nance series with about 30 maxima, we find that the orbit circumference must be about $30\pi\lambda = 0.4$ cm. The maxima of both phenomena sharpened and the background attenuation shifted as the temperature was reduced from 4.2°K to 1.5°K, suggesting that the resistivity of the Ga was changing. Using these measurements and extrapolating the resistivity measurements of Olsen-Bär and Powell,[6] one finds that $\rho_{1.5°\mathrm{K}}$ must be of the order of 10^{-12} ohm cm. Recent measurements by Weisberg and Josephs[7] also indicate that very low resistivity values may occur in Ga samples at 1.26°K. These four observations suggest that a mean free path, l, of order 1 cm is obtained in these Ga crystals at 1.5°K. Associated values of $\omega\tau \sim 5$ and $ql \sim 1500$ follow, where q equals $2\pi/\lambda$. The condition for geometric resonance is $ql > 1$ and is much less stringent than the condition for observing cyclotron resonance.

The topology of the Fermi surface is yet to be established, but the geometric resonance presented here suggests an extremal dimension (1.3×10^{-20} g cm/sec) consistent with regions of electrons in the seventh zone as predicted by the nearly free electron model.[8] The effective electronic mass, m^*, derived from the cyclotron resonance for the same orientation is $0.8_4 m_0$.

Further study of the Ga Fermi surface topology and related effective masses is under way.

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GENERATION-RECOMBINATION NOISE IN p-TYPE GOLD-DOPED GERMANIUM

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Preliminary estimates of the capture cross sections of free holes by the 0.041-ev donor level and the 0.145-ev acceptor level in p-type gold-doped germanium have been obtained from measurements of the generation-recombination noise spectrum in the range 10^4-10^7 cps. At 97°K the Fermi level of the sample studied was located midway between the two gold levels, such that the noise spectrum simultaneously exhibited relaxation times characteristic of both levels. A two-impurity level model appropriate to the gold-doped germanium system was employed to predict the observed noise in terms of the capture cross sections. Good agreement was obtained between theory and experiment, using values for the total cross sections of $\sigma_A = 1.55 \times 10^{-12}$ cm^2, $\sigma_D = 3.75 \times 10^{-18}$ cm^2. To our knowledge this work represents the first experimental observation of two-level generation-recombination noise in gold-doped germanium and the development of a theory capable of accounting for all of the observed noise in interacting two-level systems of this type.

Generation-recombination noise in semiconductors arises from the modulation of the conductivity by the random thermal fluctuation of charge carriers between the valence (or conduction) band and the impurity levels. Thus, for an extrinsic semiconductor the time-dependent second moment $\langle \Delta J(0)\Delta J(t)\rangle$ of the current fluctuations is simply related to the corresponding moment $\langle \Delta p_V(0)\Delta p_V(t)\rangle$ of the fluctuations in the valence band population:

$$\langle \Delta J(0)\Delta J(t)\rangle = (J^2/p_V^{\,2})\langle \Delta p_V(0)\Delta p_V(t)\rangle, \quad (1)$$

where J is the average current and p_V the equilibrium valence band population. The generation-recombination noise spectrum $G(f)$ is the Fourier transform of Eq. (1).

Van Vliet[1] has developed a general prescription for obtaining the free carrier fluctuations in multilevel systems by computing the linear response of the various level populations to the thermodynamically conjugate applied Fermi potentials and has applied this technique to several simple systems and to the Shockley-Read model. We shall adopt the more straightforward approach of calculating the free carrier fluctuations directly, which is, of course, equivalent to the fluctuation-dissipation theorem of Callen and co-workers.[2] The model of p-type gold-doped germanium we consider is indicated in Fig. 1, which shows the impurity level scheme, and also the Fermi level as a function of temperature for the sample studied, as given by Klein and Debye.[3] The shallow acceptor level is assumed to be completely ionized at 97°K, so that only two independent variables need be considered. These we take to be the hole populations p_V and p_D of the valence band and gold donor level, respectively. The customary assumption is made regarding the interaction of the gold levels, wherein the number $N_{Au}{}^0$ of neutral gold atoms corresponds simultaneously to the number of ionized donor states and the number of filled acceptor states.

The linear kinetic equations for the displacements Δp_V and Δp_D of the hole populations of the valence band and donor level, respectively, have the form

$$d\Delta p_V(t)/dt = -A_{VV}\Delta p_V(t) - A_{VD}\Delta p_D(t), \qquad (2)$$

$$d\Delta p_D(t)/dt = -A_{DV}\Delta p_V(t) - A_{DD}\Delta p_D(t), \qquad (3)$$

where the A_{ij} are functions of the equilibrium statistics and the capture cross sections σ_A and σ_D. The lifetimes are the usual roots of the secular determinant obtained from these equations. $\Delta p_V(t)$ is taken to be a linear combination of decaying exponentials,

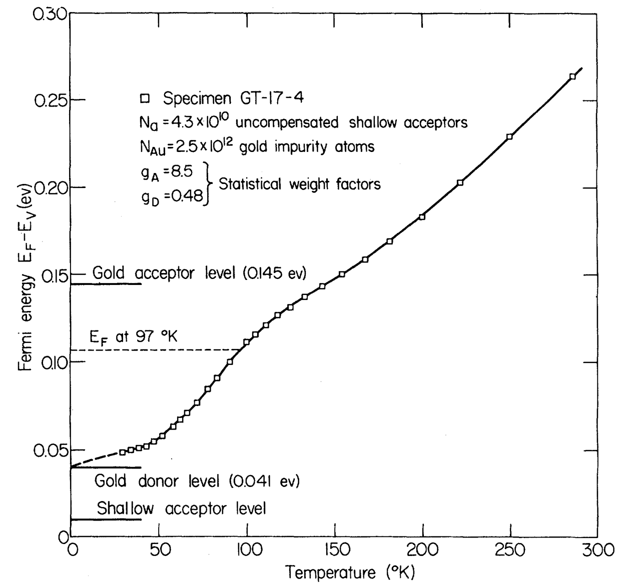

FIG. 1. Energy level scheme for Au-doped p-type germanium.

$$\Delta p_V(t) = C_1 e^{-t/\tau_1} + C_2 e^{-t/\tau_2}, \qquad (4)$$

the expansion coefficients being determined by the initial conditions. The free carrier fluctuation moment $\langle \Delta p_V(0)\Delta p_V(t)\rangle$ is obtained from Eq. (4) by multiplying through by $\Delta p_V(0)$ and averaging over the equilibrium system. Substituting this result into Eq. (1) and taking the Fourier transform yields the noise spectrum in the form

$$G(f) = 4\frac{J^2}{p_V{}^2(\tau_1-\tau_2)}\left\{\frac{\tau_1{}^2[(1-\tau_2A_{VV})\langle(\Delta p_V)^2\rangle - \tau_2A_{VD}\langle\Delta p_V\Delta p_D\rangle]}{1+(2\pi f\tau_1)^2} - \frac{\tau_2{}^2[(1-\tau_1A_{VV})\langle(\Delta p_V)^2\rangle - \tau_1A_{VD}\langle\Delta p_V\Delta p_D\rangle]}{1+(2\pi f\tau_2)^2}\right\}. \qquad (5)$$

The fluctuation moments $\langle(\Delta p_V)^2\rangle$ and $\langle\Delta p_V\Delta p_D\rangle$ appearing in Eq. (5) can be computed from the free energy, according to the equilibrium fluctuation theory of Greene and Callen,[4] by solving the equations

$$\sum_k \frac{\partial^2 F}{\partial p_i \partial p_k}\langle \Delta p_k \Delta p_j\rangle = kT\delta_{ij}. \qquad (6)$$

The free energy appropriate to the interacting gold levels in germanium is

$$F = -p_V E_V - kTp_V \ln\frac{N_V}{p_V} - p_D E_D - kT\ln\frac{g_D{}^{p_D}(N_{Au}{}^0+p_D)!}{p_D!\,N_{Au}{}^0!}$$
$$-(N_{Au}{}^0+p_D)E_A - kT\ln\frac{g_A{}^{(N_{Au}{}^0+p_D)}(N_{Au})!}{(N_{Au}{}^0+p_D)!\,(N_{Au}-N_{Au}{}^0-p_D)!}. \qquad (7)$$

That Eq. (7) is correct can be verified by showing

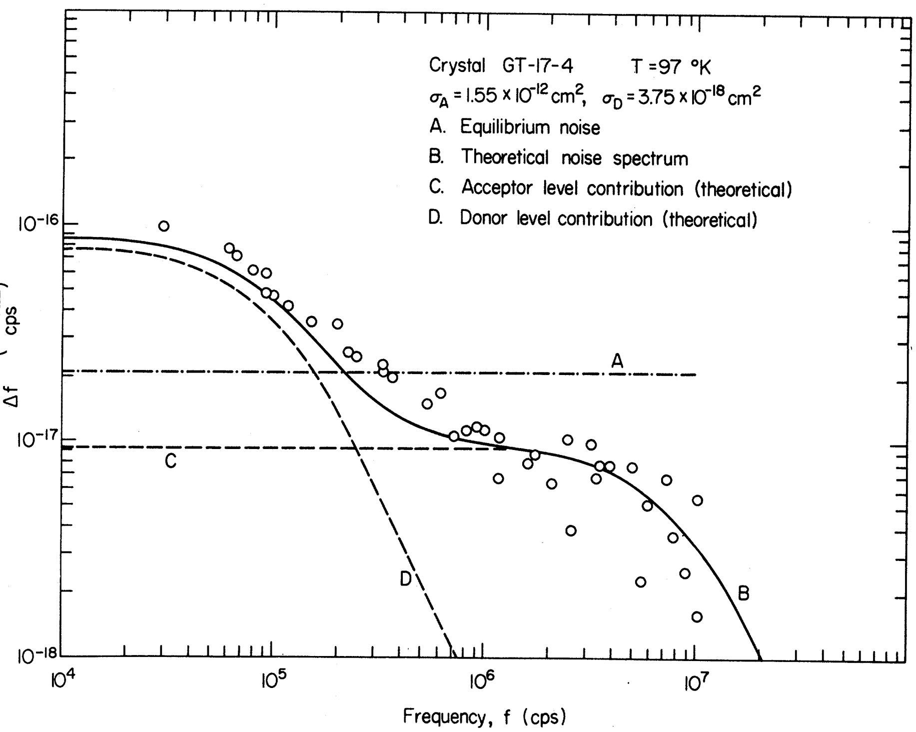

FIG. 2. Generation-recombination noise spectrum.

that it is consistent with Klein's calculation[3] of the equilibrium level populations using the grand canonical potential.

The experimental noise spectrum, obtained by standard techniques similar to those employed by van der Ziel and co-workers,[5] is shown by the circles in Fig. 2. These data have been corrected for the equilibrium noise, indicated by Curve A, and certain high-frequency characteristics of the apparatus. Two characteristic relaxation times, corresponding roughly to the two gold levels, are clearly evident. The increased scatter of the data in the high-frequency range can be attributed to the low generation-recombination noise level relative to the equilibrium noise. The best fit of Eq. (5) to the experimental points, corresponding to the total hole capture cross sections $\sigma_A = 1.55\times10^{-12}$ cm^2 and $\sigma_D = 3.75\times10^{-18}$ cm^2, is shown by solid Curve B. These values are in qualitative agreement with those obtained independently using other techniques. For example, the field-effect studies of Rupprecht[6] yield the cross sections $\sigma_A = 4.85\times10^{-13}$ cm^2, $\sigma_D = 6.70\times10^{-19}$ cm^2.

From Fig. 2, the quantitative agreement between theory and experiment is seen to be quite good, especially for the acceptor level contribution, Curve C. The significance of the small discrepancy associated with the donor level contribution, Curve D, is not as yet clear. Aside from experimental causes, such a discrepancy might arise either from the incidental presence of copper, which has an acceptor level at ~0.04 ev, or from the contribution of excited gold donor states to the generation-recombination process. Further studies are planned on additional crystals as they become available, as are investigations of the temperature dependence of the noise spectrum. We wish to thank G. Rupprecht of this laboratory for lending us this particularly appropriate crystal and to acknowledge the experimental assistance of E. Wang during the early phases of the work.

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