High-Precision Anisotropy Measurement of the Lamb Shift in He^+

G. W. F. Drake, J. Patel, and A. van Wijngaarden
Department of Physics, University of Windsor, Windsor, Ontario, Canada N9B 3P4
(Received 4 January 1988)

A high-precision measurement of the $2s_{1/2}$-$2p_{1/2}$ Lamb shift in He^+ is obtained by means of the quenching anisotropy method. The measured value of 14042.22 ± 0.35 MHz is in excellent agreement with Mohr's electron self-energy calculation, but disagrees with Erickson's by more than 8 standard deviations. The accuracy of the measurement provides the most stringent available test of the order-$\alpha(Z\alpha)^6mc^2$ contributions to the calculated electron self-energy.

PACS numbers: 42.50.Wm, 32.70.Fw

In previous papers,[1] we have developed a quenching anisotropy method of measuring the $2s_{1/2}$-$2p_{1/2}$ Lamb shift in one-electron atoms. An important advantage of the method is that the accuracy is not limited by the large width of the $2p$ state relative to the Lamb shift. The present Letter reports the results of a refined series of measurements which improve the accuracy for He^+ to ± 25 ppm (parts per million). The new result is a factor of 4 improvement over previous measurements in He^+,[1-3] and it now provides the most stringent test of the electron self-energy contributions to the term $G(Z\alpha)$ below in the Lamb shift. If the Lamb shift $\mathcal{L}$ is expanded in powers of $Z\alpha$ and α in the form

$$\mathcal{L} = [\alpha(Z\alpha)^4mc^2/6\pi]\{A_{40}+A_{41}\ln(Z\alpha)^{-2}+A_{50}Z\alpha+(Z\alpha)^2[A_{62}\ln^2(Z\alpha)^{-2}+A_{61}\ln(Z\alpha)^{-2}+G(Z\alpha)] + (\alpha/\pi)[B_{40}+O(Z\alpha)]+O(\alpha^2/\pi^2)\} + \text{finite nuclear mass and size corrections}, \quad (1)$$

then the primary theoretical uncertainty comes from $G(Z\alpha)$, which represents the sum of all terms of order $\alpha(Z\alpha)^6mc^2$ and higher. Calculations of this quantity by Erickson[4] and by Mohr[5] and Sapirstein[6] differ by about 30%, which is many times larger than the estimated $\pm 4\%$ uncertainty of their calculations. Since the other terms in (1) are well established,[4,5] the present measurement determines $G(Z\alpha)$ to an accuracy of $\pm 3\%$. This is better by about a factor of 2 than previous resonance-type measurements in hydrogen or heavier ions. An advantage of our studying He^+ is that the nuclear radius is known to exceptionally high precision from muonic fine-structure measurements,[7] thereby removing this as a source of uncertainty in the interpretation of the experiment.[8]

The experimental details of the anisotropy method have recently been described,[1] and are only briefly summarized here. The anisotropy of the Ly-α quenching radiation emitted by a hydrogenic ion in the metastable $2s\,^2S_{1/2}$ state in the presence of a weak electrostatic field is defined by $R=(I_\parallel - I_\perp)/(I_\parallel + I_\perp)$, where $I_\parallel$ and $I_\perp$ are the intensities parallel and perpendicular to the electric field direction. A measurement of R determines the Lamb shift because in the lowest-order weak-field limit

$$R^{(0)} = \frac{-3\,\mathrm{Re}(\rho) - 3|\rho|^2/2}{2-\mathrm{Re}(\rho)+7|\rho|^2/2}, \quad (2)$$

with

$$\rho = \frac{E(2s_{1/2}) - E(2p_{1/2}) + i\Gamma/2}{E(2S_{1/2}) - E(2p_{3/2}) + i\Gamma/2}. \quad (3)$$

The presence of the $2p$-level width Γ in (3) changes $R^{(0)}$ by only 493 ppm and so an approximate nonrelativistic value is sufficiently accurate. The small corrections to $R^{(0)}$ required to interpret anisotropy measurement at the 1-ppm level of accuracy are summarized in Table I.

TABLE I. Systematic and higher-order corrections used to obtain the lowest-order anisotropy $R^{(0)}$ and the Lamb shift $\mathcal{L}$ from R_{exp}.

Quantity	Value[a]
Measured anisotropy R_{exp}	0.118030142(2791)
Detector nonlinearity	0.000000000(354)
$2E1$ two-photon decay	0.000002012(236)
Finite solid angle of detectors and deflections of ion beam	0.000150115(259)
Relativistic angular shift	0.000006980(56)
11.8-G Zeeman splitting	0.000000230(2)
$\mathbf{v}\times\mathbf{B}$ electric field[b]	0.000000125(1)
Finite quenching field effects	−0.000232222(60)
Magnetic quadrupole transitions	0.000007715
Mixing with higher np states and final-state perturbations	0.000002796
$(\alpha Z)^2$ terms in matrix elements	−0.000000755
$R^{(0)}$ (sum of above)	0.117967137(2836)
$E(2p_{3/2}) - E(2p_{1/2})$	175593.55(3) MHz
Γ_{2p}	$1.00307\times10^{10}\ \mathrm{s}^{-1}$
$\mathcal{L}$ [from Eqs. (2) and (3)]	14042.220(349) MHz

[a]Numbers in parentheses indicate the uncertainties in the final figures quoted.
[b]This term arises from the transverse deflection of the ion beam in an axial magnetic field.

 © 1988 The American Physical Society

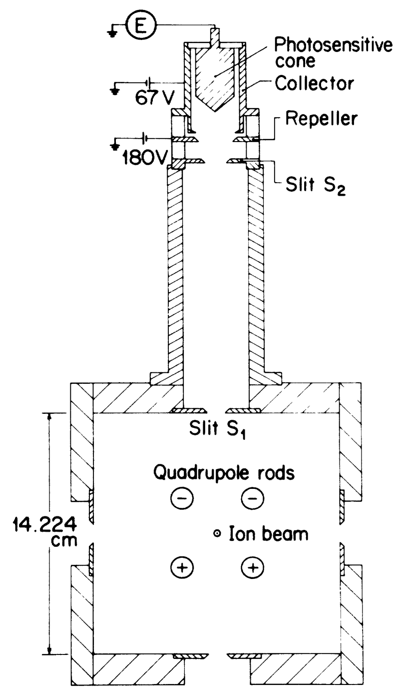

FIG. 1. Diagram of the apparatus showing one of four identical symmetrically placed photon-detection systems.

Figure 1 shows a cross section of the observation cell drawn to scale, with one of four symmetrically placed photon-detection systems in the upper part of the diagram. A 125-keV He^{+}-ion beam ($\simeq 5\ \mu A$) containing about 0.5% metastables travels out of the page in the electrostatic field produced by the quadrupole rods. The 304-Å quenching radiation which passes through the slits S_1 and S_2 produces a photoelectron current of about 10^{-13} A when it strikes the MgF_2-coated cones at the end. The current is measured with an electrometer, E (Keithley model 642 LNFA), and the amplified output digitized by a voltmeter (Hewlett Packard model HP 3457 A). We have shown the detection system to be linear to within 1 ppm over the range of currents required for the experiment. It is this linearity at high photon fluxes which makes possible a dramatic improvement in accuracy over what we earlier achieved[9] by standard photon-counting techniques. Secondary charged and neutral particles are prevented from reaching the cones by repeller plates and a thin Al film covering S_2. Also, an axial magnetic field of 11.8 G is applied to the observation cell to deflect charged particles accelerated toward the detectors by the transverse electric quenching field. A second identical detection system is mounted 3.048 cm further downstream so that the quenching radiation is monitored by eight detectors in all. This improves the combined ratio of signal to dark current ($\simeq 1000$) by nearly a factor of 2 over our earlier work. On our rotating the electric field in steps of 90° by switching potentials on the rods, the intensity ratios $I_{\parallel}/I_{\perp}$ can be combined in such a way that the relative sensitivities of the detectors cancel out.

In our previous work,[1] we defined the background noise to be the signal still observed in the absence of a quenching field. Since this definition contains a correc-

TABLE II. Comparison of theory and experiment for the total Lamb shift, and the derived electron self-energy part G_{SE} of the term $G(Z\alpha)$ in Eq. (1). R_N is the nuclear radius used.

Ion	R_N (fm)	$\mathcal{L}_{expt}$	$\mathcal{L}_{theor}$	$(G_{SE})_{expt}$ [a]	$(G_{SE})_{theor}$ [b]	$(G_{VP})_{theor}$ [c]
^{1}H	0.862(20)	1057.845(9)[d]	1057.869(11) MHz	$-26.71\pm1.25\pm0.93$	−23.4(1.2)	−0.517(20)
^{4}He	1.673(1)	14042.22(35)[e]	14042.26(50) MHz	$-22.99\pm0.76\pm0.03$	−22.9(1.0)	−0.508(17)
^{6}Li	2.56(5)	627615(21)[f]	62737(6) MHz	$-17.17\pm4.0\pm0.77$	−22.49(88)	−0.500(16)
^{16}O	2.711(14)	2192(15)[g]	2196.14(21) GHz	$-22.91\pm7.9\pm0.03$	−20.72(45)	−0.473(12)
		2215.6(7.5)[h]		$-10.45\pm4.0\pm0.03$		
		2203(11)[i]		$-17.26\pm5.8\pm0.03$		
^{19}F	2.900(15)	3339(35)[j]	3343.0(1.8) GHz	$-21.45\pm9.0\pm0.03$	−20.42(39)	−0.469(7)
^{31}P	3.197(5)	20.13(20)[k]	20.25(1) THz	$-20.32\pm2.4\pm0.004$	−18.81(14)	−0.449(2)
^{35}Cl	3.335(18)	31.19(22)[l]	31.34(2) THz	$-19.22\pm1.3\pm0.01$	−18.34(8)	−0.444(1)
^{40}A	3.428(8)	37.89(38)[m]	38.24(2) THz	$-19.55\pm1.6\pm0.005$	−18.11(6)	−0.442(1)
^{238}U	5.751(50)	70.4(8.3)[n]	75.3(4) eV	$-8.32\pm0.46\pm0.02$	−8.050(4)	−0.600(8)

[a]The first uncertainty listed is due to the experimental uncertainty in $\mathcal{L}$, and the second to the nuclear radius uncertainty. Nuclear size corrections (Ref. 13) to the self-energy and vacuum polarization terms have been subtracted for the high-Z ions.

[b]Reference 5.

[c]Includes Uehling (Ref. 5) and Wichmann-Kroll (Ref. 13) vacuum polarization contributions. The total $G(Z\alpha)$ for a point nucleus is $G_{SE}+G_{VP}$.

[d]Reference 15.

[e]Present work.

[f]Reference 16.

[g]Reference 17.

[h]Reference 18.

[i]Reference 19.

[j]Reference 20.

[k]Reference 21.

[l]Reference 22.

[m]Reference 23.

[n]Reference 24.

tion which increases linearly with residual gas pressure, we now define the noise to be the normal quenching signal which remains when the $2s_{1/2}$ ions are removed from the beam by prequenching. This definition must be corrected for a small spontaneous $2E1$ two-photon decay component in the original quenching signal. Since the $2E1$ decay rate of $8.23Z^6$ s^{-1} is smaller by a factor of 4.74×10^4 at our quenching field of 631 V/cm, the correction can be estimated to the necessary $\pm10\%$ precision from the known $2E1$ spectral distribution[10] and the relative MgF_2 photoelectron yield.[11] The correction is listed in Table I, along with other small systematic effects which have been discussed previously.[1]

Additional radiative corrections to the Bethe-Lamb quenching theory used to interpret the experiment are discussed by Lévy.[12] Corrections of $O(\alpha^3Z^2)$ to the transition-matrix elements are j-independent multiplying factors in the nonrelativistic electric dipole approximation which cancel out. The largest effect appears to be an anomalous-magnetic-moment correction to the $2s_{1/2}$-$2p_{3/2}$-$1s_{1/2}$ magnetic quadrupole transition amplitude, which is negligibly small.

The average anisotropy obtained from a total of 9930 individual measurements is $R_{\rm exp}^{(0)}=0.118\,030\,14 \pm 0.000\,002\,79$, where the error is the statistical scatter in the results. Since this is the largest source of uncertainty, further improvements in the precision are evidently possible. After we allow for the small corrections and systematic effects shown in Table I, the above reduces to a lowest-order anisotropy of $R_{\rm exp}^{(0)}=0.117\,967\,14 \pm 0.000\,002\,83$. The corresponding Lamb shift from (2) and (3) is $14\,042.220\pm0.35$ MHz. The additional error arising from the ±0.03-MHz uncertainty in the theoretical $2p_{1/2}$-$2p_{3/2}$ energy splitting of 175 593.55 MHz needed in (3) is only ±0.002 MHz. Our result is consistent with the older resonance measurement of Lipworth and Novick[2] ($14\,040.2\pm1.8$ MHz), but disagrees with the more recent value of Narasimham and Strombotne[3] ($14\,046.2\pm1.2$ MHz).

With the use of a nuclear radius of 1.673 ± 0.001 fm,[7] our recalculated Lamb shift is $14\,042.26\pm0.50$ MHz, in excellent agreement with experiment. The uncertainty comes almost entirely from[5] $G_{\rm SE}(Z\alpha)=-22.9\pm1.0$ in (1). The calculation includes all terms evaluated by Johnson and Soff,[13] as well as all the new $\alpha(Z\alpha)^5(m/M)mc^2$ relativistic recoil corrections (beyond reduced-mass corrections) recently calculated by Bhatt and Grotch.[14] These terms decrease $\mathcal{L}$ by 0.030 MHz.

Since the primary theoretical uncertainty in $\mathcal{L}$ comes from the self-energy part $G_{\rm SE}$ of $G(Z\alpha)$, it is of interest to take the other terms in (1) as correct and extract an experimental value for $G_{\rm SE}$. This is done in Table II for all the ions where high-precision measurements are available. (Not included in the table is the work of Sokolov and co-workers,[25,26] who effectively measure the ratio $\mathcal{L}/\Gamma$ to very high precision.) The present result for He^+ of $G_{\rm SE}=-22.99\pm0.76$ provides the most stringent test of theory and is in excellent agreement with Mohr's calculation. However, it differs by more than 8 standard deviations from Erickson's value of -16.9. This clear disagreement confirms the trend already evident for the higher-Z ions ^{31}P, ^{35}Cl, ^{40}A, and ^{238}U in Table II, all of which favor Mohr's values.

In view of the close agreement obtained in the present work for ^{4}He, it is difficult to understand the apparent discrepancies with Mohr's calculation for ^{1}H found by Lundeen and Pipkin[15] and Sokolov and co-workers.[25,26] Since the value of the proton radius is a major source of uncertainty,[15] a remeasurement of this quantity is clearly necessary.

Research support by the Natural Sciences and Research Council of Canada is gratefully acknowledged.

[1]J. Patel, A. van Wijngaarden, and G. W. F. Drake, Phys. Rev. A **36**, 5130 (1987), and earlier references therein.

[2]E. Lipworth and R. Novick, Phys. Rev. **108**, 1434 (1957).

[3]M. Narasimham and R. Strombotne, Phys. Rev. A **4**, 14 (1971).

[4]G. W. Erickson, J. Chem. Phys. Ref. Data **6**, 831 (1977).

[5]P. J. Mohr, Phys. Rev. A **26**, 2338 (1982), and in *Beam Foil Spectroscopy,* edited by I. A. Sellin and D. J. Pegg (Plenum, New York, 1976), p. 89.

[6]J. Sapirstein, Phys. Rev. Lett. **47**, 1723 (1981).

[7]E. Borie and G. A. Rinker, Phys. Rev. A **18**, 324 (1978).

[8]Recent work has failed to confirm evidence for metastable muonic He^+ at high pressures. See, for example, M. Eckhause, P. Guss, D. Joyce, J. R. Kane, R. T. Siegel, W. Vulcan, R. E. Welsh, and R. Whyley, Phys. Rev. A **33**, 1743 (1986).

[9]G. W. F. Drake, S. P. Goldman, and A. van Wijngaarden, Phys. Rev. A **20**, 1299 (1979).

[10]S. P. Goldman and G. W. F. Drake, Phys. Rev. A **24**, 183 (1981).

[11]L. B. Lapson and J. G. Timothy, Appl. Opt. **12**, 388 (1973), and **15**, 1218 (1976).

[12]J. P. Lévy, Phys. Rev. A **29**, 3189 (1984).

[13]W. R. Johnson and G. Soff, At. Data Nucl. Data Tables **33**, 405 (1985).

[14]G. C. Bhatt and H. Grotch, Ann. Phys. (N.Y.) **178**, 1 (1987).

[15]S. R. Lundeen and F. M. Pipkin, Metrologia **22**, 9 (1986).

[16]M. Leventhal, Phys. Rev. A **11**, 427 (1975).

[17]B. Curnette, C. L. Cocke, and R. D. DuBois, Nucl. Instrum. Methods **202**, 119 (1981).

[18]G. P. Lawrence, C. Y. Fan, and S. Bashkin, Phys. Rev. Lett. **28**, 1612 (1972).

[19]M. Leventhal, D. E. Murnick, and H. W. Kugel, Phys. Rev. Lett. **28**, 1609 (1972).

[20]H. W. Kugel, M. Leventhal, D. E. Murnick, C. K. N. Patel, and O. R. Wood, II, Phys. Rev. Lett. **35**, 647 (1975).

[21]P. Pellegrin, Y. El Masri, and L. Palffy, Phys. Rev. A **31**, 5 (1985).

[22]D. R. Wood, II, C. K. N. Patel, D. E. Murnick, E. T. Nelson, M. Leventhal, H. W. Kugel, and Y. Niv, Phys. Rev. Lett. **48**, 398 (1982).

[23]H. Gould and R. Marrus, Phys. Rev. A **28**, 2001 (1983).

[24]C. T. Munger and H. Gould, Phys. Rev. Lett. **57**, 2927 (1986).

[25]Yu. L. Sokolov and V. P. Yakovlev, Zh. Eksp. Teor. Fiz. **83**, 15 (1982) [Sov. Phys. JETP **56**, 7 (1982)]; V. G. Palchikov, Yu. L. Sokolov, and V. P. Yakovlev, Lett. J. Tech. Phys. **38**, 347 (1983).

[26]If the theoretical value for Γ used by Sokolov and coworkers in Ref. 25 is correct, then their Lamb-shift measurement in H of 1057.8514(19) MHz would correspond to $G_{SE} = -25.82 \pm 0.26$.