ing to E_{p_1}' and E_{p_2}', respectively, and

$$\vec{p}_1 \cdot \vec{p}_2 = -\alpha + \frac{(2\alpha - \Delta_1^2)(2\alpha - \Delta_2^2)}{4\omega^2}, \tag{13}$$

with $\alpha = W^2/2 - m^2$. Finally we have for the upper limit of integration

$$g(T) = \left[m^2 - W^2\frac{T}{m} + W(W^2 - 4m^2)^{1/2}\left(\frac{T^2}{m^2} + \frac{2T}{m}\right)^{1/2}\right]^{1/2}. \tag{14}$$

The predictions of formula (4) at 970 Mev are shown in Fig. 2 and compared with the experimental spectrum.[9] As is seen, the agreement is good while a statistical spectrum fails to explain the three main features of the experimental distribution; these are: the strong peaking at low (~50 Mev) energy, the broad bump at high (~500 Mev) energy, and the minimum between them. These features are, however, quantitatively predicted from our formula (4) and are typical effects of the peripheral interaction (see reference 1 for a more detailed discussion).

Theoretically, the total cross section[10] is predicted to be 12 mb; its experimental value is 16.4 ± 0.7 mb.[2] This indicates that something more than the diagrams of Fig. 1 contributes to process (1). However, this "something," whatever it is, is not more than 30% of the peripheral contribution. In view of these results it seems very desirable to have a deeper comparison between this model and the experimental data.[11]

The author is indebted to Professor A. Stanghellini and Dr. E. Ferrari for very useful suggestions, and also to many theoreticians of CERN for discussions.

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[9]This spectrum was already used in reference 1 and was originally a private communication from Dr. L. Riddiford to Professor A. Minguzzi.

[10]Calculations of the total cross section for process (1) under similar assumptions have already been performed by T. Kobayashi, Progr. Theoret. Phys. (Kyoto) 18, 318 (1957), who, however, did not take into account the effects of the Pauli principle, and by G. Da Prato, University of Rome, thesis (unpublished).

[11]This work was conceived as a collaboration with Professor A. Stanghellini, Dr. S. Bergia, and Dr. B. Bortolani of Bologna University who are now working on other aspects of the problem.

INTERMEDIATE BOSON PRODUCTION IN PION-PROTON COLLISIONS*

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The current-current hypothesis has been applied in recent years to the study of the weak interactions. The conserved vector current[1] hypothesis and the pionic character of the divergence of the axial vector current[2] have evolved in its wake. So has the idea of a boson field which mediates all weak interactions.[3] This note deals only with the vector part of the current, leaving for another time similar calculations with the axial vector part; and examines the possibility of producing the boson W, in pion-nucleon collisions, in case of W having relatively low mass.[4]

For definiteness let us consider the reaction

$$\pi^+ + p \to W^+ + p, \tag{I}$$

and concentrate solely on the vector part of the interaction. Then, on inverting incoming and outgoing particles, it is apparent that the process is almost exactly the same as electroproduction of pions (i.e., production of pions by photons off the mass shell). The differences are: (*i*) The coupling constant is not the electromagnetic coupling constant. (*ii*) The four-momentum of the "photon" is timelike, not spacelike. But the analysis of electroproduction in papers A and B[5] is in fact relevant provided that instead of $e^2=1/137$, we write[3] $e^2=(1/\pi\sqrt{2})(N^2/M^2)\times 10^{-5}$, where N is the boson mass and M is the proton mass. This simple substitution is the content of the conserved vector current hypothesis. The problems raised by (*ii*) imply the extrapolation of the analysis in A and B to a new region.

The total amplitude $\mathfrak{M}$ may be written in the form

$$\mathfrak{M}=AM_A+BM_B+CM_C+\cdots+FM_F, \tag{1}$$

where the six functions A, B, $\cdots$, F depend on the invariants

$$\nu=-\frac{P\cdot k}{M},\ \nu_B=\frac{q\cdot k}{2M},\ k^2=-N^2; \tag{2}$$

here $P=\frac{1}{2}(p_1+p_2)$; p_1, p_2 are the final and initial momenta of the proton; q, k the momenta of the π and W. M_A, $\cdots$, M_F are defined by Eq. (4) in A in terms of γ matrices. It is also possible to write $\mathfrak{M}$ in terms of two-component Pauli spin matrices:

$$\mathfrak{M}=\mathfrak{F}=\sum_{i=1}^{6}\Sigma_i\mathfrak{F}_i, \tag{3}$$

where the Σ_i and the linear connection between the $\mathfrak{F}_i$ and the A, B, $\cdots$, F are given by Eqs. (8) and (9) in B. The cross section in the center-of-mass system is then given by

$$\frac{d\sigma}{d\Omega}=\frac{|\vec{\mathrm{k}}|}{|\vec{\mathrm{q}}|}\frac{M^2}{W^2}|\langle 2|\mathfrak{F}|1\rangle|^2, \tag{4}$$

where W is the total energy in the c.m. system, and the matrix element includes the sum over final and average over initial spin states.

As the energy for reaction (I) is much above the 3-3 resonance of the pion-nucleon system, the calculation first will be done using only the Born terms. Here

$$A=-ef\,|F_1^{\,p}(k^2)|\left(\frac{1}{\nu_B-\nu}+\frac{1}{\nu_B+\nu}\right),$$

$$B=\frac{ef}{2M\nu_B}|F_1^{\,p}(k^2)|\left(\frac{1}{\nu_B-\nu}+\frac{1}{\nu_B+\nu}\right),$$

$$C=\mu_p'f\,|F_2^{\,p}(k^2)|\left(\frac{1}{\nu_B-\nu}-\frac{1}{\nu_B+\nu}\right),$$

$$D=\mu_p'f\,|F_2^{\,p}(k^2)|\left(\frac{1}{\nu_B-\nu}+\frac{1}{\nu_B+\nu}\right),$$

$$E=F=0,\ f^2=0.08,\ \mu_p'=1.78(e/2M); \tag{5}$$

$F_1^{\,p}$ and $F_2^{\,p}$ are the Hofstadter form factors of the proton. Note that the form factors are of opposite sign in argument to those determined experimentally. The result of the calculations done with $F_1^{\,p}(k^2)=F_2^{\,p}(k^2)=1$ for varying N at fixed energy is shown in Fig. 1. The cross section varies slowly with the energy for a given W mass.

It has been pointed out by Frazer and Fulco[6] that a π-π resonance in the state $I=1$, $J=1$ would give theoretical agreement with the experiments for the isotopic vector form factors. A more recent paper by Bowcock, Cottingham, and Lurié[7] fits the position and width of such a resonance to the data on π-N scattering and electromagnetic nucleon structure.

From the dispersion relations (with subtractions where necessary)

$$F_i^{\,V}(t)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{g_i^{\,V}(t')dt'}{t'-t},\ (i=1 \text{ or } 2) \tag{6}$$

with

$$g_i^{\,V}(t')=|F_\pi(t')|^2[g_i^{\,V}(t')]_0, \tag{7}$$

where the $[g_i^{\,V}(t')]_0$ are given in the papers on nucleon structure neglecting the π-π resonance,[8] the real and imaginary parts of $F_i^{\,V}(t=+N^2)$ can be calculated. The pionic form factor is given (for t not too large) by

$$|F_\pi(t)|^2=\frac{t_r^{\,2}}{(t-t_r)^2+\gamma^2v^3},\ v=\tfrac{1}{4}t-\mu^2, \tag{8}$$

when the π-π scattering amplitude is written in resonance form (for $I=1$, $J=1$)

$$f_{\pi\pi}=\frac{\gamma}{t_r-t-i\gamma v^{3/2}}.$$

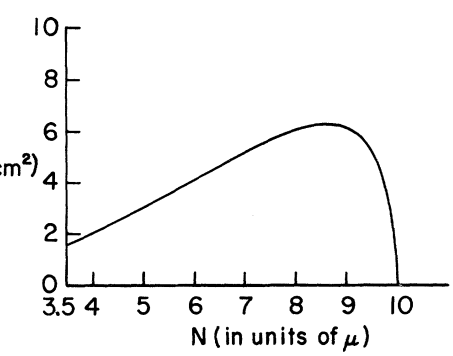

FIG. 1. Cross section σ as a function of boson mass N in uncorrected Born approximation. Momentum of pion in c.m. system is 980 Mev/c.

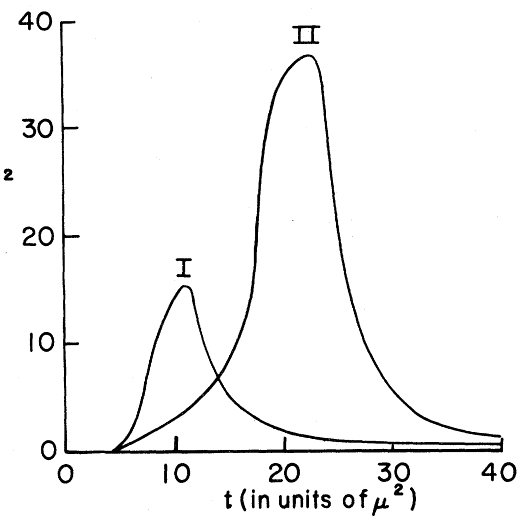

FIG. 2. Pionic form factor about resonance region. Curve I given by Frazer and Fulco. Curve II given by Bowcock, Cottingham, and Lurié.

In reference 7, the values for γ and t_γ are given as

$$\gamma = 0.376\mu^{-1}, \quad t_\gamma = 22.4\mu^2. \tag{9}$$

The two alternative curves for $|F_\pi(t)|^2$ are given in Fig. 2. The multiplicative factor (taking the charge and magnetic moment form factors to vary similarly in the low-energy unphysical region as well as the physical) to the curve in Fig. 1 is given in Table I.

An estimate of the effect of the 3-3 resonance was attempted using the dispersion relations for $A, \cdots, F$ given in A and B and replacing the resonance by a delta function, i.e.,

$$(1/\pi)\mathrm{Im}A(x) = A_{33}(x)\delta(1-\omega/\omega_\gamma), \text{ etc.,} \tag{10}$$

where $x = \nu - \nu_B$, $\omega = W - M$, ω_γ refers to the position of the resonance, and $A_{33}(x)$ is the 3-3 projection of the Born term for A, and is given in paper B. As the 3-3 resonance occurs well into the unphysical region for reaction (I), the amplitudes $A_{33}, \cdots, F_{33}$ must be analytically continued. If this is done and the calculation carried out for the total cross section, a very large increase is obtained (>100). There is no reason, however, to take this calculation seriously; but it should be viewed as an indication that the cross section may yet be larger than that given by the Born term with or without corrections.

Table I. Multiplicative factor given by π-π resonance to curve in Fig. 1 for various values of N.[a]

N	3.5	4	4.5	5	6	7	9
Frazer and Fulco	5	~1	~1	~1	<1	<1	<1
Bowcock, Cottingham, and Lurié	20	45	150	60	15	5	~1

[a] $F_i^S(N^2)$ is neglected.

So, in summing up, it can be said that if the Bowcock, Cottingham, and Lurié fit to the π-π resonance is correct, then the cross section should be of the order of 10^{-30} cm^2 for values of the W mass between four and five pion masses. It is also possible, if the 3-3 resonance is taken into account, that the cross section is greater still; and it must be remembered that the axial vector contribution has been completely neglected.

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