available at present around the $\pi^-\pi^+$ threshold are poor and consequently the discussed threshold effect is not seen in the experimental π^+ distribution, we cannot give here any quantitative estimate of $\sigma(\pi^-\pi^+\to\pi^0\pi^0)$. We note, however, that the threshold effect can in general be quite visible for this kind of interaction. A qualitative estimate of its size can be given by using Eq. (2). Assuming that

$$(k_{2b}/k_{2a})dw(K^+\to\pi^+\pi^-\pi^+)/dE_1$$
$$\cong 3dw(K^+\to\pi^+\pi^0\pi^0)/dE_1,$$

as the experimental results on the integrated spectra suggest, taking for $(k_{2a}/k_{2b})\sigma(\pi^-\pi^+\to\pi^0\pi^0)$ a value around 10 mb as expected from other work,[1] and supposing that the above-mentioned second derivative is appreciably small over a range of a few Mev around the $\pi^-\pi^+$ threshold, we get a threshold effect of 5-10% in the π^+ distribution for decay (b) of the K^+ meson. Obviously, this argument can be reversed to give a qualitative estimate of $\sigma(\pi^-\pi^+\to\pi^0\pi^0)$ once the threshold effect is found experimentally.

[1]See, for example, J. A. Anderson, P. G. Burke, D. D. Carmony, and N. Schmitz, Proceedings of the 1960 Annual Conference on High-Energy Physics at Rochester (Interscience Publishers, New York, 1960), p. 58; Yu. A. Batusov, S. A. Bunyatov, V. M. Sidorov, and V. A. Yarba, ibid., p. 79; and the copious references contained in these two reports.

[2]L. Fonda and R. G. Newton, Phys. Rev. 119, 1394 (1960).

[3]L. I. Lapidus and Chou Kuang-chao, J. Exptl. Theoret. Phys. (U.S.S.R.) 39, 364 (1960) [translation: Soviet Phys. – JETP 39(12), 258 (1961)].

[4]The same procedure will apply also to the photoproduction of two pions on hydrogen and in other cases.

[5]Coulomb effects cover a very small energy region around $\bar{E}_1$ and have been neglected.

[6]Equations similar to (2) and (3) have been obtained for two-particle → two-particle reactions by R. G. Newton, Phys. Rev. 114, 1611 (1959); and by L. Fonda and R. G. Newton, Nuovo cimento 14, 1027 (1959).

π-Λ RESONANCE AND THE SIGMA HYPERON

S. Barshay* and H. Pendleton, III†
Physics Department, Brandeis University, Waltham, Massachusetts
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A resonance in the π-Λ system has been reported at a center-of-mass total energy of 1385 ± 15 Mev.[1,2] In this note we would like to consider a possible connection between this resonance and the sigma hyperon, in the case in which the latter particle is hypothesized to be a bound state of a pion and a Λ with binding energy of ~ -65 Mev.[3]

Dalitz and Tuan[4] have shown that the coupling of the π-Λ system to the $\bar{K}$-nucleon system at threshold can lead to a resonance in π-Λ scattering at some energy below the $\bar{K}$-nucleon threshold if the so-called (a-) solution[4] turns out to describe the $\bar{K}$-nucleon system at moderate energies above threshold. In an approximation in which we neglect π-Σ production in the isotopic spin one state in K^--p reactions at threshold,[5] we may write a formula for the π-Λ S-wave phase shift, δ, according to the zero-range formulation of Dalitz and Tuan[4]:

$$(q/E)\cot\delta=(q_t/E_t)\cot\delta_t \times\{[1+\kappa(a+b\tan\delta_t)]/[1+\kappa(a-b\cot\delta_t)]\}. \tag{1}$$

In this formula q and E are the center-of-mass momentum and total energy, respectively, in the π-Λ system; the subscript t denotes an evaluation of the various quantities at the $\bar{K}$-nucleon threshold; κ is defined in terms of q by the following equation:

$$(q^2+m_\pi^{\ 2})^{1/2}+(q^2+m_\Lambda^{\ 2})^{1/2} =(-\kappa^2+m_K^{\ 2})^{1/2}+(-\kappa^2+m_p^{\ 2})^{1/2}, \tag{2}$$

where the various masses are denoted by an appropriate subscript. The a and b are taken to be the parameters of the (a-) solution of Dalitz,[6] $a=-1.09$ fermis, $b=0.20$ fermi. We note that in the limit of $b\to0$ (i.e., the coupling of the π-Λ system to the $\bar{K}$-nucleon system at threshold going to zero), we have the zero-range formula:

$$(q/E)\cot\delta=(q_t/E_t)\cot\delta_t=\text{constant}. \tag{3}$$

The phase δ_t depends on the various dynamic properties of the π-Λ system (the various mechanisms giving rise to the forces between a pion and a Λ), of which the coupling to the $\bar{K}$-nucleon system may be but one. Implicit in the following discussion is the assumption that there is more to the dynamics of the π-Λ system than is contained in its coupling to the $\bar{K}$-nucleon system at threshold. In particular, the exchange of two pions through the vertices $\Lambda \rightarrow \Lambda + 2\pi$ and $\pi + 2\pi \rightarrow \pi$ will give rise to an attraction if the two vertices have positive couplings. The zero-range formula for $q\cot\delta$ may reasonably be expected to be a fair approximation since presumably the longest range force between a pion and a Λ is mediated by the exchange of two pions (the effective range is likely to be less than $1/2m_\pi$), while q varies from 0 to $q \sim 1.5\, m_\pi$ at resonance. We would like to determine δ_t by requiring that the scattering resonance be at its observed position, ~1385 Mev. We then ask what is $\lim_{q \rightarrow 0} q \cot\delta = -1/A$, as determined by Eq. (1). Here A is the scattering length in the π-Λ system. We find $\delta_t \sim 154.5°$ or $-25.5°$ and $-1/A \sim +82m_\pi$. It is of interest to note that in the limit as $b \rightarrow 0$ (this corresponds to a very narrow π-Λ scattering resonance[4]) under the assumption that δ_t remains near the value determined above, $-1/A \rightarrow -3.47m_\pi$. The negative sign in the latter result is of particular interest, for it allows for the following possible interpretation of the scattering length behavior: In the limit of $b \rightarrow 0$, the zero-energy π-Λ S-wave phase shift starts at π and the system exhibits a scattering length characteristic of an attractive potential more than deep enough to have one bound state, but perhaps not deep enough to have a second bound state; for $b = 0.20$, the coupling of the π-Λ system to the $\bar{K}$-nucleon system at threshold results in an even stronger attraction, as indicated by a change in the scattering length from a positive value, $A \sim 1/3.47m_\pi$, to a very small negative value. The phase shift starts at π, goes through a resonance at $3\pi/2$, and approaches $\delta_t \sim -25.5°$ at the $\bar{K}$-nucleon threshold. The suggested bound state would be interpreted as the sigma hyperon; the quantum numbers of the π-Λ scattering resonance should be identical with those of the sigma hyperon, in other words the Y^* (the π-Λ resonance) and Σ should have the same spin, relative parity, strangeness, and isotopic spin. It is perhaps interesting to note that if the mass of the Y^* were at ~1395 Mev, we would obtain $\delta_t \sim 127.5°$ or $-52.5°$ and A would go from $\sim 1/3.43m_\pi$ to $\sim 1/1.27m_\pi$ as b goes from 0.20 to zero. The value, $A \sim 1/1.27m_\pi$, is not too far from the value one would obtain by assuming, in this case, that the zero-energy wave function was not too different from the bound-state wave function, $A \sim R$, where $R \sim 1/1.07m_\pi$, as determined by a binding energy of -65 Mev for the sigma hyperon. This somewhat academic remark is made simply because of the apparent tendency for the experimental resonance position to rise slightly and we would like to see the modification of the argument in this case. For a resonance at ~1385 Mev, the value of $\delta_t \sim -25.5°$ causes a relatively slight distortion of the symmetry of the resonance shape; however, for a resonance at ~1395 Mev, the value of $\delta_t \sim 127.5°$ or $-52.5°$ would imply a considerably slower falloff on the high-energy side of the resonance than on the low-energy side.

It is clear that an alternative interpretation of the zero-energy behavior of $q\cot\delta$ is possible, namely, that the extremely small negative scattering length, $A \sim -1/82m_\pi$, implies a very weak attraction (the zero-energy S-wave π-Λ phase shift starts at zero), which in the limit of $b \rightarrow 0$ goes to a repulsion characterized by $A \sim 1/3.47m_\pi$. However, the latter limit includes the assumption that δ_t remains close to its value as determined by the position of the π-Λ resonance with $b = 0.20$. Since in this case there would seem to be very little dynamics to the π-Λ system other than its coupling to the $\bar{K}$-nucleon system at threshold, δ_t probably cannot be considered to remain reasonably constant as $b \rightarrow 0$, but rather δ_t may $\rightarrow 0$ $(A \rightarrow 0)$ as $b \rightarrow 0$.

The position of the resonance is not very sensitive to the introduction of dynamics in the π-Λ system through a nonzero δ_t; for $\delta_t = 0$ or π, the resonance is at ~1378 Mev. However, the zero-energy scattering length is quite sensitive to δ_t (for $\delta_t = 0$ or π, $A \sim -1/3.27m_\pi$). In this sense, the precise position of the resonance, which is predicted in general for $b \neq 0$,[4] may well reflect additional dynamics in the π-Λ system. It is interesting that an extrapolation of the $(a+)$ solution[4] of Dalitz to zero energy in the π-Λ system (with $\delta_t = 0$) would imply a weak repulsion. If we assume in the bound-state model that the coupling of the π-Λ system to the $\bar{K}$-nucleon system should contribute an attraction, then the existence of the bound state would imply the $(a-)$ solution.

In conclusion we state that the value of the above rather speculative remarks could be most decisively tested by a determination and comparison of the spins and relative parities of the π-Λ resonance and the sigma hyperon. Since the spin of sigma hyperon is $\frac{1}{2}$, the spin of the π-Λ resonance must be $\frac{1}{2}$. It has been suggested[3] that the bound-state hypothesis is most attractive for an S-state π-Λ system. This implies that the Σ-Λ relative parity is odd[7] and that the π-Λ resonance is in the S wave. Given certain assumptions, the bound-state model makes a prediction[7] with regard to the relative sign of the asymmetry parameter, α_Λ, in the decay $\Lambda \to \pi^- + p$, and the asymmetry parameter, α_0, in the decay $\Sigma^+ \to \pi^0 + p$, namely $\alpha_\Lambda \alpha_0 \sim +1$ for an $S_{1/2}$ bound state (a $P_{1/2}$ bound state would give a negative relative sign). The positive relative sign is to be contrasted with the negative relative sign predicted by certain theories[8, 9] of strong and weak interactions which assume even Σ-Λ relative parity. These latter theories also predict $\alpha_\Lambda \alpha_\Xi \sim +1$, where α_Ξ is the asymmetry parameter in the decay $\Xi \to \pi^- + \Lambda$. This result is in contradiction to recent experiment.[10]

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[1]M. H. Alston et al., Phys. Rev. Letters 5, 520 (1960).

[2]Private communication from A. Rosenfeld.

[3]S. Barshay and M. Schwartz, Phys. Rev. Letters 4, 618 (1960).

[4]R. H. Dalitz and S. F. Tuan, Ann. Phys. 10, 307 (1960).

[5]The (Σ/Λ) ratio in the isotopic one state is apparently at least 0.2 at threshold, but falls off sharply below the $\bar{K}$-nucleon threshold, the region in which we will be applying formula (1).

[6]R. H. Dalitz, Phys. Rev. Letters 6, 239 (1961).

[7]S. Barshay, Phys. Rev. Letters 1, 97 (1958).

[8]S. Treiman, Nuovo cimento 15, 916 (1960).

[9]A. Pais, Nuovo cimento 18, 1003 (1960).

[10]W. B. Fowler et al., Phys. Rev. Letters 6, 134 (1961).

ELECTRODYNAMIC PROPERTIES OF BARYONS IN THE UNITARY SYMMETRY SCHEME

Sidney Coleman* and Sheldon Lee Glashow
California Institute of Technology, Pasadena, California
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Gell-Mann[1] recently introduced a theory of strong interactions involving a new symmetry, called "unitary symmetry." The principal purpose of this note is to use this symmetry to express the magnetic moments of all the baryons in terms of those of the neutron and proton. We also derive a relation among baryon electromagnetic mass splittings.

The unitary symmetry scheme proposes that the elementary particles may be represented as tensors in a three-dimensional (generalized isospin) space, and that the strong interactions are invariant under unitary transformations in this space. In particular, the eight baryons form the components of a traceless matrix ψ,

$$\psi = \begin{pmatrix} -(\frac{2}{3})^{1/2}\Lambda & p & n \\ \Xi^- & (\frac{1}{6})^{1/2}\Lambda + (\frac{1}{2})^{1/2}\Sigma^0 & \Sigma^- \\ \Xi^0 & \Sigma^+ & (\frac{1}{6})^{1/2}\Lambda - (\frac{1}{2})^{1/2}\Sigma^0 \end{pmatrix}, \tag{1}$$

while the seven known pseudoscalar mesons (plus a predicted new pseudoscalar meson, χ^0) form the components of a traceless Hermitian matrix ϕ,

$$\phi = \begin{pmatrix} -(\frac{2}{3})^{1/2}\chi^0 & K^+ & K^0 \\ K^- & (\frac{1}{6})^{1/2}\chi^0 + (\frac{1}{2})^{1/2}\pi^0 & \pi^- \\ \bar{K}^0 & \pi^+ & (\frac{1}{6})^{1/2}\chi^0 - (\frac{1}{2})^{1/2}\pi^0 \end{pmatrix}. \tag{2}$$

The scheme also proposes the existence of eight vector mesons which transform in the same way as ϕ. Although of great importance in Gell-Mann's theory, they will not be described here, for their symmetric interactions do not affect our conclusions.

If the strong interactions are to be invariant under unitary symmetry, the possible forms of the Lagrangian density are (assuming ps-ps meson-nucleon coupling)

$$L = \mathrm{tr}\bar{\psi}(i\partial_\mu\gamma^\mu - m_0)\psi + \mathrm{tr}(\tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{2}M_0^2\phi^2) + g\,\mathrm{tr}(\bar{\psi}\gamma_5\psi\phi) + g'\,\mathrm{tr}(\bar{\psi}\gamma_5\phi\psi) + L', \tag{3}$$