# BINDING ENERGY OF THE TRITON

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We have made a variational calculation of the binding energy of the triton, using the same two-body nuclear force law which was used by Brueckner and Gammel in their calculation of the properties of nuclear matter,[1] and assuming, as they did, that there are no specifically many-body forces. Our results disagree with the experimental data on the binding energy of the triton and on the Coulomb radius of $He^3$, thereby excluding the Brueckner-Gammel force law.

Of the 10 different states which contribute to the ground-state wave function of the triton,[2] we have included 8. The two states omitted have fully antisymmetric "internal" wave functions (states Nos. 2 and 5 of reference 2), and thus high kinetic energy.

The trial wave functions were chosen as follows. For each state $i$ ($i = 1, 2, \ldots, 10$), let

$$y_i \equiv \exp[-\gamma_i(r - r_0)], \tag{1}$$

where $r_0$ is the core radius, and $\gamma_i$ is an adjustable parameter. Let $a_{i3}$, $a_{i5}$, $a_{i7}$ be further parameters, and let

$$u_i(r) = \frac{1}{r}[y_i + a_{i3}(y_i)^3 + a_{i5}(y_i)^5 + a_{i7}(y_i)^7 - (1 + a_{i3} + a_{i5} + a_{i7})(y_i)^9]. \tag{2}$$

Then the trial wave function for the dominant, symmetric, $^2S_{1/2}$ state has the form

$$\psi_1 = A_1 u_1(r_{12}) u_1(r_{23}) u_1(r_{31}) + \bar{A}_1 \bar{u}_1(r_{12}) \bar{u}_1(r_{23}) \bar{u}_1(r_{31}), \tag{3}$$

where $A_1$, $\bar{A}_1$ are parameters, and $\bar{u}_1(r)$ differs from $u_1(r)$ only through the parameter values $\bar{\gamma}_1$, $\bar{a}_{13}$, $\bar{a}_{15}$, $\bar{a}_{17}$.

Wave functions of "mixed" symmetry, e.g., the mixed-symmetry $^2S_{1/2}$ state (state 3 of reference 2) are constructed as follows: Let

$$z_i = \exp[-2\delta_i(r - r_0)], \tag{4}$$

where $\delta_i$ is a parameter, let $a_{i4}$, $a_{i6}$, and $a_{i8}$ be additional parameters, and let

$$v_i(r) = \frac{1}{r}[z_i + a_{i4}(z_i)^2 + a_{i6}(z_i)^3 + a_{i8}(z_i)^4 - (1 + a_{i4} + a_{i6} + a_{i8})(z_i)^5]. \tag{5}$$

Then the internal part of the final wave function for the mixed-symmetry $^2S_{1/2}$ state has the two components

$$\psi_{3,1} = A_3(3)^{-1/2}\{u_3(r_{12})[v_3(r_{23})u_3(r_{31}) + u_3(r_{23})v_3(r_{31})] - 2v_3(r_{12})u_3(r_{23})u_3(r_{31})\},$$

$$\psi_{3,2} = A_3 u_3(r_{12})[v_3(r_{23})u_3(r_{31}) - u_3(r_{23})v_3(r_{31})]. \tag{9}$$

Trial functions for $P$ states and $D$ states are constructed in an analogous fashion, with the appropriate factors in front so as to give the correct dependence on Euler angles and spins and the correct asymptotic behavior in the singular configurations (equilateral triangle, and triangle degenerating into a straight line).[3] However, the doubling-up of wave functions for one state, as in (3), is employed only for the dominant $^2S_{1/2}$ state, not for any other states.

The matrix elements of the Hamiltonian, using these states, involve sums over spin and isospin indices, integrals over Euler angles, and integrals over $r_{12}$, $r_{23}$, $r_{31}$. The spin sums and Euler angle integrations are done analytically,[3,4] the integrations over $r_{12}$, $r_{23}$, $r_{31}$ numerically.[5] Depending upon the fineness of the net and upon the number of states used, the numerical integrations take between 10 minutes and 1 hour on the IBM 704. The number of netpoints ranges between 9000 and 30 000, and the upper limit on $r_i$ between $R = 6.4 \times 10^{-13}$ cm and $R = 9.6 \times 10^{-13}$ cm. The dominant $^2S_{1/2}$ state integrals are done with a much finer net which gives 5-figure accuracy; the other integrals have numerical errors of the order of 1%. The numerical error in the expectation value of the Hamiltonian is estimated to be less than 0.3 Mev.

After varying some 40 parameters, we find for the ground-state energy of the triton

$$E_0 < -10 \text{ Mev}, \tag{7}$$

and for the Coulomb radius

$$R_c = 3.2\times10^{-13} \text{ cm}. \quad (8)$$

These values disagree with the experimental values, -8.49 Mev and $2.26\times10^{-13}$ cm, respectively. Since this is a conventional variation calculation, any improvement in the wave function would increase the disagreement in the energy.

We do not believe that the upper bound (7) for $E_0$ is close to the mathematically correct ground-state energy. Every time we varied new parameters, or revaried ones varied much earlier, the bound on $E_0$ decreased another few tenths of an Mev. We merely quit varying parameters when the discrepancy with experiment became sufficiently large to be significant.

The $D$-state probability is about 7%; no direct comparison with experiment is possible for this quantity.

The $P$ states are completely unimportant. States 4, 6, and 7 of reference 2 have probabilities $3.8\times10^{-4}$, $0.8\times10^{-4}$, and $1.9\times10^{-4}$, respectively, adding up to a total $P$-state probability of $6.5\times10^{-4}$. The contribution of the $P$ states to the binding energy is less than 100 kev.

The $\vec{L}\cdot\vec{S}$ force contributes significantly, but not through the $P$ states. Rather, the matrix elements of the $\vec{L}\cdot\vec{S}$ force between the $D$ states must be included in the calculation, and these matrix elements contribute several Mev, in a repulsive direction.

The sharp variation of the trial functions near the core radius, especially for state 8 of reference 2, requires a reasonably fine net for the numerical quadratures.

The large number of states required, and the sensitivity of the result to the values of many of the parameters in each state, makes it appear doubtful that any simple approximation can give accurate results in a three-body problem with tensor forces. By analogy, simple approximations in the many-body problem may well turn out to be less accurate than supposed.

We conclude that the combination of the Gammel-Brueckner potential with the assumption of superposition of 2-body forces, is inconsistent with experimental data on the triton and $He^3$. This does not necessarily contradict the result of reference 1, since one and the same potential may disagree with $H^3$ and $He^3$ and yet agree with nuclear matter. However, it is not impossible that the accuracy of the Brueckner theory of nuclear matter is somewhat less good than had been hoped.

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[4]G. Derrick, Nuclear Phys. 18, 303 (1960).

[5]J. M. Blatt and D. Mustard, Mathematics of Computation (to be published).

## THE REACTION $\gamma+\gamma\rightarrow\nu+\bar{\nu}$[†]

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In a current-current picture of the weak interactions, a coupling term of the form $(e\nu)(e\nu)$ arises.[1,2] The contribution from charge-exchange currents is an interaction Lagrangian

$$\frac{G}{\sqrt{2}}[\bar{e}\gamma_\alpha(1+\gamma_5)\nu][\bar{e}\gamma_\alpha(1+\gamma_5)\nu]^+, \quad (1)$$

in the case of local coupling. [If there are charge-retention currents involving leptons, other terms may occur, of course, conceivably even cancelling (1).]

In reference 2, the cross section for neutrino-electron scattering is calculated using the interaction (1); the result is too small for an experi-