}0}$ is decreased to 1, the phase shift becomes small and also depends on the time t_0 , as shown by open circles. However, in both cases we obtain results similar to those shown in Fig. 2(b).

In the experimental results in Fig. 1(b), the cell is injected at a random phase relative to the background fluctuations. This might be realized if the signals in Fig. 4(b) are shifted in time to get the same injection time t_0 and then averaged. Figure 4(c) shows the signal thus obtained. Here, we have $t_0 = 20$. For $t > t_0$ the phase of the random fluctuation is locked and an organized structure evolves clearly. For $t < t_0$, on the other hand, all the phases are mixed and cancel out completely. These results agree well with those in Fig. 1(b).

In conclusion, we have observed the forced organization of the flute-type turbulence by convective-cell injection. The bulk-mode evolution follows the Van der Pol-type equation with a single pulse forcing. Also the microscopic behavior was clarified. That is, the energy of the inherent mode is transferred to a lower mode number through the interaction with the externally injected cells. This interaction thus takes the form of an inverse cascade. This might also correspond to a quenching of the initial spatial mode of the turbulence. Since these phenomena occur within the lifetime of a cell, the turbulence is rearranged in time and as a result an organized structure appears in the turbulence. Similar results may appear also in the broad-band case. This investigation is in progress.

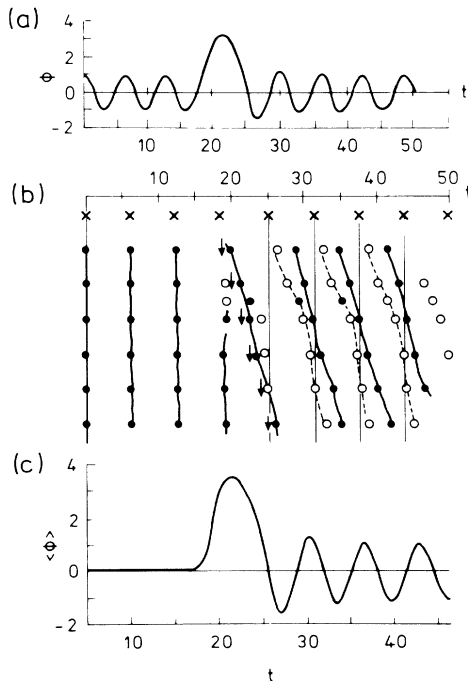


FIG. 4. (a) Numerical results from Eqs. (1) and (2). $\phi_{\text{ext}0} = 5$ and $t_0 = 20$. (b) Time phases of peaks of the signals. t_0 is varied as $t_0 = 20 + j(T_0/5)$, where $j = 1, 2, \dots, 6$, from the second trace to the lowest one as shown by the arrows. Top trace shows the phase without external force. Filled and open circles correspond to $\phi_{\text{ext}0} = 5$ and 1, respectively. (c) Averaged signal obtained by our adding the data in (b) after shifting the signals in time to obtain the same injection time $t_0 = 20$. In all cases $\epsilon = 0.1$ and $\beta = 4$.

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