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ELECTROSTATIC SOUND WAVE MODES IN A PLASMA*

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The dispersion relation for ion waves in an infinite plasma is[1,2]

$$\omega^2 = \frac{\omega_{pi}^2}{1+(\omega_{pi}^2 m_p/\beta^2\gamma k T_e)}, \qquad (1)$$

where m_p is the positive-ion mass, β is the phase constant, T_e is the electron temperature, zero ion temperature is assumed, and k is Boltzmann's constant. The constant γ depends on the assumptions made concerning collisions and will lie between 1 and 3.[2] The ion plasma frequency is given by

$$\omega_{pi} = (n_p e^2/\epsilon_0 m_p)^{1/2}, \qquad (2)$$

where n_p is the ion density, e is the electronic charge, and ϵ_0 is the permittivity of free space.

In the long-wavelength limit $\omega/\beta \to (\gamma k T_e/m_p)^{1/2}$, giving rise to the so-called electrostatic sound wave mode of propagation[1] with velocity independent of n_p. A low-frequency cutoff dependent on tube dimensions might be expected.

Ion waves have often been suggested as a source of low-frequency fluctuations occurring in the parameters of dc discharges.[3] To test this hypothesis, dc mercury-vapor discharges at a pressure of $\sim 1\mu$ have been studied in tubes of the type shown in Fig. 1. Each contained an oxide-coated spiral cathode, mounted transversely in a molybdenum cup. One tube of uniform diameter was available, and eight others of various column diameters, but roughly similar construction in the enlarged section containing the cathode assembly. The dimensions were comparable to the electron-neutral mean free path. Spectra of anode voltage fluctuations were observed to contain peaks varying slowly with current, and typical results are shown in Fig. 1. Similar spectra were picked up on a ring sliding along the outside of the glass. Attempts to

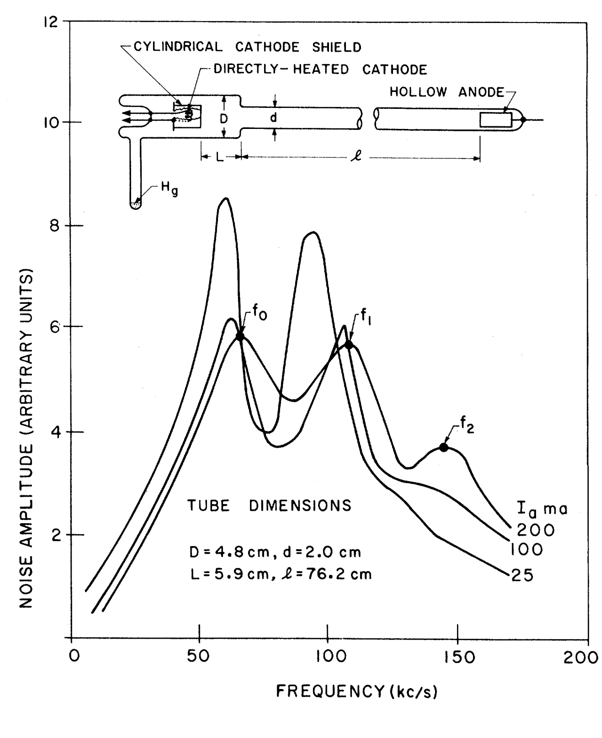

FIG. 1. Experimental discharge tube, and typical spectra of low-frequency fluctuations.

measure phase shift have indicated that the fluctuations must travel along the column at velocities $>10^9$ cm/sec over most of its length, but that over the first few centimeters the phase velocity is of the order of 10^7 cm/sec.[4]

Although it is doubtful whether longitudinal ion wave modes have ever been observed, it is possible that radial modes, generated near the cathode, and near cutoff within the column, could explain these results. Their theory may be derived from the cylindrical-polar form of the wave equation, and if such solutions are valid the frequencies of higher order modes should bear fixed numerical relations to the frequency of the lowest mode. For a mode in which the potential varies as $\cos n\phi$, and is assumed to have a node at the wall, β will be determined by the zeros of $J_n(\beta d/2)$.

Data on the relations between the frequencies of peaks observed in Tubes I-IX are shown on Fig. 2(a) over a wide range of current and number density ($\sim 10^8 - 10^{10}$ ions/cc). Most of the frequency ratios are distributed about the average values 1.68 and 2.34 which are close to the ratios suggested by our simple theoretical considerations. The occasional second harmonics observed could easily arise through nonlinearities.

Assuming that radial electrostatic sound waves are involved, we may substitute for the constants in the expression for the frequency,

$$f_0 = (2.405/\pi d)(\gamma k T_e/m_p)^{1/2},$$

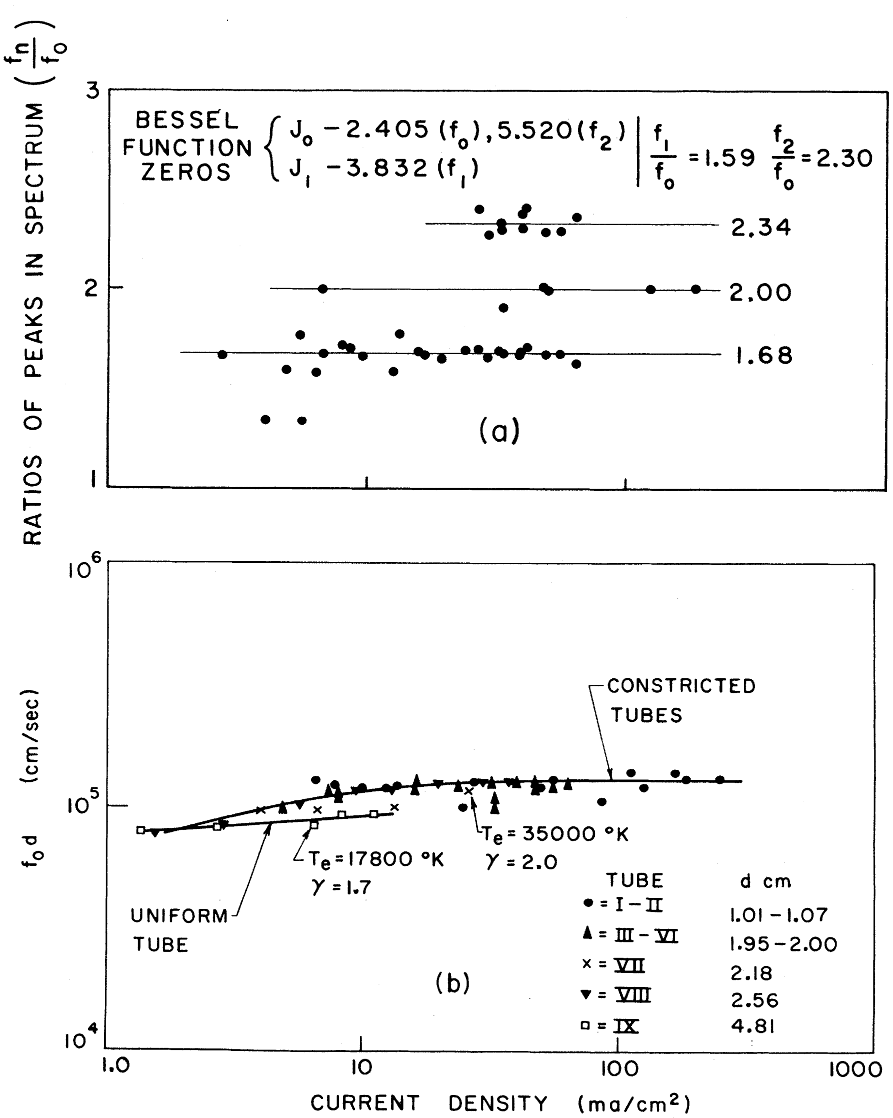

FIG. 2. (a) Frequency ratios of peaks in fluctuation spectra. (b) ($f_0 d$) variation with column current density.

to obtain the result

$$f_0 d = 490(\gamma T_e)^{1/2} \text{ cm/sec}, \qquad (3)$$

where f_0 is in cps, d is in cm, and T_e is in °K.

Figure 2(b) shows $(f_0 d)$ plotted against column current density. It has been assumed that the plasma column fills the tube and no correction has been attempted for sheath thickness. Langmuir probe measurements of T_e were made at the points marked. Substitution of these values in Eq. (3) gives values of γ in the range expected. The temperatures were measured nearer to the anode than the cathode so that the values may not be appropriate to the actual generation region.

The results suggest that an electrostatic sound wave mechanism may be operating, and that it is enhanced by the presence of a constriction. The Bessel function relations are appropriate to the first and second radially varying modes, and the first azimuthally varying mode, though it is difficult to see why an azimuthally varying mode should be excited so strongly and detected so easily. To determine the ratios more accurately, modification of Eq. (1) is required to include the axial and radial drift velocities of the ions and electrons.

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ENHANCED DIFFUSION AND OSCILLATIONS IN WEAKLY IONIZED PLASMAS

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The present study deals with observations on enhanced diffusion performed on a weakly ionized plasma.

Experiments have been performed with a Reflex-type discharge in H_2, N_2, and Ar, the pressure, density, and magnetic field conditions of which are variable. The discharge works in continuous mode. The diffusion is measured by means of the escape flux of the ions transversally to the magnetic field B_0. The measurement is made with a probe external to the plasma, negatively polarized with regard to the plasma potential. The variation of the escape flux as a function of the magnetic field for different values of the pressure parameter is shown in Figs. 1 and 2.

All curves show three separate regions:

1. $B < B_C$: The escape flux decreases with the magnetic field and increases with the pressure according to the classical laws of diffusion confirmed experimentally by several researchers[1-3] (Fig. 1).

2. $B_C < B < B_M$: A new diffusion mode appears in which the escape flux increases with the mag-

	PRESSURE mm Hg	B_c (CRITICAL FIELD)
1	1.6×10^{-2}	230 gauss
2	2.4×10^{-2}	320 gauss
3	3×10^{-2}	360 gauss

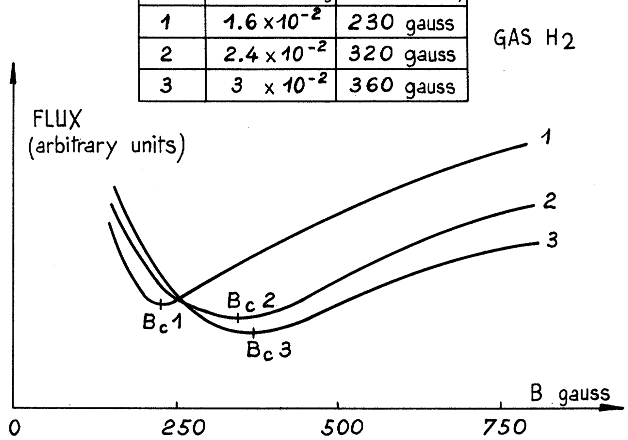

FIG. 1. Escape flux of ions transversally to the magnetic field; $0 < B < 1000$ gauss. These results have been obtained with a cold-cathode P.I.G. discharge, working with hydrogen, of length 110 mm, diameter 30 mm, voltage between anode and cathode 400 volts, current 200 ma.