ROLE OF THE PERIPHERAL INTERACTION IN PROTON-PROTON COLLISIONS

Franco Selleri
CERN, Geneva, Switzerland
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It has been suggested in a recent paper by Bonsignori and the writer[1] that the inelastic scattering of any particle on a target nucleon may be dominated by the exchange of one single virtual pion. Analyzing some known experiments,[2,3] it was there shown that in the Bev region there is good evidence for such a mechanism, when the incoming particle is a π meson or a nucleon. Further experiments[4] have shown the same qualitative evidence which consists essentially in a low-energy peaking of the laboratory spectrum of the recoil nucleon. It seemed therefore desirable to go further and calculate the cross section for the process

$$p+p\rightarrow p+n+\pi^{+} \qquad (1)$$

under the assumption of dominating peripheral interaction. We wish to stress that this dominance is assumed throughout the whole physical region and not only for very small momentum transfers as has been done in recent work.[5] The reason for choosing reaction (1) is that final-state interactions between the nucleons are negligible at high energy, while the (3, 3) interaction of the pion is likely to involve only one of the nucleons and thus not to destroy the typical effects of the one-pion exchange. A typical peripheral diagram for this process is shown in Fig. 1. p_1 and p_2 are the 4-momenta of the incoming protons; q, q_1, and q_2 those of the outgoing pion, proton, and neutron, respectively. Together with this, one should consider the other three diagrams obtained by exchanging the initial protons and the final nucleons.

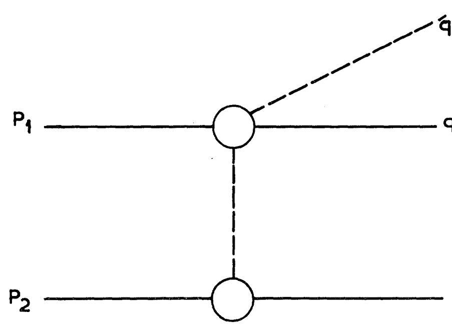

FIG. 1. Peripheral diagram contributing to process (1). Full and dashed lines represent nucleons and pions, respectively.

However, simple isotopic spin arguments show that the separate contribution to the total cross section for process (1) of the two diagrams in which the π^{+} and the neutron emerge from the same vertex is one ninth that of the diagram of Fig. 1. Therefore we can consider them of secondary importance and neglect them in our calculation. Furthermore, we shall be forced to calculate the nucleon-nucleon-pion vertex and the pion propagator to lowest order, and to substitute for the four-particle matrix element its expression known phenomenologically for the case in which all four particles are on the mass shell. We will see a posteriori upon comparison with experiments that these approximations probably are not too bad.

In the stated approximation the S-matrix element for process (1) can be written

$$S_{fi}=-(2\pi)^{7/2}\frac{m^2}{(2\omega_q E_{p_1}E_{p_2}E_{q_1}E_{q_2})^{1/2}}\delta^4(P_f-P_i)\sqrt{2}\,G_r\frac{1}{\sqrt{2}}[M(p_1p_2)-M(p_2p_1)], \qquad (2)$$

where

$$M(p_1p_2)=\frac{\bar{u}(q_2)\gamma_5 u(p_2)}{(q_2-p_2)^2+\mu^2}\bar{u}(q_1)[-A+i\vec{q}B]u(p_1). \qquad (3)$$

In (2), $\omega_q(E_l)$ is the zero component of the four-vector $q(l)$, $P_f=q+q_1+q_2$, $P_i=p_1+p_2$, and $\sqrt{2}\,G_r$ is the rationalized and renormalized coupling constant for emission of a charged pion. The factor $2^{-1/2}$ and the minus sign are typical effects of the identity of the initial protons.[6] In (3), A and B are the well-known invariant amplitudes for pion-nucleon scattering (in our case in the state of isospin 3/2) as defined for example by Chew et al.,[7] and m (μ) is the nucleon (pion) mass. From (1) and (3) it is in principle possible to calculate any cross section at any energy. We choose to calculate $d\sigma/dT$, T being the lab kinetic energy of the neutron. The importance of this variable was stressed in reference 1 and is essentially due to its simple relation with the 4-momentum transfer to the target nucleon: $(p_2-q_2)^2=2mT$. In performing the actual calculation we had in mind a comparison with the 970-Mev experiment of the Birmingham group.[2] At this energy, the relative energies of the π^+ and the proton are in the region where the (3, 3) resonance dominates. We have therefore kept only the $J=3/2$ P-wave part of A and B. For future reference we write down the result of our calculation:

$$\frac{d\sigma}{dT}=\frac{2f^2m}{\pi p_i^2\mu^2}\int_{m+\mu}^{g(T)} wR(w)\sigma(w)[a(T)+b(T,w)+c(T,w)]dw, \tag{4}$$

where $b^2=\mu^2(16\pi m^2)^{-1}G_r^2=0.08$, p_i is the lab momentum of the incoming proton, $w=[-(q+q_1)^2]^{1/2}$ is the total energy of π^+ and p in their c.m. system, and $\sigma(w)$ is the π^+p cross section at a c.m. energy w.[8] R, a, b, and c are the following functions:

$$R(w)=\tfrac{1}{2}[w^4-2w^2(m^2+\mu^2)+(m^2-\mu^2)^2]^{1/2}, \tag{5}$$

$$a(T)=\frac{\Delta_2^2}{(\Delta_2^2+\mu^2)^2}, \tag{6}$$

$$b(T,w)=\frac{\Delta_1^2}{(\Delta_1^2+\mu^2)^2}, \tag{7}$$

$$c(T,w)=\frac{\phi}{(\Delta_1^2+\mu^2)(\Delta_2^2+\mu^2)}, \tag{8}$$

$$\phi=(w-m)[(E_{p_2}'+m)(E_{p_1}'+m)]^{1/2}\cos\theta_i+(w+m)[(E_{p_2}'-m)(E_{p_1}'-m)]^{1/2}\frac{3\cos^2\theta_i-1}{2}. \tag{9}$$

In the preceding formulas Δ_1^2 and Δ_2^2 are defined by

$$\Delta_1^2=(p_2-q_1)^2=m^2+2m(T_i-T)-w^2,$$

$$\Delta_2^2=(p_2-q_2)^2=2mT,$$

T_i being the lab kinetic energy of the incoming proton and E_{p_1}', E_{p_2}', θ_i the energies of the initial protons and the angle between their directions, all in the c.m. system of the π^+ and the final proton. These three quantities can be expressed as a function of W, total c.m. energy for reaction (1), w^2, Δ_1^2, and Δ_2^2 through the following relations:

$$E_{p_1}'=(w^2+m^2+\Delta_2^2)/2w, \tag{10}$$

$$E_{p_2}'=(w^2+m^2+\Delta_1^2)/2w, \tag{11}$$

$$\cos\theta_i=(\vec{p}_1\cdot\vec{p}_2)/p_1p_2, \tag{12}$$

where $\vec{p}_1$ and $\vec{p}_2$ are three-momenta correspond-

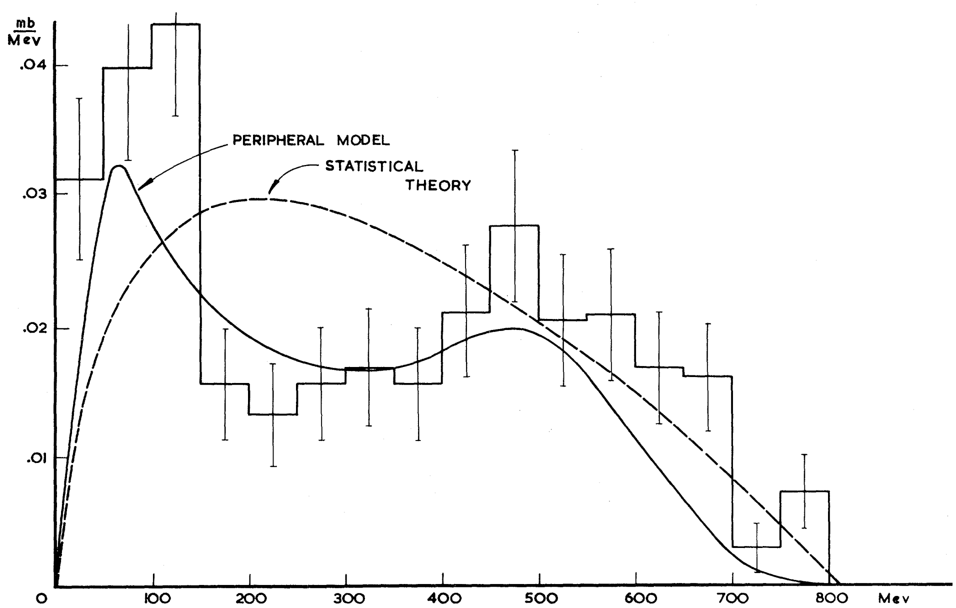

FIG. 2. Experimental energy spectrum in the lab system for neutrons from reaction (1) as compared with statistical theory (dashed line) and our formula (4) (full line).

ing to E_{p_1}' and E_{p_2}', respectively, and

$$\vec{p}_1 \cdot \vec{p}_2 = -\alpha + \frac{(2\alpha - \Delta_1^2)(2\alpha - \Delta_2^2)}{4\omega^2}, \qquad (13)$$

with $\alpha = W^2/2 - m^2$. Finally we have for the upper limit of integration

$$g(T) = \left[m^2 - W^2 \frac{T}{m} + W(W^2 - 4m^2)^{1/2} \left(\frac{T^2}{m^2} + \frac{2T}{m} \right)^{1/2} \right]^{1/2}. \qquad (14)$$

The predictions of formula (4) at 970 Mev are shown in Fig. 2 and compared with the experimental spectrum.[9] As is seen, the agreement is good while a statistical spectrum fails to explain the three main features of the experimental distribution; these are: the strong peaking at low (~ 50 Mev) energy, the broad bump at high (~ 500 Mev) energy, and the minimum between them. These features are, however, quantitatively predicted from our formula (4) and are typical effects of the peripheral interaction (see reference 1 for a more detailed discussion).

Theoretically, the total cross section[10] is predicted to be 12 mb; its experimental value is 16.4 ± 0.7 mb.[2] This indicates that something more than the diagrams of Fig. 1 contributes to process (1). However, this "something," whatever it is, is not more than 30% of the peripheral contribution. In view of these results it seems very desirable to have a deeper comparison between this model and the experimental data.[11]

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[1]F. Bonsignori and F. Selleri, Nuovo cimento 15, 465 (1960).
[2]A. P. Batson et al., Proc. Roy. Soc. (London) 251, 218 (1959).
[3]V. Alles-Borelli et al., Nuovo cimento 14, 211 (1959).
[4]I. Derado, Nuovo cimento 15, 953 (1960); E. Pickup et al., Phys. Rev. Letters 5, 161 (1960).
[5]S. D. Drell, Phys. Rev. Letters 5, 278 (1960); 5, 342 (1960).
[6]N. N. Bogoliubov and D. V. Shirkov, Introduction to the Theory of Quantized Fields (Interscience Publishers, New York, 1959), p. 259.
[7]G. Chew et al., Phys. Rev. 106, 1337 (1957).
[8]In practical calculations one can use for $\sigma(w)$ either directly its experimental value or a phenomenological fit like the Breit-Wigner formula given by M. Gell-Mann and K. Watson, Annual Review of Nuclear Science (Annual Reviews, Inc., Palo Alto, California, 1954), Vol. 4, p. 219.
[9]This spectrum was already used in reference 1 and was originally a private communication from Dr. L. Riddiford to Professor A. Minguzzi.
[10]Calculations of the total cross section for process (1) under similar assumptions have already been performed by T. Kobayashi, Progr. Theoret. Phys. (Kyoto) 18, 318 (1957), who, however, did not take into account the effects of the Pauli principle, and by G. Da Prato, University of Rome, thesis (unpublished).
[11]This work was conceived as a collaboration with Professor A. Stanghellini, Dr. S. Bergia, and Dr. B. Bortolani of Bologna University who are now working on other aspects of the problem.

INTERMEDIATE BOSON PRODUCTION IN PION-PROTON COLLISIONS*

Norman Dombey
California Institute of Technology, Pasadena, California
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The current-current hypothesis has been applied in recent years to the study of the weak interactions. The conserved vector current[1] hypothesis and the pionic character of the divergence of the axial vector current[2] have evolved in its wake. So has the idea of a boson field which mediates all weak interactions.[3] This note deals only with the vector part of the current, leaving for another time similar calculations with the axial vector part; and examines the possibility of producing the boson W, in pion-nucleon collisions, in case of W having relatively low mass.[4]

For definiteness let us consider the reaction

$$\pi^+ + p \rightarrow W^+ + p, \qquad (\mathrm{I})$$

and concentrate solely on the vector part of the interaction. Then, on inverting incoming and outgoing particles, it is apparent that the process is almost exactly the same as electroproduction of pions (i.e., production of pions by photons off the mass shell). The differences are: (*i*) The coupling constant is not the electromagnetic coupling constant. (*ii*) The four-momentum of the "photon" is timelike, not spacelike. But the anal-