Since this structure is oriented along the axis of the wire, it is somewhat surprising that such large differences between the parallel and transverse orientations were found.

The critical current at zero external field was also measured with direct currents in the samples. The direct-current values were 1/3 lower than those obtained with pulsed techniques. This is attributed to heating at the current joints or in the cladding which surrounds the Nb_3Sn core. This heating is substantially reduced with pulse techniques.

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†Operated for the U. S. Atomic Energy Commission by the Union Carbide Corporation.

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FERMI SURFACE TOPOLOGY OF EVEN-VALENT METALS FROM THEIR MAGNETORESISTANCE ANISOTROPY

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The anisotropy of the magnetoresistance of Au and Cu has recently been successfully interpreted by employing the concept of open orbits on a multiply-connected Fermi surface in a periodically extended zone scheme.[1] Open orbits occur for a limited range of field directions, and the transverse magnetoresistance $\rho_{\alpha\alpha}$ in a sufficiently high field H with the current making an angle α with the direction of the open orbits is then of the form,[2]

$$\rho_{\alpha\alpha} = A + BH^2\cos^2\alpha. \qquad (1)$$

A plot of the resistance in a fixed high field whose direction varies is characterized by sharp maxima for field directions permitting some open orbits (with $\alpha \neq \pi/2$), which rise above the low resistance obtained for most field directions for which only closed orbits occur and the resistance saturates.

This behavior is in marked contrast with the anisotropy of the magnetoresistance observed for Mg,[3] Zn,[4] Cd,[4] Pb,[5] and Sn[5] (the transition metals are not considered here and insufficient data are available for other even-valent metals). For these metals the resistance varies quadratically with field and is uniformly high for most field directions, falling to sharp minima for certain field directions where it varies less rapidly.

The proposed explanation of this behavior in the even-valent metals is as follows. Consider the range of field directions for which closed cyclotron orbits occur over the whole Fermi surface, and fields so large that for all such orbits $\omega_c\bar{\tau} \gg 1$, ω_c being the cyclotron frequency and $\bar{\tau}$ the relaxation time averaged over the orbit. If the component σ_{xy} of the conductivity tensor with field H along the z axis is expanded in ascending powers of $1/H$, the leading term is of the form,[2]

$$\sigma_{xy} = -\frac{e}{cH}\frac{2}{h^3}\sum_i \int S_i(p_z)dp_z. \qquad (2)$$

Here the area $S_i(p_z)$ enclosed by the intersections with the sheet i of the Fermi surface by the plane p_z = constant will, in general, be the sum of positive terms for closed orbits containing occupied electron states of lower energy within and immediately adjacent to each orbit, and of negative terms for orbits containing hole states. For the case where all sheets of the Fermi surface are simply connected, the range of integration need be only over a single Brillouin zone (with suitable remapping), and Eq. (2) results in the well-known expression,[2-6]

$$\sigma_{xy} = -\frac{e}{cH}(n_e - n_h)N_a. \qquad (3)$$

Here N_a is the number of atoms per unit volume, and n_e and n_h are the numbers of electron and hole states per atom, which are well defined since each sheet consists of closed surfaces containing either electrons or holes.

But Eq. (3) also applies to the case where one or more sheets of the Fermi surface are multiply connected, provided no open orbits occur in the range of field directions considered and $\omega_c\bar{\tau} \gg 1$ for all closed orbits. This may be seen by extending the range of integration to sufficient periodically extended zones to include the most extended orbits of any practical significance. The value of the integral for each sheet in Eq. (2) is still independent of the field direction: Its sign determines the electron or hole character of the sheet, and its numerical value is accordingly the electron volume $n_e N_a h^3/2$ or the hole volume $n_h N_a h^3/2$ enclosed by the sheet. This is true despite the fact that on the same multiply-connected sheet both electron and hole orbits may occur, since in the case of an electron sheet every hole orbit will be surrounded by an electron orbit and vice versa for a hole sheet.[7] This provides a useful criterion for determining the electron or hole character of a multiply-connected sheet.

Moreover, inspection of the proposed Fermi surface topologies for the metals considered here according to this criterion shows that n_e and n_h are equal (and it seems a reasonable hypothesis to suppose this to be true for any even-valent metal). It follows that the leading term in σ_{xy} vanishes for all field directions for which only closed orbits with $\omega_c\bar{\tau} \gg 1$ occur, and the components ρ_{xx} and ρ_{yy} of the resistivity tensor then rise quadratically with the field.[6]

The transverse magnetoresistance in a fixed high field will therefore be uniformly high except for minima in field directions where there are open orbits making an angle $\alpha \sim \pi/2$ with the current direction, or where there are closed orbits for which $\omega_c\bar{\tau} \lesssim 1$. In the first case the field dependence of the resistance is given by Eq. (1), saturating at the highest fields when $\alpha = \pi/2$, while in the second case the resistance will saturate at intermediate fields but again rise quadratically at fields sufficiently high to make $\omega_c\bar{\tau} \gg 1$ for all orbits.

The multiply-connected hole surface for Mg is shown in Fig. 1; another hole surface is split off by the spin-orbit coupling,[8] but this need not be drawn since it is simply connected, as are the electron surfaces.[9] Measurements of the de Haas-van Alphen effect in Mg and Zn are consistent with this picture of the Fermi surface, while similar measurements for Cd suggest that the arms of the "ring" parallel to the basal plane are pinched off.[10]

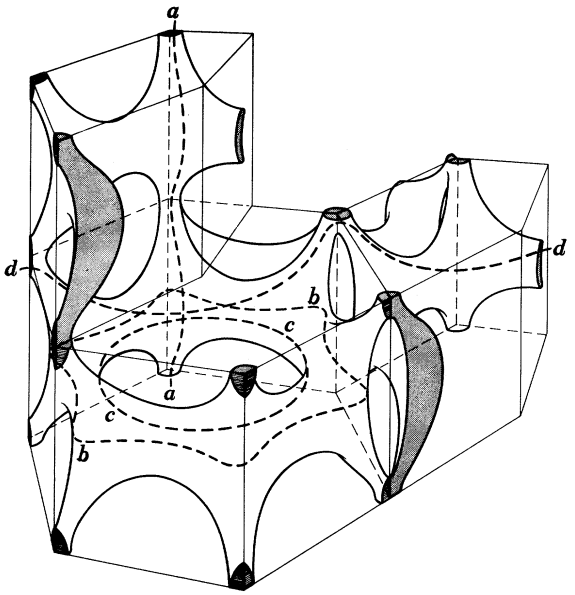

FIG. 1. The multiply-connected Fermi surface containing holes in the first Brillouin zone for Mg[9] (and probably Zn[10]), drawn in a periodically extended zone scheme. *aa*—open orbit along the hexagonal axis; *bb*, *cc*—closed orbits around the "ring"; *dd*—open orbit parallel to the basal plane.

The connectivities of these hole surfaces are consistent with most features of the observed anisotropy and field dependence of the transverse magnetoresistance. For example, the experimental results illustrated in Fig. 2, with the current J near the basal plane and the field H making an angle θ with the basal plane, show a sharp minimum in the resistance at $\theta = 0°$ in both Zn and Cd, which is due to open orbits along the hexagonal axis (for example orbit *aa* in Fig. 1). A minimum is observed in Zn also at $\theta = 90°$, which is attributed to the lower value of ω_c for orbits around the circumference of the ring (orbits *bb* and *cc* in Fig. 1) than for orbits embracing its arms; the absence of this minimum confirms that the ring is pinched off in Cd. Measurements in this orientation have not been published for Mg, but the minimum observed when H and J are coplanar with the hexagonal axis and J makes an angle of 25° with the basal plane (and in several other specimens of unspecified orientations[3]) may be due to open orbits parallel to the basal plane (orbit *dd* in Fig. 1). For Mg and Cd (and probably also for Zn) with J along the hexagonal axis and H in the basal plane, the resistance is roughly isotropic and varies quadratically with H due to open orbits along the hexagonal axis.[8]

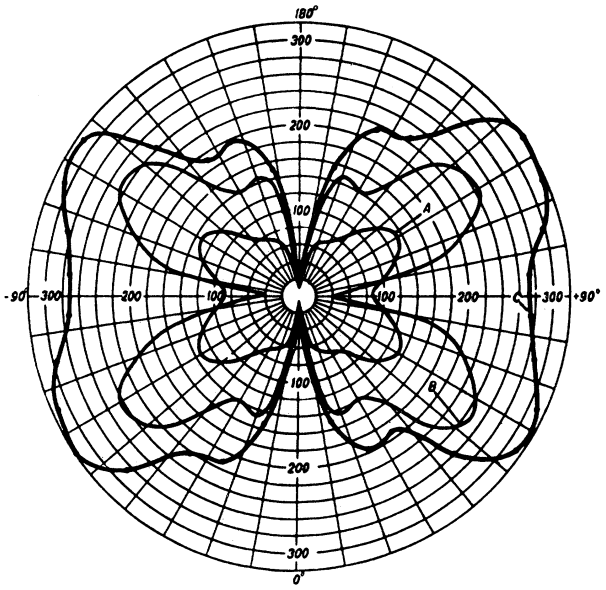

FIG. 2. Polar diagram of the transverse magnetoresistance, $\Delta\rho_H/\rho_0$, in Zn and Cd at 4.2°K. α is the angle between the current J and the hexagonal axis, and r the resistance ratio, $\rho_{273}/\rho_{4.2}$, in zero field (after Justi et al.[4]).

A—Cd2,	H=12.9 kgauss,	α=87°,	r= 800.
B—Zn10,	H=12.9 kgauss,	α=88°,	r=2100.
C—Cd47,	H=14.8 kgauss,	α=84°,	r=2000.

A multiply-connected electron surface in the third Brillouin zone for Pb, consisting in a periodically extended zone scheme of thick cylinders along the edges of the zone intersecting at its corners, was proposed by Gold.[11] With J along both [111] and [001] the transverse magnetoresistance of Pb is uniformly high and varies quadratically with the field, except for sharp minima where it saturates when H is along [110]. It is difficult to explain this result employing open orbits since, for any direction of the orbit, the angle α in Eq. (1) must differ from $\pi/2$ for one or the other of these current directions. But with H along [110] a large closed orbit exists around the inside of the multiply-connected surface in a single zone, of which there is some evidence from magnetoacoustic measurements.[12] One might expect a large cyclotron mass and hence small ω_c for this orbit, and if $\omega_c\bar{\tau}\lesssim 1$ for the fields and specimens used, one has a ready explanation of the observed anisotropy of the magnetoresistance. But this interpretation is necessarily tentative since, although the de Haas—van Alphen measurements indicate that the hole surface in the second zone is closed,[11] the behavior of the magnetoacoustic attenuation in high fields suggests that this surface also is multiply connected.[12] In any case open orbits on the surface in the third zone probably occur for some field directions.

The Fermi surface of Sn is so complex, consisting of several sheets one or more of which may be multiply connected, as to have evaded attempts at its determination using several different experimental methods.[13] The anisotropy of the magnetoresistance suggests that open orbits occur [for example, the resistance at the minimum for one field direction changes according to Eq. (1) as the current direction is varied in a plane normal to the field[11]], and the predominantly quadratic field dependence of the resistance is consistent with the hypothesis that $n_e=n_h$ for all even-valent metals.

The author has benefited from stimulating discussions with his colleagues at the Royal Radar Establishment, particularly Dr. J. M. Lock and Dr. M. D. Sturge; he is also indebted to Dr. D. H. Parkinson for his encouragement and to D. W. Charles for drawing Fig. 1.

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[7]This statement is valid even for those field directions in a plane of high symmetry for which open orbits of type A [6] occur, if such an orbit is regarded as the limiting form of a highly extended orbit. It is not true when the field is in the isolated direction of high symmetry which lies at the center of a solid angle of field directions for which open orbits of type B [2,6] occur (for example, the [100], [110], and [111] axes in Au and Cu[1]). In fact the integral in Eq. (2) cannot be defined where orbits of type B occur, and one may envisage the transformation of an electron sheet into a hole sheet as the volume occupied by electrons is expanded to fill more than half the zone by supposing the sheet to pass through intermediate stages for which orbits of type B occur for all field directions.

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RESOLVED ISOTROPIC HYPERFINE STRUCTURE OF THE ELECTRON PARAMAGNETIC RESONANCE ABSORPTION OF F CENTERS IN NaH†

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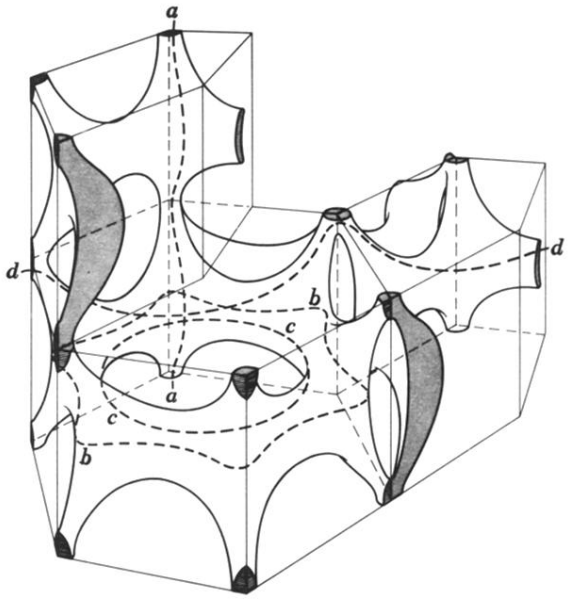
The wide, inhomogeneously broadened electron paramagnetic resonance lines of F centers in alkali halides are due to the hyperfine interactions between the center electron and the surrounding nuclei.[1,2] These lines are commonly structureless and the hyperfine interactions must be studied using the electron nuclear double resonance (ENDOR) method.[3] When the first shell contact interaction predominates, however, and the nuclear spin of the alkali ion is 3/2, one expects a nineteen-line spectrum with relative intensities of the components of 1, 6, 21, 56, 120, 216, 336, 456, 546, and 580, corresponding to total shell nuclear magnetic quantum numbers of $\pm 9, \cdots, 0$, respectively.[1] In a few crystals, viz., LiF and NaF,[4,5] and RbCl,[6] a partially resolved spectrum has been observed. The structure of the LiF line, originally reported to be isotropic,[4,5] is now known to possess many more than the predicted nineteen lines[7] and to be anisotropic.[8] ENDOR studies[9] have shown that the first shell interaction does not predominate in this case, but subsequent calculations[10] have established that the resolved spectrum is consistent with the ENDOR results and with the de Boer model. It seems likely that in RbCl, too, the spectrum is due to more than the first shell nuclei alone, since the spectrum could not be reconciled with the known isotopic abundances of the alkali ions.[6] In NaF, on the other hand, we believe that the resolved spectrum is due to the first shell interaction as originally reported.[4,5] We have found that the resolution of the NaF spectrum is unimpaired when the sample is in the form of a powder, whereas in LiF the resolution is washed out when the sample is powdered. NaF is thus the only alkali halide in which the hyperfine interaction with the first shell nuclei has been seen directly. However, even in NaF not all of the nineteen lines have been observed, and the signals observed were neither well enough resolved nor strong enough to permit a direct comparison with the predicted relative strengths of the nineteen lines. Since the favorable circumstances in NaF may be traced in part to the relatively large hyperfine interaction constant of sodium, and in part to the low atomic number of fluorine, we were led to examine the spectrum of the very similar crystal NaH, in which it was to be expected that the situation should be even more favorable for the appearance of a resolved first shell structure.

It is known that the hydrides may be colored by radiation and that in LiH the principal product



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