# CONSEQUENCES OF A SCALAR $\Sigma\Lambda\pi$ COUPLING

Jeremy Bernstein*† and Reinhard Oehme‡
Institute for Theoretical Physics, University of Vienna, Vienna, Austria
(Received May 2, 1961)

It is the purpose of this note to point out some new experimental and theoretical possibilities which follow from the assumption that the relative $\Sigma\Lambda$ parity is odd.[1] Using a crude dispersion-theoretic estimate of the scalar $\Sigma\Lambda\pi$-coupling constant, we discuss the rates for the decay processes $\Sigma^{\pm}\rightarrow e^{\pm}\pm\nu+\Lambda^0$ and $\Sigma^0\rightarrow\gamma+\gamma+\Lambda^0$, and we remark briefly upon some other consequences of a possibly strong scalar $\Sigma\Lambda\pi$ coupling.

Let us begin with the evaluation of the constant $g_{\Sigma\Lambda\pi}$. Because the $\Sigma\Lambda$ parity is odd, the $\Sigma$ can be considered as a composite $\Lambda\pi$ system in an $s$ state.[2] The $s$-wave amplitude for $\pi\Lambda$ scattering, which is defined by $f_0(s)=q^{-1}(s)\sin\delta_0(s)\times\exp i\delta_0(s)$, with $q^2(s)=(4s)^{-1}[s-(m_\Lambda+m_\pi)^2]\times[s-(m_\Lambda-m_\pi)^2]$, has a pole at $s=m_\Sigma^2$. The residue of this pole may be expressed in terms of $g_{\Sigma\Lambda\pi}$; we find

$$f_0(s)=\frac{g_{\Sigma\Lambda\pi}^{\;2}}{4\pi}\frac{(m_\Sigma+m_\Lambda)^2-m_\pi^{\;2}}{2m_\Sigma}\frac{1}{m_\Sigma^{\;2}-s}+\text{other (nonpole) terms.} \quad (1)$$

In order to make a simple first approximation, let us assume that $f_0[s(q)]$ is a regular function of the c.m. momentum $q$, except for the $\Sigma$ pole at $q=q(s=m_\Sigma^2)\equiv i\kappa\approx i(2m_{\rm red}B)^{1/2}$. From the unitarity condition we find

$$f_0(s)=\frac{-\kappa+iq(s)}{q^2(s)+\kappa^2}. \quad (2)$$

If we compute the residue[3] of the pole at $s=m_\Sigma^2$ from Eq. (2) and compare the result with Eq. (1), we obtain[4]

$$\frac{g_{\Sigma\Lambda\pi}^{\;2}}{4\pi}\approx\frac{16m_\Sigma^{\;5}\kappa}{[(m_\Sigma+m_\Lambda)^2-m_\pi^{\;2}][m_\Sigma^{\;4}-(m_\Lambda^{\;2}-m_\pi^{\;2})^2]}\approx 1.8. \quad (3)$$

How good is this "zero-range" approximation? Presumably the most important correction is due to the $\Sigma$ pole in the crossed reaction, the residue of which is again determined by $g_{\Sigma\Lambda\pi}^{\;2}$. For the $s$-wave amplitude $f_0(s)$, the relevant contribution from this pole appears in the form of a short logarithmic branch cut,[5]

$$(m_\Lambda^{\;2}-m_\pi^{\;2})^2/m_\Sigma^{\;2}\leq s\leq 2m_\Lambda^{\;2}+2m_\pi^{\;2}-m_\Sigma^{\;2}. \quad (4)$$

Preliminary estimates seem to indicate that the effect of this branch line is to increase the value (3) of the coupling constant. Aside from the $\pi\Sigma$ and $\bar{K}N$ thresholds for $s>(m_\Lambda+m_\pi)^2$, we have also left out the circular branch line $|s|=m_\Lambda^{\;2}-m_\pi^{\;2}$, which is due to the intermediate pion states of the reaction $\pi+\bar{\pi}\rightarrow\Lambda+\bar{\Lambda}$. This neglect seems to be reasonable, provided there are no low-mass two-pion resonances with $T=0$.

In what follows we make the working assumption that the scalar coupling constant is of the order one, say $g_{\Sigma\Lambda\pi}^{\;2}/4\pi=1.5$. As a first application, we estimate the rate for the decay $\Sigma^-\rightarrow\Lambda^0+e^-+\bar{\nu}$ on the basis of several assumptions.

We assume that the strangeness-conserving vector current is conserved. Therefore we may neglect the vector contribution to the decay rate since the $\Sigma\Lambda$ transition has $\Delta T\neq 0$. The vector part vanishes in the allowed approximation in accordance with the selection rule $\Delta T=0$. If the vector current is not conserved, one would expect that its over-all contribution amounts to an increase of the total decay rate.

The matrix element of the axial vector current may be written in the form (noting the assumption of odd $\Sigma\Lambda$ parity),

$$\langle\Lambda|P_\alpha|\Sigma^-\rangle=\tfrac{1}{\sqrt{2}}\bar{u}_\Lambda[\gamma_\alpha a(-p^2)-ip_\alpha b(-p^2)+\sigma_{\alpha\beta}p_\beta c(-p^2)]u_\Sigma, \quad (5)$$

where $p=p_\Sigma-p_\Lambda$, and $a(0)=G_A{}'$ is the corresponding renormalized axial coupling constant. The coefficients in Eq. (5) are real analytic functions which are regular in the complex $p^2$ plane except for the branch lines[6] $-p^2\geq(3m_\pi)^2$ and, in the case of $b(-p^2)$, the pion pole at $-p^2=m_\pi^{\;2}$. We assume that in the neighborhood of the interval $0<-p^2\leq(m_\Sigma-m_\Lambda)^2$ the function $(m_\Sigma-m_\Lambda)a(-p^2)-p^2b(-p^2)$ is dominated by this pion pole. By well-known methods we find the relation[7]

$$(1/\sqrt{2})G_A{}'=g_{\Sigma\Lambda\pi}F(m_\pi^{\;2})/(m_\Sigma-m_\Lambda), \quad (6)$$

where $F(m_\pi^2)$ is determined by the pion lifetime. We may compare Eq. (6) with the Goldberger-Treiman formula for nuclear $\beta$ decay. Denoting the usual axial vector coupling constant by $G_A$, we can write

$$G_A'/G_A = (g_{\Sigma\Lambda\pi}/\sqrt{2}\, g_{NN\pi})[2m_N/(m_\Sigma - m_\Lambda)].$$

According to our zero-range calculation we may take $g_{\Sigma\Lambda\pi}{}^2/g_{NN\pi}{}^2 \approx 1/10$ and find $G_A'/G_A \approx 6$. We assume that in the region of interest the function $a(-p^2)$ is slowly varying so that $a(-p^2) \approx G_A'$; then we obtain

$$b(-p^2) \approx G_A'(m_\Sigma - m_\Lambda)/(m_\pi{}^2 + p^2). \qquad (8)$$

In the computation of the decay rate for $\Sigma^- \to \Lambda^0 + e^- + \bar{\nu}$, we can neglect the $b$ term in Eq. (5) because its contribution becomes proportional to the electron mass. The $c$ term in Eq. (5) is difficult to estimate; it is unlikely to make a very large contribution, and we neglect it. With these assumptions, the rate is given in terms of our constant $G_A'$ by the "allowed" $\beta$-decay formula,

$$\Gamma_{\Sigma^- \to \Lambda^0 + e^- + \bar{\nu}} = (G_A'^2/60\pi^3)(m_\Sigma - m_\Lambda)^5. \qquad (9)$$

We compare this decay rate with the experimental rate for the strangeness-changing process $\Sigma^- \to n + e^- + \bar{\nu}$, which is taken to be about 1/10 of the rate obtained from a straightforward calculation using the universal constant. With our value $G_A' \approx 6G_A$ we find $\Gamma_{\Sigma^- \to \Lambda^0 + e^- + \bar{\nu}}/\Gamma_{\Sigma^- \to n + e^- + \bar{\nu}} \approx 1/10$, a number which is about 30 to 40 times larger than one would predict on the basis of phase-space considerations. This larger rate is a reflection of the renormalization effects due to the rather large scalar $\Sigma\Lambda\pi$ coupling. We believe that, as far as our approximations are concerned, we have consistently underestimated this effect. We note that the states $|\bar{\Sigma}^-\Lambda^0\rangle$ and $|\Sigma^+\bar{\Lambda}^0\rangle$ are related by $G$ conjugation. If the current $P_\alpha$ is such that it transforms like $\bar{n}\gamma_\alpha\gamma_5 p$ under $G$ conjugation, then we have $G_A'(\Sigma^- \to \Lambda^0 + e^- + \bar{\nu}) = -G_A'(\Sigma^+ \to \Lambda^0 + e^+ + \nu)$. Consequently, within our approximations, the rate for the decay $\Sigma^+ \to \Lambda^0 + e^+ + \nu$ should be the same as that for the $\Sigma^-$ decay.

We hope that there will soon be enough data on $\beta$ decays of $\Sigma$ hyperons to make possible a serious test of our considerations.[8]

It has been mentioned by Okun and Rudik[9] that for odd $\Sigma\Lambda$ parity an intermediate $\pi^0$ state may play an important role in the calculation of the process $\Sigma^0 \to \Lambda^0 + \gamma + \gamma$. Since we propose a rather large value for the scalar $\Sigma\Lambda\pi$ coupling, it seems reasonable to estimate the rate on the basis of this contribution. We have

$$\Gamma_{\Sigma^0 \to \Lambda^0 + \gamma + \gamma} = (4/945)(1/\pi)(g_{\Sigma\Lambda\pi}{}^2/4\pi)\Gamma_{\pi^0 \to 2\gamma} + \text{magnetic terms},$$

and with a $\pi^0$-decay rate[10] of $0.3 \times 10^{16}$ sec$^{-1}$ we see that, even for $g_{\Sigma\Lambda\pi}{}^2/4\pi \approx 10$, the rate for the decay $\Sigma^0 \to \Lambda^0 + \gamma + \gamma$ is negligible compared to that for the decay $\Sigma^0 \to \Lambda^0 + \gamma$, which may be of the order of $10^{19}$ sec$^{-1}$. The "magnetic terms" have been discussed in reference 9; they do not change the situation.

A strong, scalar $\Sigma\Lambda\pi$ coupling gives rise to a corresponding spin-independent two-pion exchange force between $\Lambda$ and $N$. Spin-dependent forces must have a somewhat shorter range because they require the exchange of three pions or a $K$ meson.

Finally we would like to remark that a large value of $g_{\Sigma\Lambda\pi}$ may well give rise to a $J = 1/2$ resonance in the $\pi\Lambda$ system.[11]

It is a pleasure to thank Professor R. Karplus and Professor W. E. Thirring for stimulating conversations.

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*Permanent address: Brookhaven National Laboratory, Upton, New York.

†National Science Foundation Post-Doctoral Fellow.

‡Permanent address: Enrico Fermi Institute for Nuclear Studies, University of Chicago, Chicago, Illinois.

[1]For a discussion of arguments in favor of odd $\Sigma\Lambda$ parity, as well as for references to the relevant papers, see Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 6, 377 and 506(E) (1961). See also S. Barshay and M. Schwartz, Phys. Rev. Letters 4, 618 (1960).

[2]S. Barshay and M. Schwartz (reference 1).

[3]M. L. Goldberger, Y. Nambu, and R. Oehme, Ann. Phys. 2, 226 (1957); this paper contains an analogous determination of the residue of the deuteron pole in $np$ scattering.

[4]A similar determination of $g_{\Sigma\Lambda\pi}$ has been made by Nambu and Sakurai, reference 1. Essentially, these authors evaluate the pole term in Eq. (1) at threshold and equate it to $-\kappa^{-1}$. It is easily seen from Eq. (2) that the extrapolation to $q = 0$ gives rise to a factor $\frac{1}{2}$ in the expression for $g_{\Sigma\Lambda\pi}{}^2/4\pi$.

[5]It should be noted that the branch line (4) corresponds to a force with maximal range $r_0 = (2m_\pi)^{-1}$. This is because, for systems of two particles with unequal mass, the "force range" should be determined from the position of the left-hand singularities in the $q^2$ plane. The branch line (4) lies inside the circle $|s| = m_\Lambda{}^2 - m_\pi{}^2$; it is mapped into the second sheet of the $q^2$ plane and determines the weight function along the cut $-m_\Lambda{}^2 \leq q^2 \leq -m_\pi{}^2$ [see R. Oehme, Phys. Rev. Letters 4, 246 (1960)]. The upper limit of this branch line corresponds to a range $r_0 = (2m_\pi)^{-1}$.

[6]We have $\langle 0|\partial_\alpha P_\alpha|2\pi\rangle = 0$ if we assume that $\partial_\alpha P_\alpha$ transforms like the pion field under $G$ conjugation. Note that this requires that $P_\alpha$ contains the $\Sigma\Lambda$ terms in the combination $\bar{\Lambda}\gamma_\alpha\Sigma^+ - \bar{\Sigma}^-\gamma_\alpha\Lambda$.

[7]Equation (6) and related relations have been written down by many authors. See, for instance, L. Okun, J. Exptl. Theoret. Phys. (U.S.S.R.) 39, 214 (1960) [translation: Soviet Phys.—JETP 12, 154 (1961)];

J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo cimento 17, 757 (1960); Chou Kuang-Chao, Dubna Report D514, 1960 (unpublished); these papers contain further references. It is the purpose of this note to use Eq. (6) in order to produce a number for the decay rate.

[8]It may be that some of the observed $\Sigma\beta$ events are due to $\Sigma \rightarrow \Lambda + e + \nu$; it will be difficult to separate $\Sigma\Lambda$ events from $\Sigma n$ events unless the $\Lambda$ decay is seen.

[9]L. Okun and A. Rudik, J. Exptl. Theoret. Phys. (U.S.S.R.) 39, 378 (1960) [translation: Soviet Phys.– JETP 12, 268 (1961)].

[10]R. F. Blackie et al., Phys. Rev. Letters 5, 384 (1960).

[11]F. Duimio and G. Wolters, Nuovo cimento (to be published) and CERN preprint.

## POSSIBLE MECHANISM FOR THE PION-NUCLEON SECOND RESONANCE

Ronald F. Peierls*
Department of Mathematical Physics, The University, Birmingham, England
(Received February 17, 1961)

Several phenomenological models[1] have been proposed to discuss pion-nucleon interactions near the higher resonances. There seems to be considerable evidence from these analyses that final states described by the 33 isobar model[2] play an important role, but so far there has been little quantitative discussion or explanation of the resonant peaks. In the present note it is shown that a simple model can give a good quantitative description of the second resonance, a rough description of the third, and a plausible explanation of the peak above 1.0 Bev. The model consists in taking seriously the concept of the 33 isobar as an unstable particle ($N^*$) of spin and isospin $\frac{3}{2}$ and having a complex mass. This means that we assume the low-energy pion-nucleon scattering to be dominated by the one $N^*$ direct pole. The position and width of the 33 resonance then enable us to find the mass of the isobar $M(N^*)$:

$$[M(N^*)]^2 = M^2 + i\Delta,$$
$$M = 8.94\,\mu_\pi, \quad |\Delta| = 8.05\,\mu_\pi^{\,2}, \tag{1}$$

where $\mu_\pi$ is the pion mass.

We consider next the reaction

$$\pi + N \rightarrow \pi + N^*. \tag{2}$$

This is strongly coupled to the process

$$\pi + N^* \rightarrow \pi + N^*. \tag{3}$$

Let us try to apply the usual formalism to reaction (3), pion-isobar scattering. For the moment we ignore the spin and isospin and assume that the reaction is described by some amplitude $A(s, \bar{s}, t)$ where $s$, $\bar{s}$, $t$ are defined in terms of the pion energy $\omega$, the real part $E$ of the isobar energy, and the center-of-mass momentum and scattering angle $q$, $\theta$:

$$s = (E+\omega)^2 + i\Delta(1+\omega/E),$$
$$\bar{s} = (E+\omega)^2 - 2q^2(1+\cos\theta) + i\Delta(1-\omega/E),$$
$$t = -2q^2(1-\cos\theta),$$
$$s + \bar{s} + t = 2M^2 + 2\mu_\pi^{\,2} + 2i\Delta. \tag{4}$$

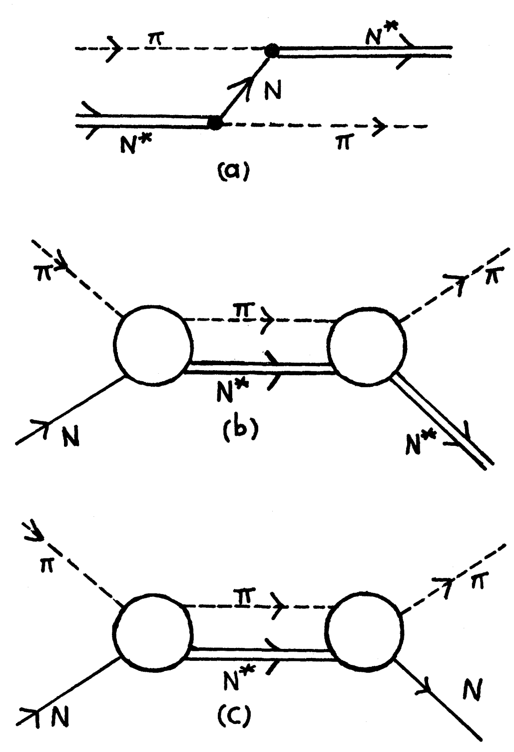


FIG. 1. (a) Crossed one-nucleon diagram for $\pi N^*$ scattering. (b) Schematic coupling between $\pi N^*$ scattering and $\pi N^*$ production in $\pi N$ collisions. (c) Schematic coupling between $\pi N^*$ production and $\pi N$ scattering.