J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo cimento 17, 757 (1960); Chou Kuang-Chao, Dubna Report D514, 1960 (unpublished); these papers contain further references. It is the purpose of this note to use Eq. (6) in order to produce a number for the decay rate.

[8]It may be that some of the observed $\Sigma\beta$ events are due to $\Sigma \rightarrow \Lambda + e + \nu$; it will be difficult to separate $\Sigma\Lambda$ events from Σn events unless the Λ decay is seen.

[9]L. Okun and A. Rudik, J. Exptl. Theoret. Phys. (U.S.S.R.) 39, 378 (1960) [translation: Soviet Phys. – JETP 12, 268 (1961)].

[10]R. F. Blackie et al., Phys. Rev. Letters 5, 384 (1960).

[11]F. Duimio and G. Wolters, Nuovo cimento (to be published) and CERN preprint.

POSSIBLE MECHANISM FOR THE PION-NUCLEON SECOND RESONANCE

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Several phenomenological models[1] have been proposed to discuss pion-nucleon interactions near the higher resonances. There seems to be considerable evidence from these analyses that final states described by the 33 isobar model[2] play an important role, but so far there has been little quantitative discussion or explanation of the resonant peaks. In the present note it is shown that a simple model can give a good quantitative description of the second resonance, a rough description of the third, and a plausible explanation of the peak above 1.0 Bev. The model consists in taking seriously the concept of the 33 isobar as an unstable particle (N^*) of spin and isospin $\frac{3}{2}$ and having a complex mass. This means that we assume the low-energy pion-nucleon scattering to be dominated by the one N^* direct pole. The position and width of the 33 resonance then enable us to find the mass of the isobar $M(N^*)$:

$$[M(N^*)]^2 = M^2 + i\Delta,$$
$$M = 8.94\,\mu_\pi, \quad |\Delta| = 8.05\,\mu_\pi^{\,2}, \tag{1}$$

where μ_π is the pion mass.

We consider next the reaction

$$\pi + N \rightarrow \pi + N^*. \tag{2}$$

This is strongly coupled to the process

$$\pi + N^* \rightarrow \pi + N^*. \tag{3}$$

Let us try to apply the usual formalism to reaction (3), pion-isobar scattering. For the moment we ignore the spin and isospin and assume that the reaction is described by some amplitude $A(s, \bar{s}, t)$ where s, $\bar{s}$, t are defined in terms of the pion energy ω, the real part E of the isobar energy, and the center-of-mass momentum and scattering angle q, θ:

$$s = (E + \omega)^2 + i\Delta(1 + \omega/E),$$
$$\bar{s} = (E + \omega)^2 - 2\,q^2(1 + \cos\theta) + i\Delta(1 - \omega/E),$$
$$t = -2\,q^2(1 - \cos\theta),$$
$$s + \bar{s} + t = 2M^2 + 2\,\mu_\pi^{\,2} + 2\,i\Delta. \tag{4}$$

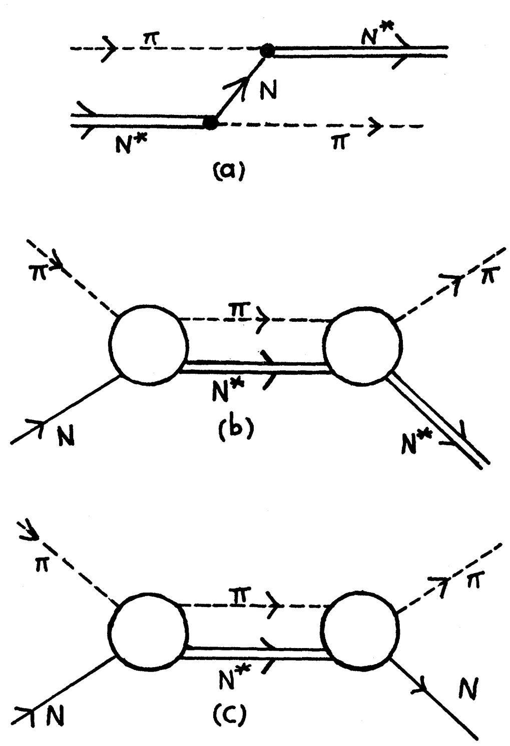

FIG. 1. (a) Crossed one-nucleon diagram for πN^* scattering. (b) Schematic coupling between πN^* scattering and πN^* production in πN collisions. (c) Schematic coupling between πN^* production and πN scattering.

The amplitude A should have a pole whenever s, $\bar{s}$, or t is equal to the mass of a possible intermediate particle. The term represented by Fig. 1(a) has the unusual feature that if $\Delta = 0$, it becomes infinite in the physical region for q, θ. The explicit contribution of this term may be taken empirically to be

$$A_0 = g^2/(\bar{s} - m^2), \tag{5}$$

where m is the nucleon mass and g is the "coupling constant" for the $NN^*\pi$ vertex[3] (determined by the π-N cross section at the 33 resonance). We can rewrite this as

$$A_0(W, \theta) = \frac{g^2}{(W^2 - W_0^2) + i\Delta(1 - \omega/E) - 2q^2(1 - \cos\theta)},$$
$$W_0^2 = 2M^2 + 2\mu_\pi^2 - m^2. \tag{6}$$

The resulting total transition probability is

$$f(W) = \int_{-1}^{+1} |A_0|^2 d\cos\theta$$
$$= \frac{g^4}{2q^2\Delta_1} \tan^{-1}\left(\frac{4q^2\Delta_1}{\Delta_1^2(W^2 - W_0^2)(W^2 - W_0^2 - 4q^2)}\right),$$
$$\Delta_1 = |\Delta|(1 - \omega/E). \tag{7}$$

This is a sharply energy-dependent effect and behaves like a resonance. This interaction is coupled to the πN inelastic scattering [reaction (2)] as indicated in Fig. 1(b) and hence, by unitarity, to the elastic scattering, Fig. 1(c). If the effect is sufficiently strong, it should produce an appreciable contribution to the total πN cross section at pion energies T_π corresponding to the center-of-mass energies W where $f(W)$ is large. Figure 2 shows f plotted as a function of T_π and compared with the experimental results on πN scattering.[4] The agreement with the position and width of the second resonance (N^{**}) in π-N scattering seems very good, in view of the fact that there are no free parameters in the model.

Let us consider the effects of spin and isospin. In the energy range in question, $2q^2 < |\Delta|$; hence we may, as a rough approximation, ignore the θ dependence of $A_0(W, \theta)$ in (6). In other words, the amplitude should contribute mainly to the s state. The only effect of taking account of the spin is to introduce several amplitudes A_i each having some angular factor $\gamma_i(\theta)$ in the numerator. But if the θ dependence of the denominator is not important, this results in multiplying $f(W)$ by some weakly energy-dependent factor, and does not change the main features of the result. Thus, we can say that this effect, looking like a "resonance" in the

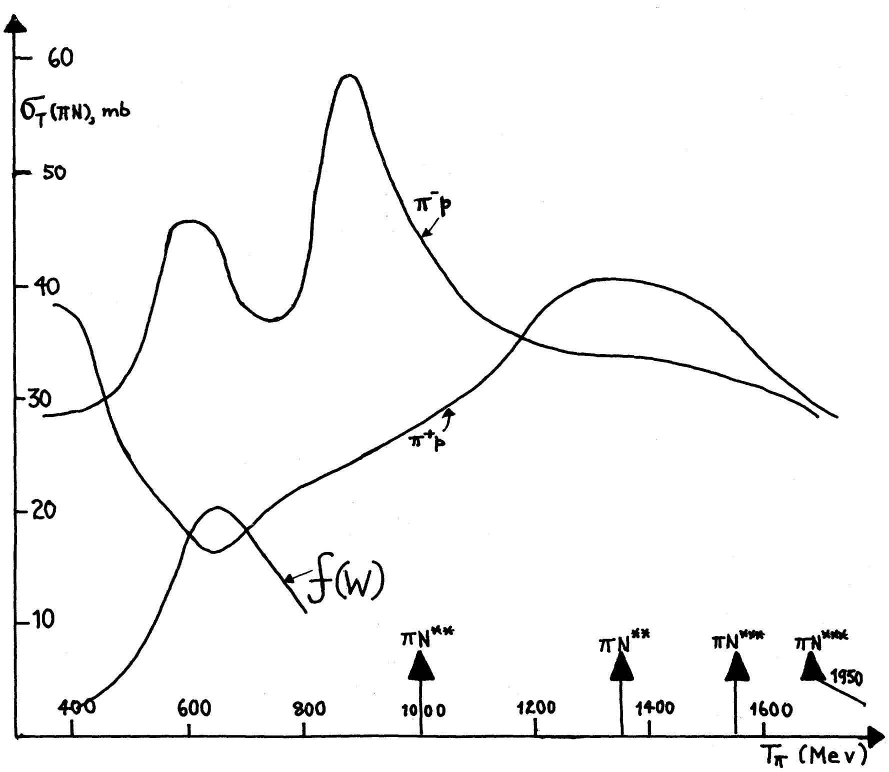

FIG. 2. The function $f(W)$ plotted as a function of the pion lab energy T_π required to produce a total energy W in the center-of-mass system. The approximate positions of the πN^{**} and πN^{***} resonances are also shown and these theoretical predictions are compared with the experimental results for the total cross sections for πN scattering.

s-wave π-N^* scattering, should produce a strong effect in the ($\frac{3}{2}$-) state of the π-nucleon scattering via the reaction of Fig. 1(b).[5] The isotopic spin dependence is not so clear; however, the contribution of A_0 to the $T=\frac{1}{2}$ state and $T=\frac{3}{2}$ state have opposite signs and the interference of these two contributions with the (appreciable) background may be constructive and destructive, respectively. This might even lead to a dip, rather than a peak, in the π^+p cross section.

The physical interpretation of the mechanism discussed above is easily understood. The isobar, being unstable, can decay into a pion and a nucleon which subsequently absorbs another pion. This [Fig. 1(a)] represents a real process (apart from the finite isobar lifetime) and it is not surprising that it should show up in the physical region. The same phenomenon should be possible whenever an unstable isobar interacts with one of its decay products and may be the explanation of the "resonance" in $\bar{K}N$ scattering due to the possible inelastic excitation of a πY^* state (Y^* being the $\pi\Lambda$ isobar whose mass lies below the $\bar{K}N$ threshold).[6]

The mechanism described corresponds to an s-wave πN^* interaction. The p wave can be studied in the static limit by the conventional effective-range theory which leads to predicted resonances in the ($\frac{1}{2}$ $\frac{5}{2}$+) and ($\frac{3}{2}$ $\frac{3}{2}$+) states. These are the states suggested by Wong and Ross,[7] who consider essentially the iteration of Fig. 1(a), but carry out the calculation in the static-nucleon, rather than static-isobar limit. (The mechanism discussed here does not occur in either static limit.) The third π-N "resonance" (N^{***}) near 900 Mev can be interpreted as the manifestation of these p-wave resonances.

Lastly, the arguments leading to the prediction of N^{**} can be applied to possible πN^{**} and πN^{***} interactions, with intermediate single-nucleon or N^* states, giving rise to four expected peaks in π-N interactions in the energy range indicated in Fig. 2. The situation is much more complex in this energy region, but this may be a partial explanation of the observed broad peak in the πN cross section above 1 Bev.

The model presented here is only approximate; it would be desirable to study it more quantitatively, in particular taking proper account of the spins and of the coupling between the various channels. Further work along these lines is being carried out and will be reported later.

I am very grateful to J. E. Paton for many valuable discussions during the course of this work and wish to acknowledge many stimulating comments and suggestions from L. Castillejo, D. J. Thouless, Professor R. E. Peierls, and Professor S. Mandelstam.

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[1]L. Landovitz and L. Marshall, Phys. Rev. Letters 4, 474 (1960); H. A. Bethe and P. Carruthers, Phys. Rev. Letters 4, 536 (1960); R. F. Peierls, Phys. Rev. 118, 325 (1960); R. F. Peierls, Phys. Rev. Letters 5, 166 (1960).

[2]S. J. Lindenbaum and R. M. Sternheimer, Phys. Rev. 106, 1107 (1957).

[3]If a one-level formula is assumed for the 33 resonance; then both g^2 and Δ are proportional to the width, so that even if $\Delta \to 0$ the cross section never becomes infinite.

[4]P. Falk-Vairant and G. Valladas, Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960).

[5]If we assume maximum inelasticity for this reaction, this fixes $\sigma_{\text{total}}(\pi N; J=\frac{3}{2}-) = 4\pi\lambda\!\!\!^{-2}$ which is the normalization used in Fig. 2.

[6]Reported at the Berkeley Conference on Strong Interactions, December, 1960 (unpublished). I am grateful to Professor S. Mandelstam for bringing the $\bar{K}N$ resonance to my attention.

[7]W.-N. Wong and M. Ross, Phys. Rev. Letters 3, 398 (1959). Y. Tomozawa (to be published) has also calculated N^* scattering in the static N^* limit but he discarded the nucleon poles which give rise to the effects considered here.