Quantum Noise Reduction via Maser Memory Effects: Theory and Applications

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In many high-precision laser measurements spontaneous-emission quantum noise determines the ultimate sensitivity. We here demonstrate that when the experimental circumstances restrict us to short measurement times, the spontaneous-emission noise can be substantially reduced by the utilization of long-lived atoms as the masing medium.

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The search for ways to reduce quantum noise and thus supersede the accepted quantum "limits" to measurement is one of the most active areas of modern quantum optics. In different forms these considerations involve the study of, for example, squeezed states,[1] back-action-evading experiments,[2] the correlated spontaneous-emission laser,[3] etc. In this paper we present another means of suppressing spontaneous-emission quantum noise via atomic memory effects[4] in lasers and masers as discussed below. Devices presently limited by such spontaneous-emission noise include the laser gravity-wave detector[5] and the laser gyroscope.[6] We here demonstrate, for the first time, that when the experimental requirements restrict us to short measurement times, then spontaneous-emission noise can be substantially reduced by use of long-lived atoms as the lasing medium. Such short measurement times may be due to the short duration of the signal, as in a gravity-wave burst (Fig. 1).

In the following we will first attempt to motivate the present studies and review the usual treatment of phase diffusion in lasers which leads to the famous Schawlow-Townes result. We then compare this to our new result for phase diffusion which takes memory effects of the lasing atoms into account. A sketch of the derivation of the present result is then given emphasizing the main ideas. Finally, we discuss the applications of our findings to problems of the type stated above, i.e., the reduction of quantum noise in, for example, a laser gravity-wave (g-wave) detector.

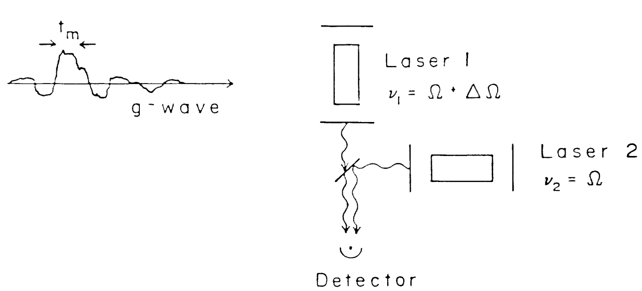

FIG. 1. Active gravity-wave detector with a short gravity wave which changes cavity frequency of laser 1. Measurement time is limited by duration t_m of the pulse.

To put the present discussion in proper perspective we recall that in an active laser interferometer the primary source of noise can be visualized as the electric vector's experiencing small impulses due to spontaneous emission which contribute a random variation to the laser phase. This leads to a gradual diffusion of the phase. In the conventional setup the atomic decay time is much smaller than the time of measurement t_m. Then the small changes in the field vector due to the spontaneous-emission noise take place on a far smaller time scale than the total evolution of the field, i.e., one considers the spontaneous-emission impulses to be δ-function-like [Fig. 2(a)]. Using such a approximation one derives[7] an expression for phase diffusion in the mean electric field,

$$\langle E(t)\rangle = E_0\langle e^{i\phi}\rangle = E_0 \exp[-\tfrac{1}{2}\langle\phi^2(t)\rangle] = E_0\exp[-\tfrac{1}{2}D_{ST}(t)]. \quad (1)$$

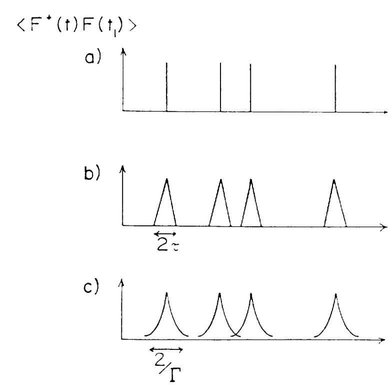

FIG. 2. Pictorial representation of spontaneous-emission noise correlation function. For atoms injected at random times t_i, the correlation is given by (a) δ functions in the usual Schawlow-Townes limit, (b) triangle functions in the case of a maser with memory, and (c) symmetric exponential functions in the case of a laser with memory.

Here $D_{ST}(t)$ is the usual Schawlow-Townes[8] phase-diffusion parameter

$$D_{ST}(t)=(\alpha/2\bar{n})t, \tag{2}$$

where α is the linear gain which goes as $2r_a(g/\Gamma)^2$ and $\bar{n}$ is the average photon number; g is the coupling constant between atoms and field and Γ denotes the atomic decay from the upper atomic level, whereas r_a specifies the atomic excitation rate. It can be readily seen from Eq. (1) that phase fluctuations go as

$$\Delta\phi_{ST}^2=\alpha t_m/2\bar{n}. \tag{3}$$

As discussed earlier, we are here considering the situation in which the individual atomic-decay events take place on a time scale short compared to the measurement time t_m. One is thus naturally led to ask, "What would happen if my measurement were carried out in a time interval which is short compared to the atomic emission times, i.e., $t_m \ll \Gamma^{-1}$?" Then the small changes in the electric field vector due to the spontaneous-emission impulses can no longer be treated as abrupt events but have a finite correlation time. Such a situation is not inconceivable. There are many atoms and molecules which exhibit extraordinary long lifetimes; for example, $O_2(^1\Delta)$ has a lifetime of $\simeq 40$ min.[9] Hence the usual adiabatic elimination of the atomic variables, as used in the derivation of Eq. (1), no longer holds.

In fact some experiments are restricted to rather short times of measurement. For example, in a gravity-wave detector one may well be interested in measuring the influence of a g-wave burst of short duration t_g; see Fig. 1. Thus the time of measurement is limited by t_g which could be short compared to the decay time of the lasing atoms.

To sum up: When an individual atom does not have time to decay spontaneously during t_m, one intuitively expects that the accumulated phase error should be reduced. In the following we show that this is indeed the case.

Motivated by the above considerations we next present the analysis of this phase-diffusion problem when atomic memory is important and obtain the mean square phase variation via an appropriate quantum Langevin analysis. In particular, we consider the simple case of maserlike activity in which atoms (whose frequency equals that of the cavity) are injected at times t_i and removed from the cavity at later times $t_i+\tau$. This enables us to carry through the problem with no unnecessary technical difficulties. We begin with the Hamiltonian for the present problem given by

$$H=H_{\text{field}}+H_{\text{atom}}+g\hbar\textstyle\sum_i\{\hat{\sigma}^i\hat{a}^\dagger N(t_i,t,\tau)+\text{adj.}\}, \tag{4}$$

where H_{field} and H_{atom} are the Hamiltonians for the field and atoms, respectively, g is the atom-field coupling constant and σ_i is the lowering operator for the i-th atom. The operators a and $a^\dagger$ represent the usual annihilation and creation operators and $N(t_i,t,\tau)$ is a function which represents the injection of an atom at time t_i and its removal at a time $t_i+\tau$ later. In this sense $N(t_i,t,\tau)$ is a notch function which has the value

$$N(t_i,t,\tau)=\begin{cases}1 \text{ for } t_i\le t<\tau,\\ 0 \text{ otherwise.}\end{cases} \tag{5}$$

Using this Hamiltonian we write the equations for the atom field operators in the interaction picture as[10]

$$\begin{aligned}\dot{\hat{a}}&=-ig\textstyle\sum_i\hat{\sigma}^i(t)N(t_i,t,\tau)-\frac{1}{2}\gamma\hat{a}(t)+\hat{F}_\gamma(t),\\ \dot{\hat{\sigma}}^i&=igN(t_i,t,\tau)\hat{\sigma}_z^i\hat{a}(t),\end{aligned} \tag{6}$$

where the effects of cavity damping are determined by the cavity decay rate γ and the associated Langevin noise source $\hat{F}_\gamma$. Integrating the equation for the atom operator and substituting it into the field operator equations we obtain

$$\dot{\hat{a}}(t)=\int_{-\infty}^{t}dt'\alpha(t,t')\hat{a}(t')-\tfrac{1}{2}\gamma\hat{a}+\hat{F}_a(t)+\hat{F}_\gamma(t), \tag{7}$$

where

$$\alpha(t,t')=g^2\textstyle\sum_iN(t_i,t,\tau)N(t_i,t',\tau)\hat{\sigma}_z^i(t'), \tag{8a}$$

$$\hat{F}_a(t)=-ig\textstyle\sum_iN(t_i,t,\tau)\sigma^i(t_i). \tag{8b}$$

In the above the noise operator (8b) may be seen to have the moments

$$\langle\hat{F}_a(t)\rangle=0, \tag{9a}$$

$$\langle\hat{F}_a^\dagger(t)\hat{F}_a(t')\rangle =g^2\textstyle\sum_{ij}N(t_i,t,\tau)N(t_j,t',\tau)\langle\hat{\sigma}^{\dagger i}(t_i)\hat{\sigma}^j(t_j)\rangle. \tag{9b}$$

Because we are injecting our lasing atoms in the upper state, the atomic average is given by $\langle\hat{\sigma}^{\dagger i}(t_i)\hat{\sigma}^j(t_j)\rangle=\delta_{ij}$. After replacing the sum upon i in (9b) by an integration over injection times t_i we find

$$\langle\hat{F}_a^\dagger(t)\hat{F}_a(t')\rangle=g^2r_a\left\{N(t'-\tau,t,\tau)[t-(t'-\tau)]-N(t',t,\tau)[t-(t'+\tau)]\right\}, \tag{10}$$

where r_a is the atomic injection rate. The phase variance can than be calculated through the noise operator product[11]

$$\langle\phi^2(t)\rangle=-\frac{1}{2\bar{n}}\int_0^t dt'\int_0^t dt''\langle\hat{F}^\dagger(t')\hat{F}(t'')e^{i[\phi(t')-\phi(t'')]}\rangle. \tag{11}$$

On insertion of (10) into (11), the expression for the generalized maser diffusion coefficient D is found to be

$$D(t)=(\alpha/2\bar{n})[(t^2/\tau - t^3/3\tau^2)\theta(\tau-t)+(t-\tfrac{1}{3}\tau)\theta(t-\tau)],\quad \text{maser.} \tag{12}$$

In the laser case involving atoms which are injected at random times t_i but which decay via spontaneous emission to far-removed ground states at a rate Γ, a similar but more complicated analysis has been carried out and will be published elsewhere.[12] The result in this case is given by

$$D(t)=(\alpha/2\bar{n})\left[t+\Gamma^{-1}(e^{-\Gamma t}-1)\right],\quad \text{laser.} \tag{13}$$

These different cases are summarized in Figs. 2(b) and 2(c). In both[13] of the above cases we find that for times $t=t_m$ small compared to the atomic lifetime the phase diffusion is quadratic in the measurement time t_m; that is, we now have a phase error which goes as

$$\Delta\phi^2=(\alpha t_m/2\bar{n})(\tfrac{1}{2}\Gamma t_m). \tag{14}$$

Therefore we see that the quantum noise due to spontaneous emission is reduced from that given by Eq. (3) by the factor $\frac{1}{2}\Gamma t_m$ which can be a significant reduction for short measurement times. For times long compared to the atomic lifetime, however, the Schawlow-Townes result is obtained from both (12) and (13) as expected.

We next apply our results to the problem of gravity-wave detection using an active interferometer system as in Fig. 1. In the figure we see that a g-wave of strength h_0 incident upon the Fabry-Perot cavity causes the resonant frequency of cavity 1 to be shifted relative to cavity 2 by an amount $\Delta\nu=\nu h_0$. Since h_0 is a number $<10^{-20}$, the frequency difference νh_0 is typically much smaller than 1 Hz. We therefore look not for a frequency difference in our heterodyne experiment, but rather for a signal phase difference[14] given by $\Delta\phi_s=\Delta\nu t_m=\nu h_0 t_m$. Here t_m is the time of measurement which is set by the g-wave pulse. During t_m additional random noise will be generated because of spontaneous-emission events in both lasers. In the usual case, where the noise is given by the Schawlow-Townes diffusion parameters, we find the related phase error $\Delta\phi_e$ between the two lasers from Eqs. (2) and (3) to be

$$\Delta\phi_e=\sqrt{2}\sqrt{D_{\rm ST}}$$
$$=(\gamma t_m/\bar{n})^{1/2}=\gamma t_m(\hbar\nu/Pt_m)^{1/2}, \tag{15}$$

where P is the power emitted by the lasers and is related to the average photon number in the cavity by $\bar{n}=P/\gamma\hbar\nu$. In Eq. (15) we have made use of the fact that the gain coefficient α is roughly equal to the cavity damping γ. Equating the signal $\Delta\phi_s$ to the noise $\Delta\phi_e$ and solving for the minimum detectable gravity-wave strength we find the minimum detectable g-wave strength in the Schawlow-Townes limit to be

$$h_{\rm ST}=(\gamma/\nu)(\hbar\nu/Pt_m)^{1/2}. \tag{16}$$

Proceeding to consider the case of current interest, namely the situation wherein the measurement time is small compared to the atomic lifetime, we now find the accumulated phase error during our measurement time t_m from Eq. (14) to be

$$\Delta\phi_e=[\gamma t_m(\hbar\nu/Pt_m)^{1/2}](\tfrac{1}{2}\Gamma t_m)^{1/2}. \tag{17}$$

Equating the noise expression (14) to the signal (15) we find the minimum detectable gravity wave to be given now by

$$h_{\rm min}=h_{\rm ST}(\tfrac{1}{2}\Gamma t_m)^{1/2}; \tag{18}$$

thus we see that our sensitivity is enhanced by the factor $(\frac{1}{2}\Gamma t_m)^{1/2}$.

It might be thought that by making Γ as small as possible we would achieve a better and better sensitivity; however, this is not necessarily the case. To see this, recall[7] that the operating frequency of the laser ν is a compromise between the atomic frequency ω and the bare-cavity resonant frequency Ω, given by

$$\nu=(\gamma\omega+\Gamma\Omega)/(\gamma+\Gamma), \tag{19}$$

and for the case of the laser gravity-wave detector of Fig. 1 in which the atomic frequency difference between lasers 1 and 2 is zero, we have

$$\Delta\nu=[\Gamma/(\gamma+\Gamma)]\Delta\Omega=[\Gamma/(\gamma+\Gamma)]\nu h_0. \tag{20}$$

Hence we see that in the general case the minimal detectable gravity wave is given by

$$h_{\rm min}=h_{\rm ST}[(\gamma+\Gamma)/\Gamma](\tfrac{1}{2}\Gamma t_m)^{1/2} \tag{21}$$

For example, if we consider a g-wave of duration 10^{-5} sec, atomic lifetimes 10^{-3} sec, and cavity lifetime of 5×10^{-3} sec we find by applying Eq. (21) that the sensitivity of our g-wave detector is enhanced by an order of magnitude.

In conclusion, the effects of atomic memory can lead to suppression of spontaneous-emission quantum noise and an enhancement of system sensitivity. More detailed analysis of potential laboratory experiments will be presented elsewhere.

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[13]We note that the atomic lifetime in Fig. 2(b) is related to that of Fig. 2(c) by $\tau = \Gamma^{-1}$.

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