

and for the Coulomb radius

$$R_c = 3.2 \times 10^{-13} \text{ cm.} \quad (8)$$

These values disagree with the experimental values, -8.49 Mev and  $2.26 \times 10^{-13}$  cm, respectively. Since this is a conventional variation calculation, any improvement in the wave function would increase the disagreement in the energy.

We do not believe that the upper bound (7) for  $E_0$  is close to the mathematically correct ground-state energy. Every time we varied new parameters, or revaried ones varied much earlier, the bound on  $E_0$  decreased another few tenths of an Mev. We merely quit varying parameters when the discrepancy with experiment became sufficiently large to be significant.

The  $D$ -state probability is about 7%; no direct comparison with experiment is possible for this quantity.

The  $P$  states are completely unimportant. States 4, 6, and 7 of reference 2 have probabilities  $3.8 \times 10^{-4}$ ,  $0.8 \times 10^{-4}$ , and  $1.9 \times 10^{-4}$ , respectively, adding up to a total  $P$ -state probability of  $6.5 \times 10^{-4}$ . The contribution of the  $P$  states to the binding energy is less than 100 kev.

The  $\vec{L} \cdot \vec{S}$  force contributes significantly, but not through the  $P$  states. Rather, the matrix elements of the  $\vec{L} \cdot \vec{S}$  force between the  $D$  states must be included in the calculation, and these matrix elements contribute several Mev, in a repulsive direction.

The sharp variation of the trial functions near the core radius, especially for state 8 of reference 2, requires a reasonably fine net for the numerical quadratures.

The large number of states required, and the sensitivity of the result to the values of many of

the parameters in each state, makes it appear doubtful that any simple approximation can give accurate results in a three-body problem with tensor forces. By analogy, simple approximations in the many-body problem may well turn out to be less accurate than supposed.

We conclude that the combination of the Gammel-Brueckner potential with the assumption of superposition of 2-body forces, is inconsistent with experimental data on the triton and  $\text{He}^3$ . This does not necessarily contradict the result of reference 1, since one and the same potential may disagree with  $\text{H}^3$  and  $\text{He}^3$  and yet agree with nuclear matter. However, it is not impossible that the accuracy of the Brueckner theory of nuclear matter is somewhat less good than had been hoped.

We are very grateful to the U. S. Atomic Energy Commission and the Institute of Mathematical Sciences, New York University, for travel allowances and a very generous allocation of machine time, which alone made this calculation possible. We greatly appreciate many helpful and informative discussions with Professor R. D. Richtmyer, Professor E. Isaacson, Professor Max Goldstein, and Mrs. B. Weitzenhoffer, and the unstinting and generous help given to us by them and by all the members of the staff of the Atomic Energy Commission Computing Center at New York University.

<sup>1</sup>K. A. Brueckner and J. L. Gammel, *Phys. Rev.* **109**, 1023 (1958).

<sup>2</sup>G. Derrick and J. M. Blatt, *Nuclear Phys.* **8**, 310 (1958); G. Derrick, Ph.D. thesis, Sydney University, 1959 (unpublished).

<sup>3</sup>G. Derrick, *Nuclear Phys.* **16**, 405 (1960).

<sup>4</sup>G. Derrick, *Nuclear Phys.* **18**, 303 (1960).

<sup>5</sup>J. M. Blatt and D. Mustard, *Mathematics of Computation* (to be published).

### THE REACTION $\gamma + \gamma \rightarrow \nu + \bar{\nu}^\dagger$

Murray Gell-Mann

California Institute of Technology, Pasadena, California

(Received December 21, 1960)

In a current-current picture of the weak interactions, a coupling term of the form  $(e\nu)(e\nu)$  arises.<sup>1,2</sup> The contribution from charge-exchange currents is an interaction Lagrangian

$$\frac{G}{\sqrt{2}} [\bar{e}\gamma_\alpha (1 + \gamma_5)\nu] [\bar{\nu}\gamma_\alpha (1 + \gamma_5)\nu]^\dagger, \quad (1)$$

in the case of local coupling. [If there are charge-retention currents involving leptons, other terms may occur, of course, conceivably even canceling (1).]

In reference 2, the cross section for neutrino-electron scattering is calculated using the interaction (1); the result is too small for an experi-

mental test to have been possible up to the present time.

Pontecorvo<sup>3</sup> has suggested that if the coupling (1) exists, it may be of astrophysical importance because it would induce reactions leading to energy loss in stars. Recently<sup>4</sup> consideration has been given to processes like

$$e^- + e^+ \rightarrow \nu + \bar{\nu}, \quad (2)$$

which is similar to electron-neutrino scattering, and

$$\gamma + \gamma \rightarrow \nu + \bar{\nu}. \quad (3)$$

It is interesting that the latter process is not, in fact, induced by the local coupling (1). The reasoning is as follows:

By a Fierz transformation, we may express the coupling as

$$\frac{G}{\sqrt{2}} [\bar{e} \gamma_\alpha (1 + \gamma_5) e] [\bar{\nu} \gamma_\alpha (1 + \gamma_5) \nu], \quad (4)$$

and we must then calculate the matrix element of  $\bar{e} \gamma_\alpha e$  and of  $\bar{\nu} \gamma_\alpha \gamma_5 \nu$  between the vacuum and a state of two photons. The first vanishes by charge conjugation invariance. The second is analogous to the amplitude for the decay of a pseudovector or a pseudoscalar meson into two photons. Since a spin 1 meson cannot decay<sup>5</sup> into two photons, only that part of the amplitude corresponding to a pseudoscalar meson survives. In the rest frame, only the time component can contribute. Thus the entire matrix element of  $\bar{e} \gamma_\alpha (1 + \gamma_5) e$  is equal to  $P_\alpha$  (the total four-momentum) times a pseudoscalar bilinear in the field strengths of the two photons. But the Dirac equation for the neutrinos tells us that  $\bar{\nu} P_\alpha \gamma_\alpha (1 + \gamma_5) \nu$  gives zero.

Thus local weak interactions, to lowest order in  $G$ , give a vanishing rate<sup>6</sup> for reaction (3).

Our proof does not hold, however, for the case of an intermediate charged vector boson<sup>2</sup> mediating the charge-exchange current-current interactions. That case must be investigated further.

The process<sup>4</sup>  $\gamma + \gamma \rightarrow \nu + \bar{\nu} + \gamma$  is not forbidden for local coupling, although the contribution from  $\bar{e} \gamma_\alpha \gamma_5 e$  vanishes by charge conjugation symmetry. The remaining amplitude, coming from  $\bar{e} \gamma_\alpha e$ , can be calculated from the amplitude<sup>7</sup> for the corresponding process where the neutrino pair is replaced by a virtual photon with "field strength" equal to the matrix element of

$$\frac{G}{e\sqrt{2}} \left\{ \frac{\partial}{\partial x_\alpha} [\bar{\nu} \gamma_\beta (1 + \gamma_5) \nu] - \frac{\partial}{\partial x_\beta} [\bar{\nu} \gamma_\alpha (1 + \gamma_5) \nu] \right\}$$

for creation of the neutrino pair.

The author wishes to thank Professor William A. Fowler and Mr. M. Levine for suggesting a study of the reactions discussed here.

<sup>†</sup>Supported in part by the Alfred P. Sloan Foundation.

<sup>1</sup>M. Gell-Mann and A. H. Rosenfeld, Annual Review of Nuclear Science (Annual Reviews, Inc., Palo Alto, California, 1957), Vol. 7, p. 407.

<sup>2</sup>R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

<sup>3</sup>B. Pontecorvo, J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 1615 (1959) [translation: Soviet Phys. -JETP **9**, 1148 (1959)].

<sup>4</sup>See, for example, H.-Y. Chiu and P. Morrison (to be published).

<sup>5</sup>C. N. Yang, Phys. Rev. **77**, 242 (1950).

<sup>6</sup>It should be noted that the important process  $\gamma + \text{Coulomb field} \rightarrow \nu + \bar{\nu}$  does occur in the case of local coupling. In the lowest order of perturbation theory, the gauge-invariant amplitude for this reaction is finite and calculable.

<sup>7</sup>R. Karplus and M. Neuman, Phys. Rev. **80**, 382 (1960).