MAPPING OF THE FERMI SURFACE BY A COMBINATION OF GEOMETRIC RESONANCE AND TILT EFFECT

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Recent experiments[1,2] have indicated an increase in the attenuation of sound waves in semimetals when there is a magnetic field which is tilted from a direction transverse to the direction of propagation. Theoretical work has confirmed that in the high-field limit,[3] the increase in the ultrasonic attenuation arises when the carriers drifting along the magnetic field have a component in the direction of propagation that is equal to the sound velocity V_S. The carrier is then in phase with the sound wave and can experience a steady acceleration, effectively absorbing energy from the wave. The onset of the increase in attenuation occurs at a critical angle ν_c given by $\sin\nu_c = V_S/V_F$, where V_F is the Fermi velocity of the fastest group of carriers in the direction of the magnetic field. The high-field tilt effect, therefore, offers the possibility of obtaining the Fermi velocity point by point on the Fermi surface.

Other experiments[1,4,5] have indicated that there are magnetoacoustical oscillations in the attenuation as a function of magnetic field in a transverse magnetic field. The theoretical explanation[6] indicates that maxima and minima in the oscillations occur when the cyclotron orbit diameter is equal to an integral or half-integral number of wavelengths, respectively. From the periods of the oscillations with magnetic field, the extremal dimensions of the Fermi surface normal to the magnetic field can be determined.

The interesting possibility of combining the tilt effect and geometric resonance experiments to obtain information about the dimensions of nonextremal orbits on the Fermi surface now presents itself. The main contribution to the attenuation, when the magnetic field is tilted towards the direction of propagation by angles larger than ν_c, comes from those particular orbits that are drifting along the field in phase with the sound wave, provided that $\omega\tau$ is large enough. By varying the angle of tilt, one can vary the orbit or orbits which dominate the attenuation. If one now simultaneously performs a geometric resonance and a tilt effect experiment, one can obtain from the period of the geometric resonances the dimensions of these nonextremal orbits as well as their drift velocity along the magnetic field. The physical situation as described above is pictured in Fig. 1(a) and (b). The drift velocity has a component along the direction of propagation equal to V_S so that its

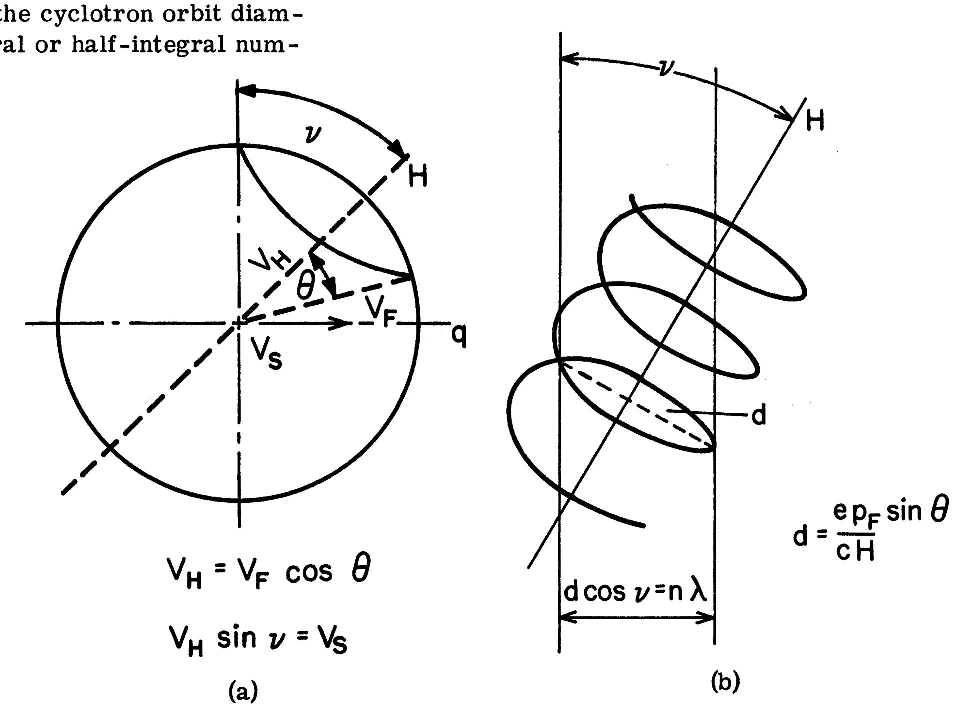

FIG. 1. (a) The orbit which dominates the attenuation when the magnetic field is tilted in the direction of the sound wave is shown to drift along the magnetic field with a velocity $V_S/\sin\nu$. (b) When the field is tilted the orbit gives rise to a maximum in the attenuation when the component of the orbit diameter in the direction of propagation is equal to $n\lambda$.

magnitude along H can be obtained from the angle of tilt, $V_H = V_S/\sin\nu$.

The conditions for this phenomenon to be observable are (1) that there exist sections of the Fermi surface with a Fermi velocity small enough so that the effect occurs at measurable large angles of tilt, and (2) that $\omega\tau$ be $\geqslant 10$ so that the contribution of the orbit drifting in phase with the sound wave dominates those from all other orbits. The first requirement would seem to be satisfied by the semimetals and by certain portions of the Fermi surfaces of Sn, Zn, Mg, Ga, etc. The second condition would require materials of ultrahigh purity together with microwave sound waves except perhaps for the semimetals, Sn, Zn, and maybe others.

Detailed calculations have shown that the geometric resonances for tilted magnetic fields arise from the field dependence of the conductivity tensor. The calculations have been made using spherical Fermi surfaces for the sake of simplicity. In the case of longitudinally polarized waves, the σ_{zz} component of the conductivity tensor plays the essential role. In the region of the geometric resonance (i.e., $X = qV_F/\omega_c \geqslant 1$) and for large $\omega\tau$ we find

$$\sigma_{zz} = [3\sigma_0/(ql\ \sin\nu)^2] \times (1 - i\omega\tau) J_0{}^2(X \cos\nu \sin\theta^*)[N - iM],$$

where N and M are functions[3] of $\omega\tau$ and ν but not of magnetic field, and

$$\theta^* = \cos^{-1}(V_S/V_F\ \sin\nu).$$

Also, q is the sound wave number and l is the mean free path of the carriers. The Bessel function gives rise to the oscillation in the attenuation with the magnetic field. In evaluating the expression for σ_{zz} we have used the presence of denominators in the integrals involved which give a resonance whenever $\cos\theta = V_S/V_F\ \sin\nu$ for large values of $\omega\tau$. This is the condition for the steady acceleration of carriers travelling on an orbit at an angle θ to the magnetic field. From the period of the oscillations, the dimensions of the Fermi surface can be determined by using

$$q \cos\nu V_F \sin\theta^* \Delta(1/\omega_c) \approx \pi.$$

The attenuation has been calculated using several of the models developed by others.[6,7] Among these models are the free-electron model, the two-band model with deformation potential for semimetals, and the case of minority and majority carriers with a positive background. All the cases considered show geometric resonance in a tilted field in a fairly similar way. In Fig. 2 we have plotted a normalized attenuation versus magnetic field for the case of the minority and majority carriers. The oscillations of the attenuation with the magnetic field are very much stronger than for transverse fields, to which all the orbits contribute. The theoretical calculations thus support in detail the physical arguments given about obtaining the dimensions of nonextremal orbits on the Fermi surface through a combined geometric-resonance tilt-effect experiment.

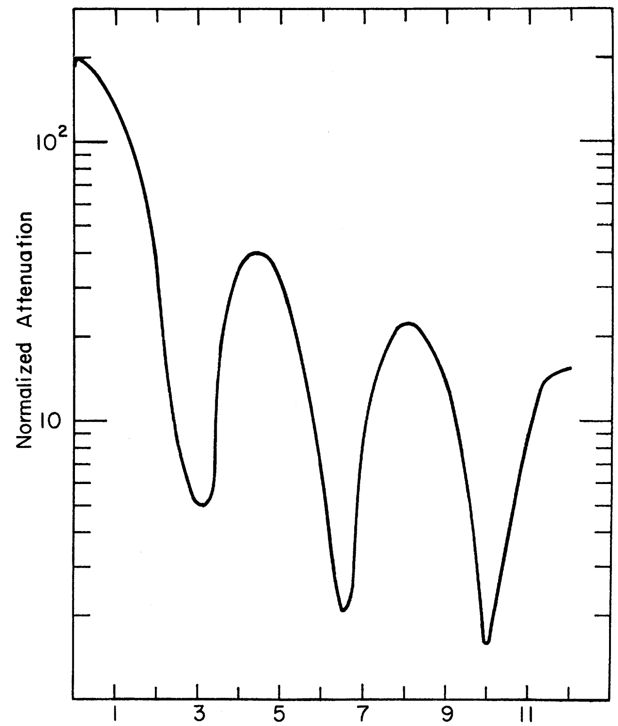

FIG. 2. The normalized attenuation is plotted versus $X = qV_F/\omega_c$ for a model consisting of minority and majority carriers with $V_F(\text{maj})/V_F(\text{min}) = 10$. The angle of tilt in this case is $\nu = 0.02$ and $\omega\tau = 10$. The plot is correct for all magnetic fields when the screening of the transverse currents breaks down and is correct for $X > 2$ when screening does not break down.

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CYCLOTRON RESONANCE IN INDIUM

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Cyclotron resonance absorption in indium reveals carriers having several values of effective mass. The results of preliminary measurements at 9.1 kMc/sec are described here, for one orientation.

A large single crystal was grown from Tadanac special research grade indium by means of the modified Bridgman technique. The experimental samples were cut, by means of a spark cutter,[1] in the shape of thin rectangular slabs, measuring approximately $2\times10\times20$ mm. Each slab was electropolished after cutting and its surface served as about 10% of the surface of a microwave cavity in the reflection spectrometer previously described.[2] The cavity was maintained reasonably close to match ($\Gamma\sim0.1$) with changes in reflection, $\Delta\Gamma$, corresponding to absorption. A 60-cps modulation of amplitude H_M was superposed on the steady field H_0. The output of the synchronous detector was presented on an X-Y recorder with X proportional to H_0 and Y to the 60-cps component of dR/dH.

Absorption signals were observed when the temperature was 2.1°K or below. Figure 1 is an example which shows well-defined peaks, together with some weak ones. This recording was obtained at 1.54°K with H_0 along the [111] axis and $H_M\sim20$ oersteds. The microwave magnetic field was parallel to H_0 and approximately parallel to the length of the sample. The large peak below 200 oe was found to be associated with the superconducting-normal transition. Repeated runs up to as high as 8000 oersteds gave reproducible series of maxima, from which the effective masses shown in the first column of Table I were calculated. In addition, there appear a number of other peaks up to 7000 oersteds to which no unambiguous assignment of masses has been made.

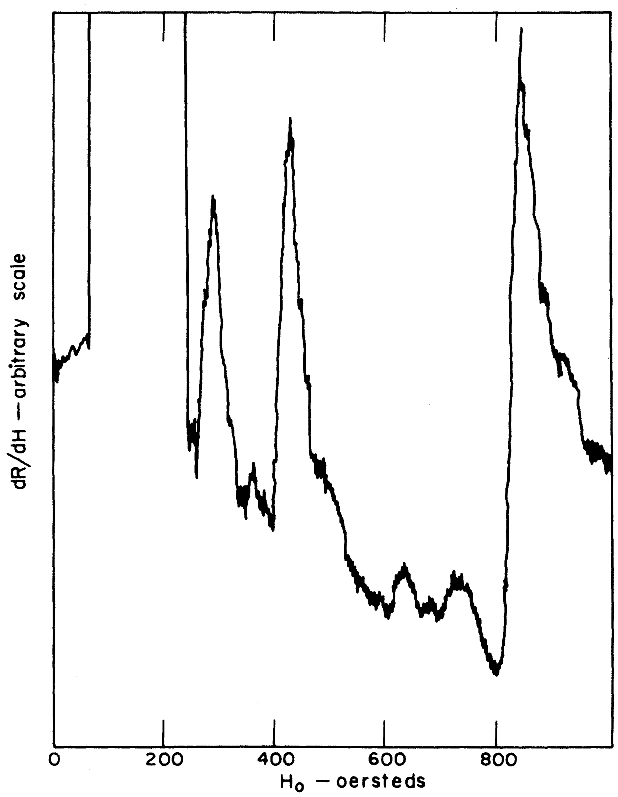

FIG. 1. Cyclotron resonance absorption in indium. The ordinate is linear with the 60-cps absorption component of $\Delta\Gamma$ of the sample cavity. Conditions: $T=1.54$°K, H_0 parallel to the [111] crystal axis, $f=9.1$ kMc/sec, and $H_M\sim20$ oe.

Indium is trivalent, and has a face-centered tetragonal structure only slightly distorted from the fcc structure of aluminum. Harrison[3] has shown that the free-electron model is reasonably successful in interpreting the cyclotron resonance data for alumium. One would therefore expect the model to be a fair approximation for indium too. The portion of the Fermi surface in the second zone in indium will be tetragonally