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ELECTROSTATIC POTENTIAL GRADIENTS IN A PENNING DISCHARGE

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It is well known that electrostatic voltage gradients over dimensions appreciably greater than a Debye length do not exist in a plasma in thermal equilibrium.[1,2] However, it has been shown that self-consistent potential gradients may be calculated for appropriate nonthermal velocity distributions of ions and electrons.[3] The purpose of this Letter is to show that the Penning or oscillating-electron discharge produces such distributions together with potential gradients in the discharge. The mechanism by which the gradients are generated and maintained is the net volume production of ion-electron pairs.

The existence of a potential gradient can be justified by the following physical argument. The density n of particles of a given type is

$$n = \int \frac{d\phi}{v} = \phi/\bar{v}, \tag{1}$$

where ϕ is the scalar particle flux and $\bar{v}$ the magnitude of the average velocity. If a potential gradient is assumed to exist initially (such as the vacuum electric field), then ions will be accelerated in the direction of the electric field and electrons will be decelerated. Consequently, if the flux remains constant for both, the density of ions will decrease and the density of electrons will increase. However, if a fraction of the electron flux is electrostatically reflected by the potential gradient, then the flux of electrons can decrease sufficiently to maintain the charge density equality between ion and electrons,[4] and the appropriate potential gradient may exist over arbitrary length scales. (A mathematical treatment of this situation in one dimension is given in reference 3.)

This argument may be generalized to show that a potential gradient not only can exist but may be self-generated and maintained in the presence of ion-electron pair production. To show that potential gradients can exist, the following physical argument may be made. Starting with an assumed potential gradient (or the vacuum electric field), the volume production of charged particles results in an increase of the total ion flux in the direction of the field, and a decrease in the electron flux. Charge density equality[4] requires that

$$\phi_e/\bar{v}_e = Z\phi_i/\bar{v}_i, \tag{2}$$

so that the average velocity of the ions has to increase and the average velocity of the electrons decrease, consistent with the potential distribution assumed. The potential distribution required to maintain equilibrium by satisfying the density equality may then be computed using Eq. (2) and the appropriate boundary conditions. This method of solution will be illustrated for a specific case.

The electrode configuration of the Penning discharge is such that the vacuum electric field traps electrons and ejects ions which are produced in the discharge (see Fig. 1). For simplicity it may be assumed that: (1) ions and electrons are essentially constrained to motion in one dimension (due in this example to the magnetic field); (2) the potential $V(x)$ is symmetric; (3) the electrons produced in the electrostatic well are trapped longitudinally and lost only by lateral diffusion across the magnetic field; (4) ions and electrons have mean free paths that are large compared to the length scale of the discharge; (5) electrons born at x make $R(x)$ electrostatic reflections before being lost by diffusion. The equality of electron and ion density, based on Eq. (1), takes the form

$$n_{ec}(x)+\left(\frac{m_e}{2e}\right)^{1/2}\int_x^L \frac{d\phi_e(x')}{[V(x)-V(x')]^{1/2}}=n(x)=\left(\frac{M_i}{2Ze}\right)^{1/2}\int_0^x \frac{d\phi_i(x')}{[V(x')-V(x)]^{1/2}}, \tag{3}$$

where $n_{ec}(x)$ is the density of electrons emitted from the cathode and the ion flux produced in dx is

$$d\phi_i(x)=[\phi_e+n_{ec}(x)\bar{v}_{ec}(x)]N(x)\sigma_i(x)dx. \tag{4}$$

N is the neutral gas density, σ_i the ionization cross section appropriate to the local velocity distribution, v_{ec} the average speed of the cathode electrons, and ϕ_e the trapped-electron flux, given by

$$\phi_e(x)=2\int_x^L R(x')[\phi_e(x')+n_{ec}(x')v_{ec}(x')] \times N(x')\sigma_i(x')dx'. \tag{5}$$

The factor 2 takes into account the electrons produced between $-x$ and $-L$. A self-consistent potential distribution $V(x)$ is in principle determined by the above equations but the solution is difficult to obtain analytically. Recourse is, therefore, made to numerical solutions obtained by an iterative procedure using finite differences.

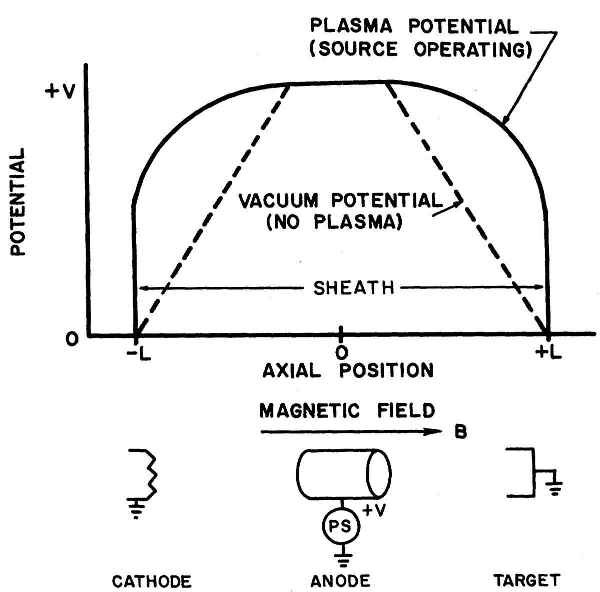

FIG. 1. Penning source electrode configuration and potential distribution.

The actual procedure is as follows: The region of interest is divided into a number of cells. Since the mean free path of particles is large compared to the dimensions of the plasma, the flux of trapped electrons characteristic of a given cell (i.e., those electrons that electrostatically reflect in that cell) is essentially constant throughout the discharge between the points of reflection. Moreover, the local potential determines the speed of these electrons and hence their density may be deduced. The production, the number of reflections, and the density of ions and electrons may then be computed for each cell. The production in the center cell is computed first. The potential required to balance the ion and electron charge density at the cell interface is determined. On the basis of the computed interface potential, the electrostatic attenuation of the electron flux is computed. This procedure is repeated for each subsequent cell. The trapped-electron flux may then be redetermined from the computed production rates in each cell and the number of reflections each group of electrons makes by the evaluation of the appropriate loss rates. This provides the input for the next iteration which can then be carried out in the same manner. A stable, self-consistent solution is obtained when the results of two succeeding iterations agree within established tolerances.

This method has been used to calculate the potential gradient in a Penning discharge. The resulting voltage distribution for a particular case (assuming only a given value of cathode current I_c, anode voltage V_0, and neutral gas density N_0, together with known collision and ionization cross sections) is shown in Fig. 2. The potential distribution measured experimentally with both emitting and conventional Langmuir probes (and verified by a momentum analysis of the ejected ions) is also plotted in Fig. 2.

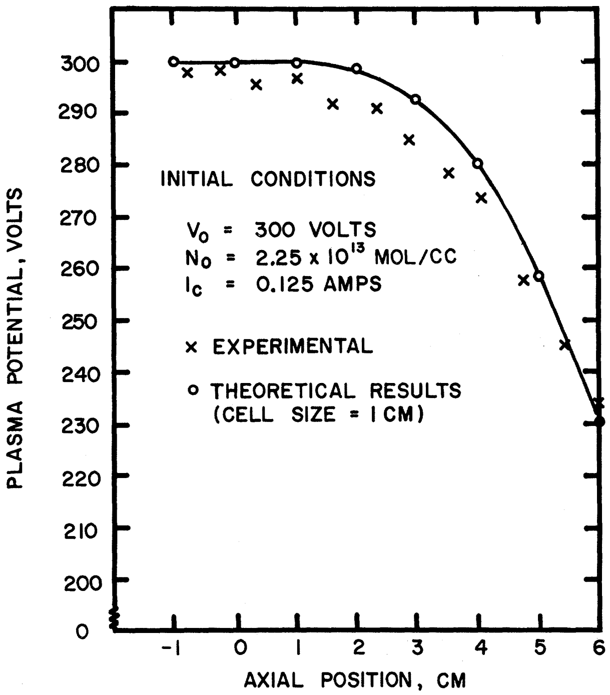

FIG. 2. Comparison of experimental and theoretical plasma potentials in the Penning discharge.

As can be seen, there is satisfactory agreement between theory and experiment.

In conclusion, it may be said that stable electrostatic potential gradients may exist in a plasma over a length scale, determined by the production mechanism, which for most cases of interest is large compared to the Debye length. These gradients are established by the nonthermal velocity distributions generated by the volume production of ion-electron pairs. The potential gradient may be determined, in general, by a simultaneous solution of Poisson's equation and the appropriate equations of motion. However, in most cases of interest, the assumption of equal ion and electron densities leads to a considerable simplification and allows the determination of the potential gradient from the flux-velocity quotient.

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[1]L. Spitzer, The Physics of Fully Ionized Gases (Interscience Publishers, Inc., New York, 1956), p. 16.

[2]L. Tonks and Irving Langmuir, Phys. Rev. 34, 876 (1929).

[3]D. J. Rose, Massachusetts Institute of Technology Quarterly Progress Report, No. 53, April, 1959 (unpublished).

[4]In general Poisson's equation must be satisfied (i.e., the difference in the densities of ions and electrons is the source of the electric field), but for most plasmas of laboratory interest the approximation of equal ion and electron densities is sufficiently accurate and leads to a considerable simplification.

EXPERIMENTS ON THE ENERGY BALANCE AND CONFINEMENT OF A MAGNETIZED PLASMA

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The possibility to confine a plasma in the magnetic field of a current loop has been discussed in a number of earlier reports.[1-4] It has been suggested[2] to supply energy and to heat the plasma in crossed electric and magnetic fields by a method used earlier in rotating plasma devices.[5,6] In a mirror machine with a rotating plasma the confinement is improved by the centrifugal force.[5,6] The large radial extensions of the current loop configuration are of special advantage when a strong centrifugal confinement has to be realized. This is indicated both by a theory on particle motion[3] and by a macroscopic approach including the energy balance.[4]

The purpose of the present experiments is to study how the energy balance and the confinement of a rotating plasma are influenced by particle losses along the magnetic field lines to a nonconducting wall. Figure 1 demonstrates the experimental arrangement which consists of a cylindrical vacuum vessel with an interior, ring-shaped magnetic coil (main coil) and two external auxiliary coils. The lines of the magnetic field $\vec{B}$ are sketched in the figure. A discharge is produced by a transverse electric field $\vec{E}$ arising from the voltage applied between the shield surrounding the main coil and the walls of the vacuum chamber. The energy is supplied from a condenser bank by means of a timed ignitron switch. A second ignitron can be used to study