PHONON MASERS AND THE PHONON BOTTLENECK*

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It was suggested some years ago by Van Vleck[1] that in some circumstances the lattice phonons cannot always carry off all the power delivered to them by a paramagnetic system, with one-phonon direct coupling. Orbach[2] has estimated under representative conditions the steady-state temperature differences of the spin, direct phonon, and heat bath systems. These differences were not impressively large, showing that the phonons are not necessarily a serious bottleneck.

The conditions for a grave phonon bottleneck are not unrelated to the basic condition for phonon maser action, with paramagnetic ions radiating phonons. The argument of Townes[3] can be readily adapted from the electromagnetic problem to the phonon problem. We consider a solid of volume V containing per unit volume N paramagnetic ions in their upper state and zero in their lower state. We suppose that a single phonon mode of frequency ω and quality factor Q lies within the linewidth $\Delta\omega$ of the paramagnetic transition. All Raman processes are neglected. The electron-phonon coupling is assumed to be described by a spin-strain Hamiltonian,[4]

$$\mathcal{H} = G_{\mu\nu} S_{\mu} S_{\nu} e_{\mu\nu}, \qquad (1)$$

linear in the strain components $e_{\mu\nu}$. This form has been demonstrated, particularly by the work of Shiren and Tucker, to account in a consistent way for the observations on direct spin-phonon interactions. The largest value of the coupling constant G reported by Shiren[5] is $\sim 10^{-13}$ erg, for Fe^{2+} in MgO; the smallest, for Mn^{2+} in MgO, is $\sim 10^{-16}$ erg. The estimates in reference 4 were made with $G = 3\times 10^{-14}$ erg.

The initial rate at which the spin system transfers energy to the isolated mode ω is of the order of

$$\hbar\omega(2\pi/\hbar)e^2|G|^2(1/\hbar\Delta\omega)NV,$$

while the rate at which the phonon energy decays is

$$\tfrac{1}{2}Ce^2V(\omega/Q),$$

where C is the elastic modulus. The condition for the buildup of stimulated phonon emission is that the gain should exceed the loss, or that

$$N > C\hbar\Delta\omega/(4\pi|G|^2Q).$$

If we take $C = 10^{12}$ ergs/cm^3, $\Delta\omega = 2\pi\times 10^7$ sec^{-1}, $G = 10^{-14}$ erg, and $Q = 100$, then for maser action $N > 0.5\times 10^{18}$ cm^{-3}. This is the value by which the concentration in the upper state must exceed that in the lower state, but our argument has assumed (for simplicity) that the latter is itself negligible. It therefore appears feasible to obtain a phonon bottleneck leading to maser action.

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[1]J. H. Van Vleck, Phys. Rev. 59, 724 (1941).

[2]R. Orbach, Proc. Roy. Soc. (London) (to be published).

[3]J. P. Gordon, H. J. Zeiger, and C. H. Townes, Phys. Rev. 99, 1264 (1955); C. H. Townes (unpublished) has independently suggested that an ultrasonic maser was feasible.

[4]C. Kittel, Phys. Rev. Letters 1, 5 (1958); a relation between $|G|^2$ and T_1T is also given in this Letter.

[5]N. S. Shiren (private communication).