LAW OF CONSERVATION OF MUONS*

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The apparent absence of muon-electron transitions without neutrinos, such as $\mu \to e+\gamma$, $\mu \to 3e$, and $\mu^- + p \to e^- + p$, leads one to suspect that there is a new conservation law forbidding them. Calculations[1] of the rate of such processes, assuming no such law exists, have indicated that it is hard to understand their absence in an intermediate boson theory of weak interactions. Even if there is no intermediate boson, the decay $\mu \to e + \nu_1 + \bar{\nu}_2$ with $\nu_1 = \nu_2$ would lead to these processes in some order of perturbation theory, and arguments have been given[2] which indicate that any field theory of weak interactions may predict unacceptably large rates for these processes in the absence of a selection rule.

If we assume that $\mu^- - e^-$ transitions are forbidden by a selection rule, the nature of the selection rule remains an open question. It has been suggested[3] that an additive quantum number exists which is always conserved, and which[4] is +1 for μ^- and zero for e^-. In order to make this consistent with known weak interactions, it is necessary to assume that there are two neutrinos, which are distinguished by their value of this quantum number. The conservation law forbids all reactions in which any nonzero number of muons change into electrons, without neutrinos.

This assumption of an additive conservation law is not the only possibility. All of the "missing reactions" involve odd numbers of muons and electrons. It is therefore possible to forbid them by a multiplicative conservation law. By this it is meant that there is a quantity we shall call "muon parity" which is -1 for the muon and its neutrino, and +1 for electrons and all other known particles.[5] The muon parity of a system of particles is the product of its values for the individual particles, and is to be universally conserved. The possibility of multiplicative conservation laws has been known for some time,[6] although no law precisely of this type is known to exist at present.

There are certain theoretical arguments in favor of a multiplicative conservation law for muons and electrons. It has recently been shown[7,8] that the symmetry in the properties of muon and electron, as well as their different mass, can be summarized by the invariance of the laws of nature under permutation of two primitive leptons (say e' and μ'). If the e' and μ' can make transitions into each other, they will not be observed as particles, but instead certain linear combinations; $e = (\mu' + e')/\sqrt{2}$, $\mu = (\mu' - e')/\sqrt{2}$, which would be stable in the absence of weak interactions, will be the observed electron and muon, and will necessarily have different mass. Invariance under the permutation symmetry $\mu' \longleftrightarrow e'$ implies invariance under the transformation $e \to e$, $\mu \to -\mu$. It is shown in reference 8 that the extension of the permutation symmetry to weak interactions requires the existence of two neutrinos ν_e, ν_μ, which also transform as $\nu_e \to +\nu_e$, $\nu_\mu \to -\nu_\mu$. This argument therefore leads directly to a multiplicative conservation law of "muon parity." If no particular model is assumed for the weak interactions, no stronger, additive, conservation law is implied. Of course, we cannot rule out the possibility that there is an additive quantum number, which would imply a stronger selection rule than the multiplicative law. This was indeed the case in the specific Lagrangian models studied in references 7 and 8. However, we believe it is worthwhile to consider the consequences of the multiplicative symmetry by itself, and to perform experiments to distinguish between the additive and multiplicative conservation laws.

The two conservation laws both forbid $\mu \to e+\gamma$, $\mu \to 3e$, and $\mu + p \to e + p$. They also imply that two different neutrinos are emitted in μ decay. However, the additive law implies that the μ^+ can only decay by $\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$, while the multiplicative law would also allow $\mu^+ \to e^+ + \nu_\mu + \bar{\nu}_e$. If the neutrinos from μ decay can be used to induce inverse transitions, the latter possibility could be tested by looking for μ^- produced by neutrinos from μ^+ decay, which is forbidden, by lepton conservation, if only $\mu^+ \to e^+ + \bar{\nu}_\mu + \nu_e$.

If one considers systems with only muons and electrons, the selection rule implied by conservation of muon parity is

$$(-1)^{n_\mu (\text{initial state})} = (-1)^{n_\mu (\text{final state})},$$

where n_μ is the number of muons of either charge present. One reaction which is allowed by it while forbidden by the additive law is $e^- + e^- \to \mu^- + \mu^-$. The cross section for this in a clashing beam experiment at 10 Bev is probably no greater than 10^{-36} cm^2.

Another reaction allowed by muon parity conservation and forbidden by the additive law is the transformation of muonium ($\mu^+ e^- \equiv M$) into antimuonium ($\mu^- e^+ \equiv \bar{M}$). This reaction was first discussed in a theory with no muon conservation law by Pontecorvo.[9] If an interaction exists which allows $M \leftrightarrow \bar{M}$ transitions with matrix element $\delta/2$, then the vacuum energy eigenstates will not be M and $\bar{M}$, but some linear combinations M_1, M_2 with different energies. Thus a system which is pure muonium at $t = 0$ will develop an admixture of antimuonium at a later time. We show in another place[10] that the probability of seeing the system in vacuum decay as antimuonium ($\bar{M} \to \underline{\text{fast}}\ e^- + \underline{\text{slow}}\ e^+ + \text{neutrinos}$) rather than as muonium ($M \to \underline{\text{fast}}\ e^+ + \underline{\text{slow}}\ e^- + \text{neutrinos}$) is given by

$$P(\bar{M}) = \tfrac{1}{2}\delta^2/(\delta^2 + \Delta^2 + \lambda^2), \tag{1}$$

where λ is the muon decay rate ($= 0.45 \times 10^{+6}$ sec^{-1}, or 3×10^{-10} ev) and Δ is any additional splitting of M and $\bar{M}$, say by external electromagnetic fields. We can estimate δ by assuming that the $M \leftrightarrow \bar{M}$ transition is produced by the Fermi-type interaction,

$$H = (C_V/\sqrt{2})\bar{\psi}_\mu \gamma_\lambda (1+\gamma_5)\psi_e \bar{\psi}_\mu \gamma^\lambda (1+\gamma_5)\psi_e + \text{H.c.} \tag{2}$$

If C_V is the vector β-decay coupling constant, we get, for hyperfine $F = 0$ and $F = 1$ ground states,

$$\delta = 16C_V/(\sqrt{2}\pi a^3) = 2.1 \times 10^{-12} \text{ ev} = 3200 \text{ sec}^{-1}, \tag{3}$$

so that if $\Delta \ll \lambda$, $P(\bar{M}) \simeq \frac{1}{2}\delta^2/\lambda^2 = 2.6 \times 10^{-5}$, which would probably be observable. An interaction of this type and magnitude would not have shown up in any previous experiments. An estimate[10] of the probability of seeing an e^- which has gained $\gtrsim 10$ Mev from the e^+ in an ordinary μ^+ decay gives a value $< 10^{-10}$.

It can be shown[10] that constant external fields do not contribute to Δ in the $F = 0$ ground state. Because of this, the splitting in this state for macroscopic external fields is negligible. Thus an experiment to detect the transition, in which the muonium-antimuonium system is in vacuum for most of the muon lifetime, will, for the $F = 0$ state, be governed by the vacuum rate (2.6×10^{-5}), even if fields are present.

On the other hand, an experiment done with muonium which remains in matter would give a much lower transition rate. This case is treated in detail in reference 10. In a solid, the energy shift Δ will be much larger than λ, which reduces the rate by a factor $(\lambda/\Delta)^2$. In a gas, the effect of collisions is to make the amplitudes, for making antimuonium in the periods between collisions, add incoherently. If one starts with muonium, the probability of seeing a μ^- decay or be captured by the nucleus of a gas atom is

$$P(\bar{M}) \simeq \delta^2/(2\lambda\omega_c), \tag{4}$$

where ω_c is the collision rate. Thus the rate is reduced by $\lambda/\omega_c = 1/N$, where N is the number of collisions, compared to the vacuum rate. In a typical experiment in a gas, $1/N$ might be 10^{-4} to 10^{-6}.

We would finally like to indicate the possible relevance of the intermediate-boson hypothesis for these considerations. If all weak interactions go via intermediate bosons, it is necessary for us to assume the existence of bosons with muon parity of -1. Such bosons are forbidden by the conservation laws to interact linearly with pions, baryons, etc. A neutral boson B^0, identical with its antiparticle, could, by interacting with μe pairs, generate the interaction (2). A charged boson B^+, interacting with the pairs $\mu\nu_e$ and $e\nu_\mu$, would generate the decay[11] $\mu^+ \to e^+ + \nu_\mu + \bar{\nu}_e$. If such a charged boson exists, it might be detected in the experiments recently proposed[12] to detect the different bosons (W^+, W^0) which may mediate the known weak interactions. Let us assume that the neutrino-scattering experiments ($\nu + n \to \text{lepton} + p$) indicate the existence of two neutrinos. (The forbidding of $\nu_\mu + n \to e^- + p$ is unaffected by the existence of B^+.) The neutrinos in $\pi^- \to \mu^- + \nu$ decay must be ν_μ. The following process may now occur if muon parity is conserved:

$$\nu_\mu + \text{nucleus} \to e^- + B^+ + \text{nucleus}.$$

The B^+ may now decay only into $\mu^+ + \nu_e$ or $e^+ + \bar{\nu}_\mu$, but not into pions, etc. The production of electrons with such a boson which decays into

μ is forbidden in a theory with an additive conservation law. If there were no muon conservation law at all, then electrons may be produced with a boson which decays into pions. The detection of electrons produced by ν_μ, together with a boson of only leptonic decay modes, would be strong evidence in favor of a multiplicative selection rule.

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[1]G. Feinberg, Phys. Rev. 110, 1482 (1958); F. J. Ernst, Phys. Rev. Letters 5, 478 (1960).

[2]N. Cabibbo and R. Gatto, Phys. Rev. 116, 1334 (1959); T. D. Lee (private communication); B. L. Joffe, Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960).

[3]K. Nishijima, Phys. Rev. 108, 907 (1957); J. Schwinger, Ann. Phys. 2, 407 (1957); S. Bludman, Nuovo cimento 9, 433 (1958).

[4]The assignment given here of the additive quantum number is conventional. A linear combination of this quantum number with lepton number would give another conserved quantum number whose physical implications are the same.

[5]The assignment of muon parity +1 to baryons, pions, etc., follows from the existence of nonleptonic transitions among such particles, and is unique up to factors $(-1)^{\text{baryon number}}$ or $(-1)^{\text{charge}}$ which have no physical significance.

[6]See, e.g., C. N. Yang and J. Tiomno, Phys. Rev. 79, 495 (1950). The properties of multiplicative conservation laws are discussed by G. Feinberg and S. Weinberg, Nuovo cimento 14, 5711 (1959).

[7]G. Feinberg, P. K. Kabir, and S. Weinberg, Phys. Rev. Letters 3, 527 (1959).

[8]N. Cabibbo and R. Gatto, Phys. Rev. Letters 5, 114 (1960).

[9]B. Pontecorvo, Zhur. Eksp. i Teoret. Fiz. 33, 549 (1957) [translation: Soviet Phys.—JETP 6(33), 429 (1958)].

[10]S. Weinberg and G. Feinberg (to be published).

[11]In this case, in order to make the total μ-decay rate come out to agree approximately with the universal coupling theory, it is necessary to reduce the coupling constant of bosons with leptons by 1/2, compared to the coupling with baryons. This may be contrary to the spirit of the universal coupling theory. However, the ambiguities in the definition of "universality" are well known, and have been emphasized recently by M. Gell-Mann.

[12]M. Schwartz, Phys. Rev. Letters 4, 306 (1960); T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960).

SPIN OF THE K'

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Recent experimental work on high-energy K-N collisions at Berkeley has been shown by Alston et al.[1] to suggest the existence of an unstable particle K',[2] formed from a K meson and a pion. They further assign to it a mass ~878 Mev, full width 23 Mev, and isospin 1/2. We shall investigate below, on the basis of the assumption that the pions produced from K-N collisions come entirely from the decay of K', whether K' is a vector or a scalar boson. We find that a vector K' fits the experimental data better than a scalar K'.

The following interactions are considered:

$$K' \text{ with spin } 0,\quad H_I = mG\phi_{K'}\phi_K\phi_\pi; \tag{1}$$

$$K' \text{ with spin } 0,\quad H_I = \frac{G}{m}\frac{\partial\phi_K}{\partial x_\mu}\frac{\partial\phi_\pi}{\partial x_\mu}\phi_{K'}; \tag{2}$$

$$K' \text{ with spin } 0,\quad H_I = \frac{G}{m}\frac{\partial\phi_{K'}}{\partial x_\mu}\frac{\partial\phi_K}{\partial x_\mu}\phi_\pi; \tag{3}$$

$$K' \text{ with spin } 0,\quad H_I = \frac{G}{m}\frac{\partial\phi_\pi}{\partial x_\mu}\frac{\partial\phi_{K'}}{\partial x_\mu}\phi_K; \tag{4}$$

$$K' \text{ with spin } 1,\quad H_I = G\left(\phi_\pi\frac{\partial\phi_K}{\partial x_\mu} - \frac{\partial\phi_\pi}{\partial x_\mu}\phi_K\right)\phi_{K'}{}^{\mu}. \tag{5}$$

For the process $K^- + p \to K'^- + p$, shown in Fig. 1, the total cross section using the first interaction is

$$\sigma_1 = \frac{g^2}{4\pi}\frac{m^2G^2}{4\pi}\frac{\pi}{4M^2|q_1|_L{}^2}\int\frac{\Delta^2}{(\Delta^2+\mu^2)^2}d\Delta^2,$$

$$\frac{m^2G^2}{4\pi} = \frac{4\lambda m_0{}^2}{3[(m_0{}^2 - m^2 - \mu^2)^2 - 4m^2\mu^2]^{1/2}},$$