approach is in no way committed to the Frazer-Fulco form.

[8]A set of relations for the S and P amplitudes at zero energy have been recently given by J. G. Taylor, Phys. Rev. Letters 6, 237 (1961). These relations are incorrect inasmuch as the contribution of the higher partial waves in the crossed channel is neglected. For example, the relation $a_0^2/a_2^2 = 5/2$ is not consistent with perturbation theory, since we expect that in the limit of weak coupling the above ratio should be $\sim(5/2)^2$, the ratio at the symmetry point, and not 5/2.

[9]Notice that the position of the resonance is not ν_R as defined by the relation (7) but it is the value of ν for which the real part of the denominator in Eq. (7) vanishes and is higher than ν_R.

[10]A bound state appears for the $I=0$ state for $\lambda \simeq -0.50$.

[11]G. F. Chew, S. Mandelstam, and H. P. Noyes, Phys. Rev. 119, 478 (1960).

[12]Such behavior was already indicated by G. F. Chew, Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, New York, 1960).

[13]A good approximation to the enhancement factor for small ν values is

$$\left\{\left[1+a_{SI}\frac{2}{\pi}\left(\frac{\nu}{\nu+1}\right)^{1/2}\ln[\nu^{1/2}+(\nu+1)^{1/2}]\right]^2+a_{SI}^2\frac{\nu}{\nu+1}\right\}^{-1},$$

where a_{SI} is the scattering length for the S wave with isotopic spin I.

[14]A. Abashian and N. E. Booth, Lawrence Radiation Laboratory, University of California, Berkeley (private communication). I am told by these authors that the experiment is being analyzed further.

[15]R. Dalitz, Phys. Rev. 94, 1046 (1954); E. Fabri, Nuovo cimento 11, 479 (1954); S. McKenna, S. Natali, M. O'Connell, J. Tietge, and N. C. Varshneya, Nuovo cimento 10, 763 (1958).

[16]B. S. Thomas and W. G. Holladay, Phys. Rev. 115, 1329 (1959); N. N. Khuri and S. B. Treiman, Phys. Rev. 119, 1115 (1960); R. F. Sawyer and K. C. Wali, Phys. Rev. 119, 1429 (1960). See also A. N. Mitra and E. Lomon, Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, New York, 1960).

LOW-ENERGY $\bar{K}$-NUCLEON INTERACTION*

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The recent discovery[1] of Y^* has emphasized the need for more complete information on the $\bar{K}$-nucleon interaction; in particular the need to differentiate between the $(a+)$ (constructive Coulomb-nuclear interference) solution and the $(a-)$ (destructive Coulomb-nuclear interference) solution which gives rise to the K^--p quasi-bound-state resonance.[2] In principle it is possible from low-energy K^--p scattering data to make this important distinction.[3-5] The interference effect should manifest itself experimentally through both the total elastic scattering cross section[4,6] and more directly in the differential elastic scattering cross section at small angles.[4,7] It is the purpose of this note to point out that, when the errors inherent in the S-wave zero-range approximation scattering lengths are included in theoretical calculations, certain ambiguities arise on comparison with experimental data which make it important to exercise great care in deciding which type of interference is correct.

Use has been made of the K-matrix formalism of Dalitz and Tuan[8] to include in the zero-range analysis of K^--p interactions at low energy the effects of $\bar{K}^0$-K^- mass difference and the long-range Coulomb interaction. The differential cross section for elastic scattering of K^- mesons incident on protons is

$$\frac{d\sigma_{el}}{d\Omega}=\left|\frac{\csc^2(\theta/2)}{2Bk^2}\exp\left[\frac{2i}{kB}\ln\sin(\theta/2)\right]+\frac{C^2[x-ik_0(x^2-y^2)]}{D}\right|^2, \quad (1)$$

where x and y are related to the complex scattering lengths $A_T = a_T + ib_T$ in the $T=1$ and $T=0$ channels by $x=\frac{1}{2}(A_0+A_1)$, $y=\frac{1}{2}(A_1-A_0)$; k and k_0 are the wave numbers in the K^--p and $\bar{K}^0$-n channels, B is the Bohr radius for the K^--p system ($B=83.4$ fermis), $C^2=(2\pi/kB)\times[1-\exp(-2\pi/kB)]^{-1}$ is the S-wave Coulomb penetration factor, and D is given by

$$D=(1-ixk_0)[1-ixC^2k(1-i\lambda)]+C^2kk_0(1-i\lambda)y^2,$$

in which $\lambda=(-2/kBC^2)[\ln(2kR)+\mathrm{Re}\,\psi(i/kB)+2\gamma]$, where $\gamma=0.5772$ (Euler's constant) and ψ is the logarithmic derivative of the gamma function. R is taken to be the K-meson Compton wave-length ($R = 0.4$ fermi).

The complex scattering lengths used in our calculations are those due to Dalitz,[9] in which an estimate of their uncertainties has been made. The scattering lengths corresponding to the so-called ($a+$) and ($a-$) solutions are given by

$$(a+)\begin{cases} A_0 = 0.05(\pm 0.2) + i\,1.10(^{+0.2}_{-0.3}), \\ A_1 = 1.45(\pm 0.2) + i\,0.35(^{+0.09}_{-0.07}); \end{cases}$$

$$(a-)\begin{cases} A_0 = -0.75(^{+0.35}_{-0.45}) + i\,2.00(\pm 0.35), \\ A_1 = -0.85(\pm 0.15) + i\,0.21(\pm 0.04). \end{cases} \qquad (2)$$

The dominant term of the elastic differential scattering cross section [Eq. (1)], except for small-angle scattering, is the nuclear term, which, it may be shown, is most sensitive to changes in $|x|$ as opposed to changes in x, y, or $|y|$. Thus, maximizing (minimizing) $|A_0 + A_1|$ will maximize (minimize) the differential cross section. Making use of this, the resulting maximum and minimum cross sections have been calculated and are denoted by $+\delta(a\pm)$ and $-\delta(a\pm)$, respectively, for the ($a\pm$) solutions. In addition, results corresponding to the "mean value" for ($a+$) and ($a-$) solutions have been calculated. The effect of these uncertainties is illustrated in Fig. 1 which gives the center-of-mass angular distribution at a K^--meson laboratory momentum of 172 Mev/c. A comparison is then made with hydrogen bubble-chamber experimental data,[7] indicating that within the limits of present experimental uncertainties it is impossible to differentiate conclusively between constructive ($a+$) and destructive ($a-$) Coulomb-nuclear interference solutions.

The experimental data for total scattering cross sections represent integrals over angles excluding a certain range of small-angle scattering. Therefore, to make a direct comparison between theory and experiment, the theoretical cross sections must be calculated by integrating between limits which are determined by the par-

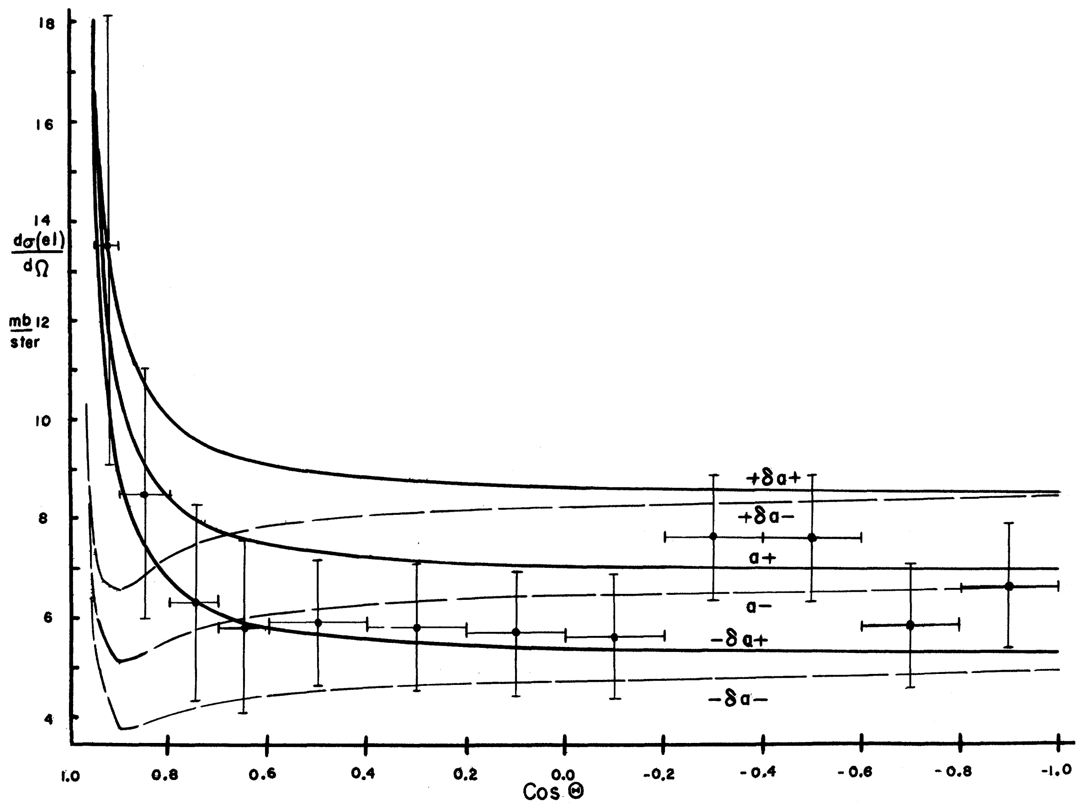

FIG. 1. Center-of-mass angular distribution for K^--p elastic scattering at laboratory momentum 172 Mev/c. The solid curves correspond to the ($a+$) constructive interference solutions, and the dashed curves correspond to the ($a-$) destructive interference solutions. Curves labelled $+\delta(a\pm)$ and $-\delta(a\pm)$ represent, respectively, the maximum and minimum cross sections for the ($a\pm$) solutions. The experimental points are hydrogen bubble-chamber data of reference 7.

ticular experiment under consideration. The different nature of the experimental techniques used in emulsion and bubble-chamber experiments require different limits of integration. The emulsion events[6,10] are selected by the requirement of a proton recoil of a certain minimum length; this results in a certain minimum incident K^- momentum below which no events are observed. The bubble-chamber events,[7] on the other hand, are not subject to the same criterion but are selected by visibility of scatter; this results in a certain minimum angle of detection at all momenta of incident K^- mesons. These differences in technique result in slightly different theoretical total elastic cross-section curves for the two cases. (The behavior at low K^- momentum shows this difference most strikingly, but it is of no importance in deciding between the solutions.)

Denoting the limiting angle of scattering by θ_m, the total elastic scattering cross section is given by

$$\sigma_{\text{el}} = \int_{\theta_m}^{\pi} 2\pi \frac{d\sigma_{\text{el}}}{d\Omega} \sin\theta d\theta,$$

and hence

$$\sigma_{\text{el}} = 2\pi |\gamma|^2 (1+\cos\theta_m) + \frac{4\pi}{\alpha^2}\cot^2(\theta_m/2)$$

$$+ \text{Re}\,\frac{16\pi\gamma}{\alpha\beta^*}\{1 - \exp[\beta^* \ln\sin(\theta_m/2)]\}, \quad (3)$$

where the asterisk denotes complex conjugate, $\alpha = 2Bk^2$, $\beta = 2i/kB$, and $\gamma = (C^2/D)[x - ik_0(x^2 - y^2)]$.

For the emulsion data the criterion for cutoff is that the laboratory momentum of the recoil proton be at least 30 Mev/c (corresponding to a range of 5 μ in emulsions).[4,6,10] For the bubble-chamber data, a cutoff corresponding to a center-of-mass scattering angle given by $\cos\theta_m = 0.7$ was used consistently for all energies of incident K^- meson.[7]

It is to be noted that the dominant term in the above expression (3) is the first, so that the limiting values for the total elastic scattering cross sections consistent with the Dalitz parameters are once again given by the $+\delta(a\pm)$ and $-\delta(a\pm)$ solutions. For these solutions and the mean value $(a\pm)$ solutions the calculated total elastic scattering cross section is compared with bubble-chamber data[7] [Fig. 2(a)] and emulsion data[6,10] [Fig. 2(b)].

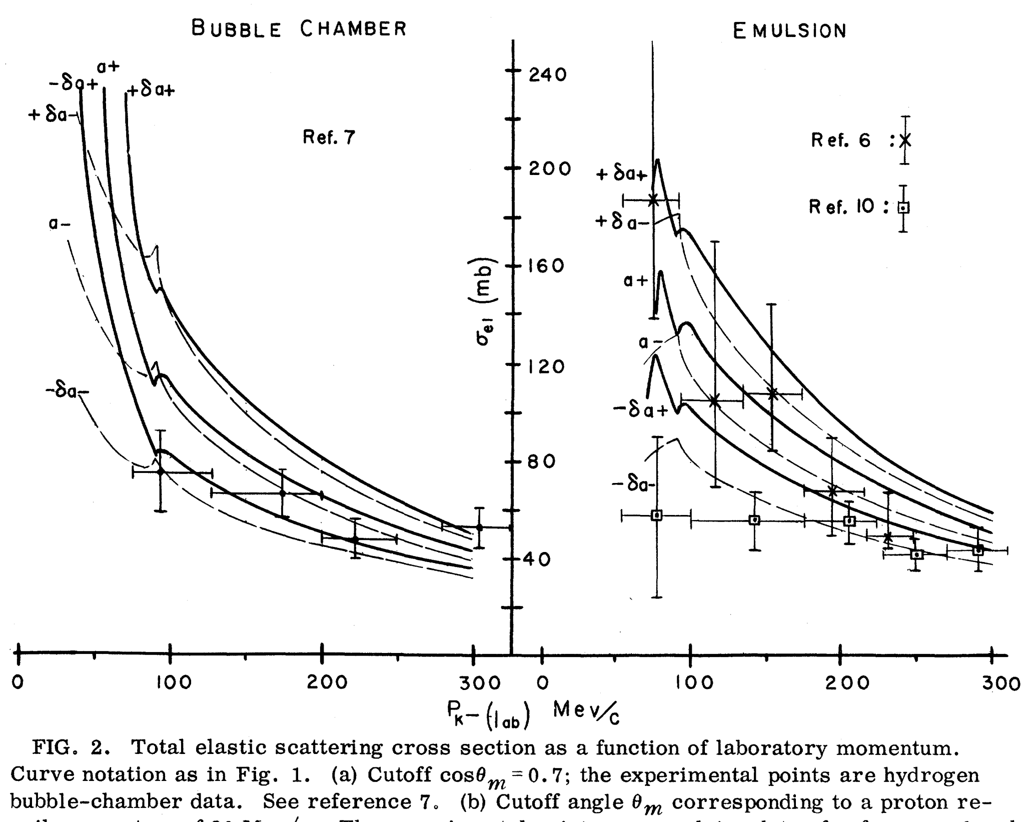

FIG. 2. Total elastic scattering cross section as a function of laboratory momentum. Curve notation as in Fig. 1. (a) Cutoff $\cos\theta_m = 0.7$; the experimental points are hydrogen bubble-chamber data. See reference 7. (b) Cutoff angle θ_m corresponding to a proton recoil momentum of 30 Mev/c. The experimental points are emulsion data of references 6 and 10.

On the basis of earlier values of the Dalitz parameters, Jackson and Wyld[4] calculated the expected variation of total elastic scattering cross section with momentum and concluded after comparison with emulsion data[10] that the Coulomb-nuclear interference is destructive. Using the same theoretical curves, Davis et al.[6] concluded that both emulsion data and bubble-chamber data suggested constructive interference. It is clear, however, from our calculations, using the latest values of the Dalitz parameters together with their errors, that no conclusive differentiation may be made between constructive $(a+)$ and destructive $(a-)$ interference on the basis of currently known experimental data.

Since the Dalitz parameters[9] are obtained by making use, among other things, of the experimental information on the total elastic scattering cross section at 172 Mev/c and the zero-range approximation neglecting Coulomb effects in the K^--p channel, the absolute magnitude of any cross sections predicted by formulas (1) and (3) will be approximately normalized to the experimental data at 172 Mev/c. Consequently, it is only the shape of both the angular distribution curves and the total cross section versus momentum curves which may be used to differentiate between the constructive and destructive solutions. Our results indicate that except in a narrow region on either side of the charge-exchange threshold, at which momentum cusp behavior is observed, the shape of the total cross sections are essentially the same for both types of solution. Thus, we make once again the oft-stated observation[3,4] that it is the differential cross section which would provide a more direct method of differentiating between $(a+)$ and $(a-)$ solutions. The shapes of angular distributions for both the $(a+)$ and the $(a-)$ curves are virtually independent of the exact values of these parameters and are sufficiently different to suggest that a conclusion might be arrived at, given a greater amount of experimental data in the essential range of $\cos\theta$ from 0.75 to 0.95.

In conclusion, we wish to emphasize (1) the need of obtaining accurate measurements of the cross sections for Λ^0 and Σ^0 reactions at the energy range of interest $[p_k(\text{lab}) \sim 200$ Mev/$c]$. A clean separation of Λ from Σ^0 events will then supplant the necessity of introducing the extrapolated assumption of constancy for the ratio[3,9,11] $\epsilon = [\Lambda/(\Sigma+\Lambda)]_{T=1} \sim 0.5$ in the low-energy region, currently employed in obtaining the theoretical solutions. (2) Our analysis has concentrated on the evaluation of relevant scattering and angular distribution information to differentiate between $(a+)$ and $(a-)$, which is of special interest (in terms of its relationship to the $T=1$ excited hyperon Y^*) because of the $T=1$ resonance present in the $(a-)$-type solution[3] below K^--p threshold. This should in no way obviate the over-all necessity of distinguishing between (a)-type solutions and (b)-type solutions. An experiment on $K_2{}^0$-p scattering would give information concerning this possibility.[12]

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[1]M. H. Alston, L. W. Alvarez, P. Eberhard, M. L. Good, W. Graziano, H. K. Ticho, and S. G. Wojcicki, Phys. Rev. Letters 5, 520 (1960); M. Ferro-Luzzi, Revs. Modern Phys. (to be published); O. Dahl, N. Horwitz, D. H. Miller, J. J. Murray, and P. G. White, Phys. Rev. Letters 6, 142 (1961); H. J. Martin, L. B. Leipuner, W. Chinowsky, F. T. Shively, and R. K. Adair, Phys. Rev. Letters 6, 283 (1961).

[2]R. H. Dalitz, Phys. Rev. Letters 6, 239 (1961); J. Franklin and S. F. Tuan, Nuovo cimento (to be published).

[3]R. H. Dalitz and S. F. Tuan, Ann. Phys. 8, 100 (1959). See also S. F. Tuan, Nuovo cimento 18, 1301 (1960); M. Ross and G. Shaw, Phys. Rev. Letters 5, 579 (1960).

[4]J. D. Jackson and H. W. Wyld, Phys. Rev. Letters 2, 355 (1959).

[5]R. Karplus, L. Kerth, and T. Kycia, Phys. Rev. Letters 2, 510 (1959).

[6]D. H. Davis, R. D. Hill, B. D. Jones, B. Sanjeevaiah, J. Zabrzewski, and J. P. Lagnaux, Phys. Rev. Letters 6, 132 (1961).

[7]L. W. Alvarez, University of California Lawrence Radiation Laboratory Report UCRL-9354, 1960 (unpublished). For the total elastic cross sections, UCRL-9354 presents the experimental points in the form $(20/17)\int_{\cos^{-1}0.7}^{\pi} 2\pi(d\sigma/d\Omega)\sin\theta d\theta$; we have removed the factor of 20/17 for standardization of emulsion and bubble-chamber results.

[8]R. H. Dalitz and S. F. Tuan, Phys. Rev. Letters 2, 425 (1959); Ann. Phys. 10, 307 (1960). See also reference 4, where Jackson and Wyld had obtained equivalent results employing the R-matrix formalism of Wigner and Eisenbud; we have adopted here the notations of Dalitz and Tuan as quoted in the references given above.

[9]R. H. Dalitz, Revs. Modern Phys. (to be published). Also private communication.

[10]W. Alles, N. N. Biswas, M. Ceccarelli, and J.

Crussard, Nuovo cimento 6, 571 (1957).

[11]Preliminary results on the $\Lambda^0/(\Sigma^0+\Lambda^0)$ ratio at 200 Mev/c suggest a fit with either of (a+) or (a-) solutions, with no indication of rapid variation for $(\Sigma/\Lambda)_{T=1}$ above K^--p threshold. (Private communication from Professor R. H. Dalitz on recent Berkeley experiments.)

[12]One such experiment is in progress in the Brookhaven National Laboratory Bubble-Chamber Group (W. J. Willis, private communication).

$\Delta I=1/2$ RULE IN Σ DECAY: A PROBLEM OF SIGN*

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Several authors[1,2] have pointed out that a certain admixture of $\Delta I=1/2$ and $3/2$ has the same physical consequences in Λ decay as the $\Delta I=1/2$ rule, and only differs from it in the sign of the $(p\pi^-)$ amplitudes relative to the $(n\pi^0)$ amplitudes. Since this sign does have physical consequences in another process, namely the nonmesonic decay of Λ hypernuclei,[2,3] the experimental data on Λ decay[4] are not conclusive evidence in favor of the $\Delta I=1/2$ rule. The purpose of this note is to indicate a similar situation in Σ decay.

Each mode of Σ decay,

$$\Sigma^+ \to p+\pi^0,$$
$$\Sigma^+ \to n+\pi^+,$$
$$\Sigma^- \to n+\pi^-, \tag{1}$$

is described by a pair of S- and P-wave amplitudes,[5]

$$N_k \equiv \begin{Bmatrix} S_k \\ P_k \end{Bmatrix}, \quad (k\equiv 0,+,-) \tag{2}$$

and the effective interaction Hamiltonian is written as

$$\mathcal{H}=a\mathcal{H}_{1/2}+b\mathcal{H}_{3/2}+c\mathcal{H}_{5/2}, \tag{3}$$

where $\mathcal{H}_{n/2}$ behaves as a quantity with isotopic spin $n/2$ ($n=1,3,5$). Since $\mathcal{H}$ is assumed to be invariant under CP, the constants a, b, and c are real, and the phases of S_k and P_k can be expressed in terms of the appropriate pion-nucleon phase shifts. From matrix elements of $\mathcal{H}$ between states of isotopic spin 1 and 3/2, we obtain the relation

$$\sqrt{2}N_0+N_+=dN_-, \tag{4}$$

where

$$d=\left\{\frac{a-b(8/5)^{1/2}+c(3/5)^{1/2}}{a+b(2/5)^{1/2}+c(1/15)^{1/2}}\right\}. \tag{5}$$

The physical properties of Σ decay[6] are not affected by the transformation

$$N_- \to -N_-, \tag{6}$$

and, consequently, cannot be used to determine the sign of d in Eq. (4). Now, it is obvious from (5) that if d is negative, at least one of b and c must be comparable in magnitude with a. Therefore, the validity of the $\Delta I=1/2$ rule depends upon a sign that has no physical significance in Σ decay itself.

To emphasize this point, let us suppose that the N_k are real, and represented by vectors in an S-P diagram.[5] Recent experiments indicate that the rates for the three decay modes are approximately equal,[7] i.e.,

$$|N_0|\approx|N_+|\approx|N_-|, \tag{7}$$

and also that $\sqrt{2}N_0$, N_+, N_- form a right-angled triangle[8]; therefore, from (4), either

$$d=+1, \tag{8a}$$

or

$$d=-1. \tag{8b}$$

If c is zero, then b is zero when $d=+1$ and $a\sqrt{(10)}$ when $d=-1$. Due to the uncertainty in the sign of N_-, no choice between (8a) and (8b), and consequently no definite conclusion about the $\Delta I=1/2$ rule, can be made.

This problem of sign can be approached in the following way.[9] When $\mathcal{H}$ is written in the form

$$\mathcal{H}=N_0\bar{\Sigma}^+p\pi^0+N_+\bar{\Sigma}^+n\pi^++N_-\bar{\Sigma}^-n\pi^-+\text{H.c.}, \tag{9}$$

it is easy to see that a change in the signs of N_- and the Σ^- field,

$$N_-\to -N_-, \quad \Sigma^-\to -\Sigma^-, \tag{10}$$

leaves $\mathcal{H}$ invariant. Therefore, insofar as the transformation (10) is permissible, the signs of