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THERMAL CONDUCTIVITY OF LIQUID He^3†

A. C. Anderson,* G. L. Salinger, and J. C. Wheatley‡
Department of Physics, University of Illinois, Urbana, Illinois
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The thermal conductivity of liquid He^3 has been measured in the temperature range from 0.026°K to 0.2°K and at pressures near 10 cm Hg by measuring the temperature difference across a column of liquid through which a constant flow of heat is maintained. The conductivity is found to increase as temperature is decreased, attaining a temperature dependence of T^{-1} below 0.04°K.

The cell used for these measurements was a cylinder, 0.50 cm i.d. by 0.025 cm wall thickness, constructed of paper impregnated with Epibond 104.[1] Two resistance thermometers were made by coating RS-12 Shielding Micropaint[2] on Epibond 104-impregnated paper rings 0.10 cm long by 0.30 cm diameter. Electrical leads of 0.003-in. diameter niobium wire supported the resistors so that they were concentric with the cell wall. The distance L between resistors was 1.10 cm. The resulting value, corrected for thermal contraction, for the ratio L/A, where A is the cross-sectional area of the He^3 column, was 5.59 cm^{-1}. Each end of the cell was closed with a plug of Epibond 104 containing about 3500 Formex-insulated 0.0024-in. diameter copper wires. The final 0.4 cm of each wire was free to extend into the He^3, thus providing a He^3-copper contact area of about 27 cm^2. He^3 entered the cell through a 1/64 in. o.d. by 0.003 in. wall, 70-30 cupro-nickel tube molded into the upper plug.

The wires emerging from the ends of the cell were formed into "foils" of 100 wires each. Those from the upper plug made direct thermal contact with the heat sink, 97 g of chromium potassium alum. The lower set of foils made direct thermal contact to 9.4 g of ~0.16-cm thick cerium magnesium nitrate crystals (CMN). These crystals constituted a thermometer for which the absolute temperature was equal to the ballistically measured magnetic temperature. The thermal time constant of the CMN was always less than that of the cell.

The heater consisted of a 100-ohm, 6-in. length of 0.002-in diameter Evanohm wire[3] woven into the lower plug in such a manner as to insure uniform heating in a plane normal to the axis of the cell. The cell was heated from below to avoid heat transfer by convection.

Resistances were measured using a pair of ac Wheatstone bridges[4] operated at 33.3 cps. At the lowest temperatures the measuring power of 10^{-14} watt to the thermometers produced no observable heating effects. The resistors were maintained below 1°K for the duration of the experiment. Thermal equilibrium times were 5000 sec at 0.03°K, 2500 sec near 0.07°K, and 6000 sec at 0.2°K. The increase at low temperatures resulted from the increasing He^3-copper thermal boundary resistance while that at the highest temperature was due to the properties of He^3 alone.

The thermal conductance of the cell wall was measured using the CMN to determine a temperature drop across the empty cell for a given heat flow. The wall contributed 1% to the total conductance at 0.2°K and a negligible correction below 0.1°K.

Measurements of the CMN temperature and of

the resistances were made during alternate periods with the heater on and off, sufficient time being allowed during each period to obtain thermal equilibrium. The smoothed data with heater off provided a relation between resistance and temperature for each thermometer. These relations, together with the change in resistance with the heater on and off, gave, for each resistor, a value for the increase in temperature, during heating, above that of the heat sink. The difference between these values for the two resistors was taken to be the temperature drop ΔT between them. The conductivity K was calculated from the relation

$$K = (\dot{Q}/\Delta T)(L/A), \qquad (1)$$

where $\dot{Q}$ is the heating rate. Typical heating rates ranged from 0.2 erg/sec at 0.03°K to 1.2 ergs/sec at 0.2°K. The calculated thermal conductivities were independent of power.

The results of this experiment are shown in Fig. 1. Below 0.04°K the smooth curve on the figure represents the relation

$$K = (48/T) \text{ ergs/cm sec}, \qquad (2)$$

which agrees with the data to within experimental error. The data at 0.2°K agree within experimental error with an extrapolation of the conductivity as measured by Lee and Fairbank[5] at 0.24°K and above. The latter measurements

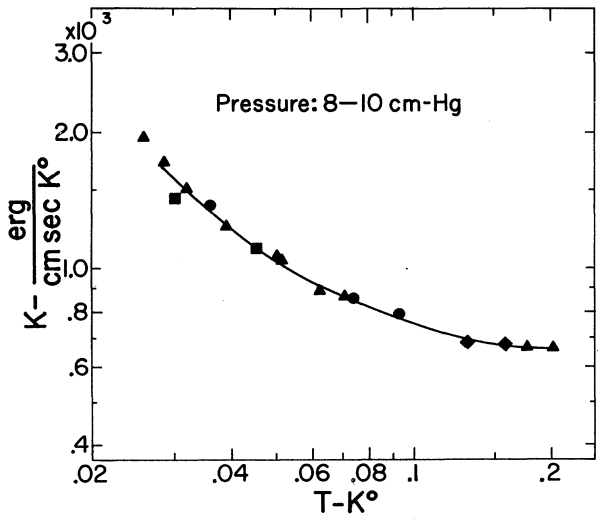

FIG. 1. Log-log plot of the thermal conductivity of He^3 vs the temperature. The points on this graph represent averaged experimental data. The various symbols represent different runs.

have been criticized by Jeener and Seidel,[6] who suggest that at 0.25°K the flow of heat from the walls of the cell into the He^3 might decrease the "measured" conductivity by 30% or more from the true value. However, we have analyzed the experiments of Lee and Fairbank along similar lines, using a range of reasonable estimates for the thermal boundary resistance, and conclude that the experimental agreement for values of K given by cells of different length is strong evidence that the effect pointed out by Jeener and Seidel is not as serious as was originally thought to be the case.

At the lowest temperatures we made an independent determination of the temperature dependence of K and of the thermal boundary resistance by using the temperature change of the CMN with the heater on and off. It is a good assumption in our apparatus that this temperature difference during heating occurred entirely across the cell. If it is assumed that K varies at T^{-1} and the boundary resistance as T^{-3},[7] the total thermal resistance of the cell may by written

$$\Delta T/\dot{Q} = a/T^3 + bT. \qquad (3)$$

Figure 2 shows a plot of $(\Delta T/\dot{Q})T^3$ vs T^4 for $T < 0.04$°K; the data are consistent with the above assumptions. Above 0.05°K the values of $(\Delta T/\dot{Q})T^3$ fall below an extrapolation of the line shown. The individual temperature drops between resistance thermometers and the ends of the cell were similarly treated and are also shown on Fig. 2. From the slopes of the three curves we obtain the thermal resistance of all the He^3, that between the upper resistor and the upper boundary, and that between the lower resistor and the lower boundary. By subtraction we obtain the thermal resistance of the column of He^3 between the two resistors. The resultant thermal conductivity is

$$K = (51/T) \text{ ergs/cm sec}, \qquad (4)$$

in good agreement with Eq. (2). From the intercepts of the three curves in Fig. 2 we obtain the thermal resistance of both boundaries in series, the upper boundary, and the lower boundary. The sum of the upper and lower boundary resistances agrees to within 2% with the independently obtained total resistance. The resultant boundary resistance between He^3 and the Formex-insulated wires is approximately $7\times10^{-6}(AT^3)^{-1}$ $(K°)^4$ cm^2 sec/erg, where A is the area of contact. This is twice the resistance reported in

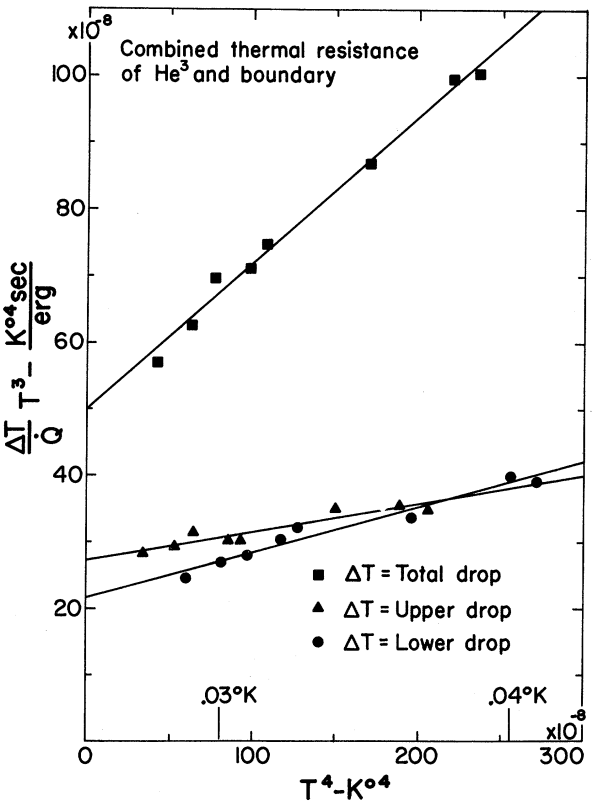

FIG. 2. Plot of $(\Delta T/\dot{Q})T^3$ vs T^4, where ΔT, the temperature difference produced by a heat flow $\dot{Q}$, is much less than T, the average temperature. Curves are shown for ΔT equal, respectively, to the temperature drop across the entire cell, to that between the upper resistance thermometer and the heat sink, and to that between the lower resistance thermometer and the heat source.

reference 7.

Abrikosov and Khalatnikov[8] derive an expression for the thermal conductivity of He^3 based on the Landau theory of a Fermi liquid.[9] An estimate of K can be made[8] by assuming that the scattering probability for two quasi-particles is given by $w(\theta,\varphi)=2\pi f^2/\hbar$, where for definition and evaluation of the quantity f and for notation we refer to reference 8. Evaluating Eq. (8.10) of reference 8, we find

$$K=(13/T)\ \text{ergs/cm sec}. \tag{5}$$

This is smaller than the observed coefficient by a factor of four. In the numerical work we used the latest empirical values for density,[10] specific heat,[7] and velocity of sound.[11]

If $w(\theta,\varphi)$ were independent of φ and of spin interactions, the ratio D/K, where D is the diffusion coefficient as calculated by Hone,[12] should be independent of w and dependent experimentally only on the value of the susceptibility. Using the low-temperature susceptibility as given by Anderson, Hart, and Wheatley[13] and the theoretical expressions[8,11] for D and K, we find

$$D/K=2\times10^{-6}\ \text{cm}^3\ \text{K°/erg}, \tag{6}$$

at 0.03°K. The observed value at 0.03°K is 1×10^{-6} cm³ K°/erg, where the empirical value for D is taken from reference 13. The difference between theory and experiment may be explained by the collision probability being dependent on φ, by different spin dependences of the w applicable to D and K, by the Fermi liquid concept being inapplicable to He^3 at 0.03°K (D appears to have a $T^{-3/2}$ temperature dependence,[13] in disagreement with theory), or by some unknown experimental errors in the temperature scales of the diffusion and conductivity experiments.

The thermal conductivity formula given in reference 8 can be expressed in terms of a relaxation time, τ_K, by the formula $K=\frac{1}{3}v_0^2\tau_K C$, where v_0 is the velocity at the Fermi surface and C is the specific heat per unit volume. Below 0.04°K the experimental data are consistent with a relaxation time $\tau_K=(6\times10^{-13}/T^2)$ sec $(\text{K°})^2$. At 0.03°K one finds $\hbar/k\tau_K=0.012$°K, where $\hbar$ is Planck's constant divided by 2π and k is Boltzmann's constant, indicating that the quasi-particle concept is probably valid at these low temperatures.

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*National Science Foundation Fellow.

‡A. P. Sloan Fellow.

[1]Furane Plastics, Inc., 4516 Brazil Street, Los Angeles 39, California.

[2]Micro-Circuits Company, New Buffalo, Michigan. We are indebted to Mr. Bradley of this firm for suggesting the addition of carbon black to RS-12 to obtain a larger temperature coefficient of resistance.

[3]Wilbur B. Driver Company, Newark, New Jersey.

[4]C. Blake, C. E. Chase, and E. Maxwell, Rev. Sci. Instr. 29, 715 (1958). This circuit was modified by using a Hewlett-Packard 200CD oscillator and a phase-sensitive detector, the circuit for which was suggested to us by D. M. Ginsberg. Although the

electrical circuits were well shielded, Joule heating of the resistors at low temperatures by $\sim 10^8$ cps FM and TV signals made measurements inaccurate, so data were obtained when the transmitters were off.

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"CORKSCREW" - A DEVICE FOR CHANGING THE MAGNETIC MOMENT OF CHARGED PARTICLES IN A MAGNETIC FIELD*

Richard C. Wingerson

Department of Nuclear Engineering and Research Laboratory of Electronics
Massachusetts Institute of Technology, Cambridge, Massachusetts

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Consider the motion of a charged particle in the magnetic configuration shown in Fig. 1. A properly designed helical field source (a "Corkscrew") can perturb an initially uniform axial field in such a way that there will be a monotonic increase (or decrease) in the transverse energy component of certain particles traversing the structure. The necessary design condition is that the force resulting from the interaction of the axial particle velocity with the transverse component of the field perturbation be always approximately in the direction of the transverse particle velocity. It follows that there must be a close match between the local pitch of the Corkscrew and that of the modified helical particle trajectory. This condition may be expressed as

$$p(z) = -2\pi m v_z(z)/qB_0, \qquad (1)$$

where B_0 is the unperturbed axial field, m, q, and v_z are the mass, charge, and axial velocity of the particle, and $p(z)$ is the Corkscrew pitch length at position z (p is negative for the left-handed structure of Fig. 1). The helical field perturbation has no over-all effect on the axial field; therefore, a change in the transverse particle energy necessitates a change in magnetic moment. The trajectory in Fig. 1 could apply to an ion moving from left to right or to an electron moving from right to left.

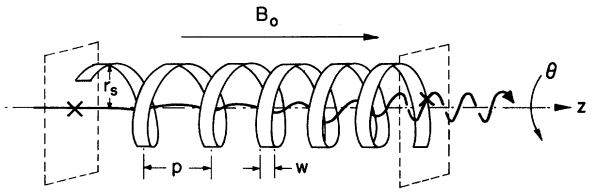

FIG. 1. Schematic diagram of a Corkscrew showing a resonant particle trajectory.

The Corkscrew may permit trapping of a high-energy beam injected axially into a magnetic mirror device. A positive particle following a path as shown in Fig. 1 could be reflected by a magnetic mirror somewhere to the right. On its return, the particle trajectory would have a handedness opposite to that of the Corkscrew, and, therefore, the particle would encounter a series of perturbations alternating in direction at a frequency higher than the cyclotron frequency. These perturbations should cancel to first order, so a mirror to the left would again reflect the particle, and trapping would appear to have been achieved. The Liouville theorem, of course, demands that some mechanism exist for particle loss. The unique feature of the Corkscrew is that this loss mechanism cannot be the same as the trapping mechanism. Trapping is achieved by what is essentially a strong resonance effect. Loss must occur by a random "scatter" effect whose exact nature has not as yet been determined. This nonreciprocal charac-