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OBSERVATIONS OF de HAAS–van ALPHEN OSCILLATIONS IN p-TYPE PbTe†

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We should like to report the results of a study of de Haas–van Alphen (dH-vA) oscillations in the magnetic susceptibility of p-type PbTe by pulsed magnetic field techniques. Although extensive studies of the dH-vA effect in metals have been made, as far as we know this is the first time that susceptibility oscillations have been observed in a semiconductor. Oscillations in transport properties such as magnetoresistance and Hall effect have, of course, been observed in semiconductors as well as in metals. Because studies of magnetoresistance and Hall effect oscillations require electrical contacts and involve the current as an additional parameter, the measurement of susceptibility oscillations by the impulse method[1] has the advantage of greater simplicity. We feel that such measurements will have wide application in the study of the band structure of semiconductors.

In each of the four p-type PbTe samples studied, more than 30 oscillations were observed. Three of the PbTe samples were cut from a single boule as close to each other as possible. Hall effect measurements made by Allgaier[2] yielded a hole density of $3\times10^{18}/\text{cm}^3$ for this boule. The fourth PbTe sample had a hole density of about $1\times10^{18}/\text{cm}^3$. These samples were circular cylinders with the cylinder axis parallel to one of the three principal symmetry directions. The magnetic field was applied along the axis of the cylinder. A typical photograph of the oscillations in dm/dt, where m is the total magnetic moment of the sample, is shown in Fig. 1 and the successive maxima and minima are plotted versus B^{-1} in Fig. 2 for two different orientations. Table I lists the periods measured in the three symmetry directions.

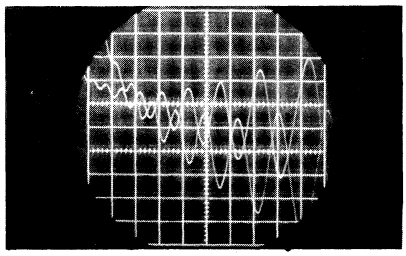

FIG. 1. dH-vA oscillations in p-type PbTe. The double trace shows the oscillations for increasing and decreasing magnetic field. The magnetic field varies from 7.3 kgauss at the left to 16.4 kgauss at the right.

It has been suggested by earlier workers that the energy surfaces at the edge of the valence band in PbTe are ellipsoids of revolution oriented along the $\langle 111\rangle$ directions.[3] Allgaier has recently suggested on the basis of his magnetoresistance and Hall effect studies at 77°K and room temperature that an additional valence band maximum appears to be involved. Our data indicate a band maximum at $\vec{k}=0$ and band maxima at the centers of the $\{111\}$ faces of the Brillouin zone, although it is not clear that the additional valence band maximum we see is the one suggested by Allgaier. The extremal cross sections associated with the $\vec{k}=0$ maximum vary

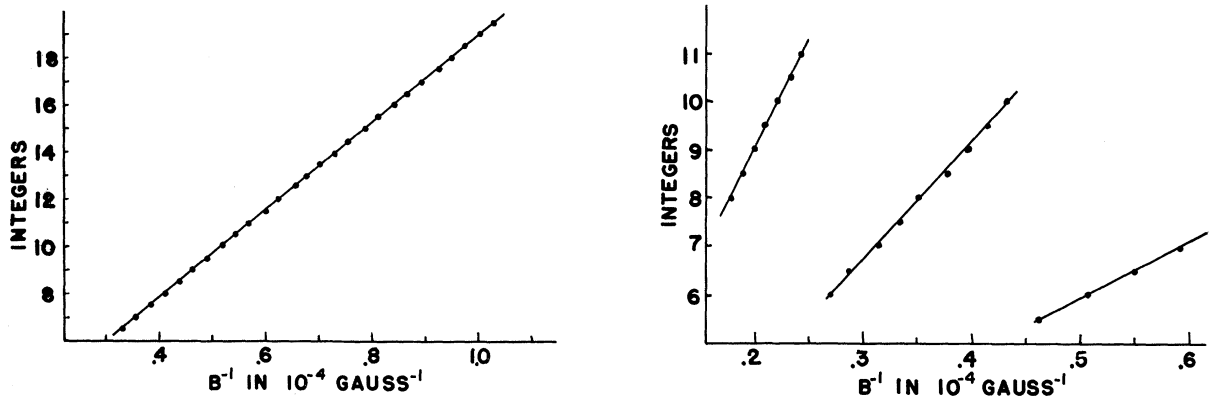

FIG. 2. Successive maxima and minima in dm/dt are plotted as integers and half integers vs B^{-1}, with $n = 3.9 \times 10^{18}$ holes/cm^3 and $T = 4.2$°K. (a) The plot for the longer period observed with $\vec{B} \| \langle 100 \rangle$. (b) The plots for all three periods observed with $\vec{B} \| \langle 111 \rangle$. Twenty-four more maxima and minima observed in the series with the smallest slope are not shown here.

with orientation by about 25%. Assuming parabolic bands, the energy surfaces at the {111} zone faces are ellipsoids of revolution with a longitudinal to transverse mass ratio of 6.4 ± 0.3. From the dependence of the oscillation amplitude on magnetic field and on temperature, we derive a transverse effective mass for the ellipsoids of $(0.042 \pm 0.006)m_0$ (m_0 = free electron mass) and an equivalent broadening temperature, T_b, of about 7°K to 10°K. T_b includes the apparent broadening due to hole density variations in the sample as well as collision broadening. We feel that the large value of T_b is primarily due to variations in hole density. On this basis the value of T_b observed in these experiments corresponds to a 10% variation in hole density, a reasonable value for such materials.

If the complete geometry of the Fermi surface is known, one can determine the total number of holes from the total volume enclosed by the surface in $\vec{k}$ space. We estimate the number of holes in the three crystals cut from a single boule to be 3.9×10^{18}/cm^3, assuming four ellipsoids and a sphere at $\vec{k} = 0$ of cross section equal to the average of the three measured cross sections. This is reasonably close to the density of 3×10^{18}/cm^3 obtained from Hall effect measurements. At this concentration the Fermi level is 0.035 ± 0.005 ev below the top of the ellipsoidal band. By comparing the results from the two samples with different hole densities, and assuming parabolic bands and our value of the ellipsoid transverse effective mass, we obtain an estimate of the effective mass of the central section, $m^* = (0.12 \pm 0.02)m_0$. We also estimate that the band edge at $\vec{k} = 0$ is higher in electron energy than the band edge at the Brillouin zone face by 0.002 ± 0.002 ev.

It is of interest to consider the factors determining the sensitivity of the impulse method for observing the dH-vA effect in semiconductors and to compare this sensitivity with that achieved

Table I. Experimental dH-vA periods.

Orientation of the magnetic field	Period[a] (gauss$^{-1} \times 10^6$) Ellipsoids	Period[a] (gauss$^{-1} \times 10^6$) $\vec{k} = 0$ section	Measured hole density[b] (cm^{-3})
⟨100⟩	13.3	6.07	9.5×10^{17}
⟨100⟩	5.37	2.63	3.9×10^{18}
⟨110⟩	7.50	2.42	
	3.70		
⟨111⟩	8.39	2.15	
	4.18		

[a] There is an error of 2% associated with each period, primarily due to the calibration of the magnetic field.
[b] Calculated from dH-vA periods.

in observations of the effect in metals. The experimentally observed signal is proportional to $dm/dt = V(dm/dH)(dH/dt)$, where V is the sample volume and M is the magnetization. In semiconductors the carrier density is orders of magnitude less than it is in metals, the oscillation period is much longer, and dM/dH may be smaller by a factor of 10^5. On the other hand, because of the longer period in semiconductors, larger field inhomogeneities due to eddy current shielding can be tolerated. Furthermore, the eddy currents are reduced by higher resistivities. Both V and dH/dt may therefore be larger for a semiconductor than for a metal and the smaller sensitivity due to the small dM/dH can be partially compensated. The signals observed in the present experiments are 10-100 times smaller than typical signals observed in experiments on metals. Nevertheless the sensitivity is quite adequate. In fact, the sensitivity of the method is such that the only limitation to observing the effect in semiconductors is the theoretical one that the materials must have $E_F > k(T + T_b)$, where E_F is the Fermi energy measured relative to the edge of the band in question. Thus, in addition to the requirement that the sample be degenerate ($E_F > kT$), one needs uniform carrier densities and long collision times ($E_F > kT_b$). The requirements of a long collision time ($E_F > \hbar/\tau$) can be expressed in terms of the density and mobility of the carrier as $n^{2/3}\mu > 10^{15}$ volt sec for the case of a single spherical energy band. There are a number of semiconductors which readily satisfy these conditions, such as the lead salt series PbS, PbSe, and PbTe,[4] which we are now investigating.

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*Philco Fellow.

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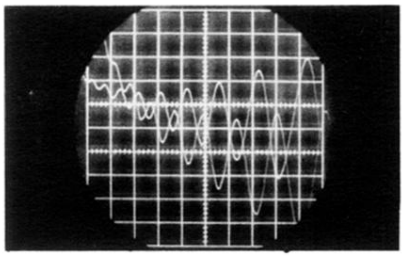

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