neutron scattering are clearly indicated. The potentially useful anomalous scatterers include Li^6, B^{10}, Cd, and a number of examples in the rare earth and actinide series. Experiments are under way to investigate Li^6 and B^{10} as well as several others.

*Operated for the U. S. Atomic Energy Commission by Union Carbide Corporation.

[1]See for example O. R. Frisch, Progress in Nuclear Physics (Academic Press, Inc., New York, 1950), Vol. 1, Chap. 8.

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THEORY OF THE PULSATION OF FLUORESCENT LIGHT FROM RUBY

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(Received November 17, 1960; revised manuscript received December 8, 1960)

It has been observed[1,2] that when one pumps a ruby crystal placed between two parallel reflecting end plates with intense light in order to create an excess of population in the fluorescent R level(s), there is a threshold of pumping power above which (a) the light from the R line is emitted in pulses accompanied by (b) a sudden spatial and spectral narrowing. This pulsed character of the output is not accounted for by the theory of Schawlow and Townes[3] for a gas Fabry-Perot type maser. We propose here a different approach to this induced fluorescence which predicts that as long as the pump power is above a certain threshold, part of the fluorescent power will occur in recurrent bursts or pulses. From the theory we derive quantitative estimates of the pulse repetition rate, the fraction of power in the pulses, and the nature of the output between pulses, all in terms of the pump power and the ordinary properties of the crystal and end plates. The equations connecting the spectral and spatial narrowing with the time-varying behavior will be developed.

We consider two infinite parallel plates of power reflection coefficient R and a distance d apart. The space between is filled with a ruby crystal. The crystal will be assumed just inhomogeneous enough and the end plates rough enough so that after many internal reflections photons will not interfere in any regular manner; therefore, we may use a straight photon model of the light. Diffraction effects will be assessed in a later article. We shall employ an optically thin crystal which can be pumped uniformly and in which the fluorescent photon density does not vary appreciably across the crystal, i.e., $1-R \ll 1$. To describe the radiation then, we may use the density of photons $u(\nu, \theta)$ traveling in the crystal at a frequency ν from the line-center frequency (measured in units of the half-width at half power) and at an angle θ to the norm. (All photon and population numbers will be measured per unit area of end plate.) We shall assume that we are dealing with a single component of the fluorescent R lines with an approximately $\cos^2\theta$ radiation pattern around a crystal axis that is normal to the end plates. (In an actual apparatus the crystal alignment need not be perfect if the crystal sides are smooth and parallel enough to form a light-pipe between the end plates.) The gain (negative absorption) coefficient $\alpha(\nu, \theta)$ associated with the R line will be assumed to result from a homogeneously broadened line and therefore may be written $[(1+\nu^2)N]^{-1}(n_2 - n_1)A\cos^2\theta$ where the peak absorption coefficient A for ruby is commonly of the order 0.4 cm^{-1} at room temperature and 10 cm^{-1} at liquid-nitrogen temperature. We shall measure the level populations and the difference $n = n_2 - n_1$ in units of the difference n_0 required for the maser material gain at $\nu, \theta = 0$ to just balance the end-plate losses. $n_0 = N(1-R)/Ad$. N is the total number of ions (per unit area) which when measured in units of n_0 will be called M. We shall also measure, for convenience, the

photon density $u(\nu, \theta)$ in units of n_0. We may write the time derivative of the photon density as

$$\dot{u}(\nu, \theta) = -u(\nu, \theta)/T(\nu, \theta) + p_i + p_s + D + F, \quad (1)$$

where $T(\nu, \theta)$ is the characteristic decay time of radiation without maser ions present. We shall assume for brevity that T is independent of ν, θ. The nature of the results does not depend much on the behavior of the relaxation times T, especially at wide angles. Then, the value of T to use is evidently $d[c(1-R)]^{-1}$, where c is the velocity of light in the crystal. The induced emission rate p_i will equal $nu(\nu, \theta)\cos^2\theta[(1+\nu^2)T]^{-1}$ with our definitions, and the spontaneous emission rate p_s will be $3n_2a(\cos^2\theta)[4\pi^2(1+\nu^2)]^{-1}$, where a is the total spontaneous emission probability per second for the R level. The terms D and F represent diffusion of photons from one angle to another and from one frequency to another as a result of crystal and end-plate imperfections, inhomogeneous broadening, and "spin-spin" interactions, etc. It has been estimated that these effects are not large enough to alter the character of the results and a discussion of them will be deferred. The time derivative of the population difference is given by

$$\tfrac{1}{2}\dot{n} = P_p - P_s - \int_{-\infty}^{\infty} d\nu \int_0^{\pi} d\theta \, \sin\theta \, p_i(\theta, \nu). \quad (2)$$

The crystal absorbs P_p pump photons per second (per unit area per n_0) and spontaneously emits P_S maser photons per second. The net contribution to the derivative of n from these terms is $\frac{1}{2}(w-a)M - \frac{1}{2}(w+a)n$, where w is the pump transition probability per second. The last term of (2) is the integrated induced transition rate. When $n<1$, the natural exponential decay time for the photon density is of the order T, which is roughly a million times shorter than any of the time constants for the decay of the populations which are of order $(wM)^{-1}$. Therefore, for $n<1$, the photon density follows very accurately and quickly any change in n, and we may use the adiabatic solution of (1) for the radiation (which reduces to the quiescent solution when n is constant):

$$\frac{u(\nu, \theta, t)}{T} \approx \frac{3(M+n)a\cos^2\theta}{8\pi^2[1+\nu^2 - n(t)\cos^2\theta]}, \quad (n<1). \quad (3)$$

If we integrate Eq. (3) over solid angle and frequency, we obtain the total output flux of photons $P_0[n(t)]$:

$$P_0[n(t)] = \tfrac{3}{4}a(M+n)[\sin^{-1}n^{1/2} - (n-n^2)^{1/2}]n^{-3/2}. \quad (4)$$

The differential equation for n in the region where the radiation can react so quickly is

$$\tfrac{1}{2}\dot{n} = P_p(n) - P_0(n), \quad (5)$$

and n changes so as to approach a value such that $P_p = P_0$. However, if in (4) we let n approach 1, P_0 approaches the value $\frac{3}{4}\pi a(M+1)$, and since clearly no quiescent solution exists for $n>1$, we have an upper limit on the output photon flux P_0 in the quiescent state. Since the output photon flux must equal the input flux from the pump (on the average), we know the pump rate w at which $n=1$, which we call the threshold pump rate w_0. For the simplifications we have made, $w_0 = 3\pi a/4$; for more accurate assumptions it is less. For $w > w_0$ we have to solve the infinite set of coupled equations (1) for $u(\nu, \theta, t)$ (one equation for each value of θ) together with Eq. (2) for the populations. Since this set is nonlinear we must proceed by approximation, the easiest method of which is to break time into intervals in which different terms may be taken as dominating, and then to connect the solutions in each interval as follows.

If the pump power is turned on suddenly so that $w > w_0$, then the population increases according to (6) until $n=1$. For a short time thereafter, of the order of $(T/wM')^{1/2}$, n continues increasing beyond unity at about the same rate as at $n=1$ to a value of the order $(wM'T)^{1/2}$ above unity. $M' \equiv M(1-w_0/w)$. At this point the positive exponential blowup of radiation quickly takes over. This wipes out the excess of n above unity by induced emission, creating momentarily $\sim\frac{1}{2}(wM'T)^{1/2}$ photons. The photon density decays in roughly Gaussian fashion, with the total decay rate being impeded by the production of more stimulated photons, and this drives n to a value $\epsilon \sim (wM'T_0)^{1/4}$ below 1 in a time $\sim T(wM'T)^{-1/4}$. By this time the photon density has locked P_0 to the value given by (4); n recommences its relatively slow recovery toward 1, and the whole process repeats cyclically. We may integrate (4) to find the recovery time between pulses, i.e., the time for n to grow from $1-\epsilon$ to 1. For any realizable pump probability w, we see that $\epsilon \ll 1$ and we may use this approximation to simplify the inte-

gration and obtain for the recovery time T_γ, $(W \equiv w/w_0)$

$$T_\gamma \approx \frac{\pi}{w_0 M(1+\sqrt{2})}\left[\epsilon^{1/2} - \frac{(W-1)\pi}{(1+\sqrt{2})2}\ln\left(1+\frac{2(1+\sqrt{2})}{\pi(W-1)}\epsilon^{1/2}\right)\right]. \quad (6)$$

A plot of $T_\gamma w_0 M$ versus W is shown in Fig. 1 with the spontaneous lifetime $a = 3\times10^{-3}$ sec, $R = 0.95$, $d = 1$ cm, and for the values $A = 0.4$ and 10 cm^{-1}. Despite the great simplifications, the results of (6) are in good agreement with the rough preliminary experimental results (giving for example $T_\gamma \approx 8$ μsec for $W = 2$ and $A = 0.4$ cm^{-1}). Fortunately the errors made in estimating ϵ fall under the fourth or eighth root and contribute correspondingly little to the error of (6).

Among the implications of the foregoing estimates of the behavior of populations and the output power for noninterfering fluorescence with negligible photon diffusion are the following: (a) If $T(\nu,\theta)\alpha(\nu,\theta)$ has a maximum in direction (and frequency) so that the total integrated output, computed from (1) with $\dot{u} = 0$ and infinite gain in the optimum direction, is finite, then there is a threshold at this output power above which pulses may be expected to appear continuously. (b) The fraction of the total output light that is emitted during the pulses is significant and is approximately $(wM'T)^{1/4}(wMT_\gamma)^{-1}$. This is of the order $\frac{1}{4}$ at $W \sim 1.1$ and approaches $(W-1)/W$ at high W. (c) Our estimate of $n(t)$ may be inserted into Eq. (1) to solve for the detailed behavior of the radiation as a function of time, frequency, and angle. The entire calculation seems too difficult without machine computation. However, the average narrowing factor during the interval between pulses has been estimated to be of the order $[Mw_0T(W-1)]^{1/8}$ both for spatial narrowing over the $\cos^2\theta$ behavior and for frequency narrowing over the natural linewidth. During the pulse the spatial narrowing is evidently great enough that interference and diffusion effects might alter the prediction of (1) for the spatial distribution significantly.

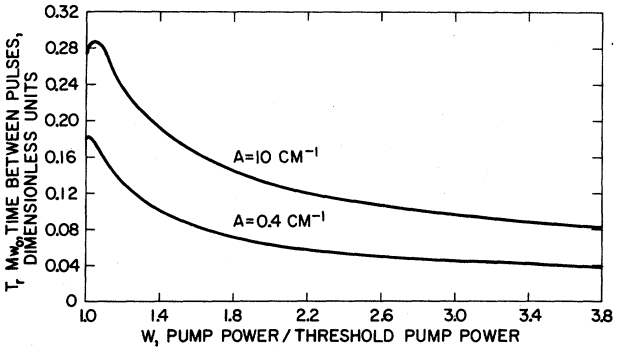

FIG. 1. Two plots of Eq. (6) for the time elapsed between pulses T_γ as a function of pump power each with $d = 1$ cm, $R = 0.95$, and $a = 3\times10^{-3}$ sec, but with different room temperature absorption coefficient A. The dimensionless units chosen make the function insensitive to changes in the above parameters.

Statz and deMars[4] have explained the possibility of a pulsed output in the simpler case of a single-mode microwave ruby maser. However, that the results of this paper differ quantitatively from those that would be obtained using the equations of Statz and deMars with new parameters appropriate to the optical case rests essentially on two things: (a) We have assumed that photons may propagate in any of a continuum of directions, which requires the "continuum" of equations of (1) and (2) rather than the two coupled equations of reference 4. Also, Eqs. (1) and (2) cannot be reduced to the form of two coupled equations in two variables. (b) Our threshold for pulsations depends on the anisotropy of the propagation constant α for ruby, or more precisely on the curvature at maximum of $\alpha(\nu,\theta)T(\nu,\theta)$. This threshold increases as the anisotropy is diminished to where, for the isotropic gas maser described by Townes and Schawlow, it might well be too high to be significant experimentally. Also for a gas maser, the medium is so much more optically homogeneous that interference phenomena would play a dominant role and the approach of Townes and Schawlow would be more direct than a modification of the approach presented here.

I would like to thank T. H. Maiman for valuable discussions of experimental results and G. Birnbaum for his interest and encouragement.

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ANGULAR DISTRIBUTION OF LYMAN-α RADIATION EMITTED BY H (2S) ATOMS IN WEAK ELECTRIC FIELDS

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H atoms in the 2S state are known to be metastable in a field-free region.[1] Application of an electric field perturbs the atoms, causing them to emit Lyman-α radiation and to decay to the 1S ground state. It is the purpose of this Letter to point out that the angular distribution of the radiation is isotropic and to examine the consequences of this conclusion for interpretation of certain electron-scattering experiments.

The electric-field-induced emission process can be considered as arising from a perturbation of the $2\,^2S_{1/2}$ state by the nearby $2\,^2P_{1/2}$ levels only. Since the $2\,^2P_{3/2}$ levels are energetically much farther away from 2S, their effect can be neglected. The effect of the perturbation of each $P_{1/2}$ level can be considered separately unless the coupling by the electric field is too strong, or unless there is accidental level crossing.[2] If one assumes that the latter condition is not present, the former can occur only if the perturbing matrix element V is comparable to the energy separation $h\nu$ between the $S_{1/2}$ and $P_{1/2}$ levels. Here $\nu = [E(S_{1/2}) - E(P_{1/2})]/h \sim 10^9$ cps; $V/h \sim ea_0E/h \sim 10^8$ cps for a typical laboratory quenching field of 50 v/cm, and the condition of weak coupling is well fulfilled.

Thus, it is only necessary to consider separately the angular distribution of the radiation emitted by an H atom in either of the two $P_{1/2}$ states. The well-known formulas[3] for the relative strengths of the π and σ lines give

$$I(\pi)/I(\sigma) = 4m^2/(J \pm m)(J \mp m + 1).$$

In the present case $J = \frac{1}{2}$, $m = +\frac{1}{2}$, and $I(\pi)/I(\sigma) = 1$. Thus the radiation is completely unpolarized and the angular distribution is isotropic.

Recently a controversy has arisen over the absolute magnitude of the cross section $\sigma(2S)$ for excitation of the 2S state of H by electron impact. Schultz and the writer[4] measured $\sigma(2S)$ from threshold (10.2 ev) to about 45 ev. The maximum value for $\sigma(2S)$ was $(0.35 + 0.05)\pi a_0^2$. Since the results depended primarily on normalization of the data to the Born approximation, the conclusions are unaffected by the present paper. A considerably less precise confirmatory absolute determination measured the number of photons emitted by electrostatic quenching. Since, in the latter experiment, the data were treated by assuming isotropic angular distribution of the photons, the conclusions of Schultz and the author rest unchanged.

Subsequently, Fite and co-workers[5] have measured $\sigma(2S)$ by comparing the intensity of photons emitted from quenched H (2S) atoms with the intensity arising from excited 2P atoms. [$\sigma(2P)$ had been measured previously by normalization to the Born approximation.] The measured $\sigma(2S)$ was consistently about one third of the results of Schultz and the writer over the common energy range. Fite et al. extended the observations to energies as high as 700 ev. Above 300 ev, the results agreed with the Born approximation. Fite et al. stated that this agreement was "thought to be undeniable evidence" for the correctness of the lower value for $\sigma(2S)$.

However, Fite et al. assumed 100% polarization of the radiation parallel to the electric field and perpendicular to the direction of observation. They multiplied their results by a factor of 2/3 to correct for anisotropy. According to the result of the present paper, this correction should not be made, since the radiation is isotropic. Thus the results of Fite et al. should be raised by 50%. This would bring their maximum cross section to $0.16\,\pi a_0^2$, in better agreement with the higher value of Schultz and the writer. Nevertheless, the disagreement is still substantial and exceeds the combined errors.

The most probable values of Fite et al. now