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[1]J. A. Anderson, P. G. Burke, D. D. Carmony, and N. Schmitz, in Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 58.

[2]G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).

[3]V. Alles-Borelli, S. Bergio, E. Perez-Ferreira, and P. Waloschek, Nuovo cimento 14, 211 (1959).

[4]I. Derado and N. Schmitz, Phys. Rev. 118, 309 (1960).

[5]F. Bonsignori and F. Selleri, Nuovo cimento 15, 465 (1960).

[6]W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1609 (1960).

[7]F. J. Bowcock, W. N. Cottingham, and D. Lurié, Phys. Rev. Letters 5, 386 (1960).

ELECTROMAGNETIC FORM FACTORS OF THE NUCLEON AND PION-PION INTERACTION

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We wish to propose a simple model for the electromagnetic structure of the nucleon, based on dispersion theory and on a strong pion-pion interaction. The model is a synthesis of several theoretical ideas proposed by Frazer and Fulco,[1] Nambu,[2] and Chew.[3]

Let us first of all summarize some general properties of the nucleon form factors. We write the interaction of the nucleon with the electromagnetic field in the form:

$$\langle p'|j_\mu|p\rangle A_\mu = i\bar{u}(p')[G_1(t)\gamma_\mu + G_2(t)\sigma_{\mu\nu}k_\nu]u(p)A_\mu, \qquad (1)$$

where p', p, and k are the four-momenta of the final nucleon, initial nucleon, and photon, respectively, and $t=k^2=(p'-p)^2$. The G_i still are operators in the isospin space:

$$G_i = G_i^S + G_i^V\tau_3,$$

and so

$$G_i^p = G_i^S + G_i^V; \quad G_i^n = G_i^S - G_i^V.$$

As is well known, the separation into the isoscalar and the isovector current is very useful because only an even number of pions contribute to G^V and an odd number to G^S. At $t=0$ the G_i functions tend to the static charge and magnetic moment of the nucleon:

$$G_1^p(0)=e, \quad G_1^n(0)=0,$$

$$G_2^p(0)=\mu_p=eg_p/2M, \quad G_2^n(0)=\mu_n=eg_n/2M,$$

$$G_1^S(0)=G_1^V(0)=e/2,$$

$$G_2^S(0)=(\mu_p+\mu_n)/2=eg_S/2M,$$

$$G_2^V(0)=(\mu_p-\mu_n)/2=eg_V/2M,$$

$$g_p=1.79, \quad g_n=-1.91,$$

$$g_S=-0.06, \quad g_V=1.85, \qquad (2)$$

The functions $G(t)$ are related to the usual Hofstadter form factors $F(t)$ by the following definitions:

$$G_i^{p,n}(t)=G_i^{p,n}(0)F_i^{p,n}(t). \qquad (3)$$

Dispersion theory allows one to write the different functions $G(t)$ in the following form[4]:

$$G(t)=\frac{1}{\pi}\int_0^\infty \frac{g(t')}{t'-t}dt'. \qquad (4)$$

The spectral functions $g(t)$ are of fundamental theoretical importance because they are related to the weight with which the different many-particle states contribute to the nucleon form factors. Therefore, if there is no bound state formed by pions, $g(t)$ will be different from zero only for t larger than $4m_\pi^2$ for the vector part and $9m_\pi^2$ for the scalar part.

If there is no strong correlation between the pions, $g(t)$ is related just to the statistical weight of the many-particle state with c.m. energy $E=\sqrt{t}$ and therefore will be a smoothly increasing function of t starting from zero.

If on the other hand, there is a strong correlation between the pions, due to a resonance with an energy E_R, the spectral function will exhibit a maximum for $t=t_R=E_R^2$. This result was first shown by Frazer and Fulco[1] and is schematically illustrated in Fig. 1.

Therefore, in a model without a strong pion-pion correlation the spectral function $g(t)$ will be dominated by the large values of t and $G(t)$ will have little dependence on t. The discrepancy of this model from the experimental data is discussed by Drell.[5]

On the other hand, a strong-correlation model leads to a rapid variation of $G(t)$ as suggested by experiment. Since for observable values of t ($t \leq 0$) the dispersion denominator in Eq. (4) is always positive, if the resonance state has a reasonable width ($<\frac{1}{2}m_\pi$), its effect can be well approximated by means of $\alpha/(t_R-t)$, where t_R is the resonance position and α is the area under the resonance curve.

Let us now discuss briefly the experimental results for the nucleon form factors at low momentum transfers. For $-t<10m_\pi^2$ the nucleon form factors were roughly given as follows:

$$F_1^p(t) \simeq F_2^p(t) \simeq F_2^n(t) \equiv F(t),$$
$$F_1^n(t) \simeq 0. \tag{5}$$

Many different analytic functions were proposed for $F(t)$ but a particularly good fit was obtained either with an exponential form or with the form proposed by Clementel and Villi[6]:

$$F(t)=0.2+\frac{1.2}{1-(t/22m_\pi^2)}. \tag{6}$$

From our theoretical point of view, it is very difficult to understand an exponential form factor; on the other hand, the Clementel-Villi model can be naturally understood on the basis of a resonant state of energy $4.7m_\pi$. The constant term appearing in Eq. (6) represents the total contribution of the higher t states which in the low-momentum-transfer region is approximately constant. Thus Eqs. (5) and (6) indicate that it is possible to interpret both isovector form factors F_1^V and F_2^V by means of the approximate form, which has a pole at $t_R \simeq 22m_\pi^2$:

$$G_1^V \simeq \frac{e}{2}\left(-0.2+\frac{1.2}{1-(t/22m_\pi^2)}\right),$$

$$G_2^V \simeq \frac{eg_V}{2M}\left(-0.2+\frac{1.2}{1-(t/22m_\pi^2)}\right). \tag{7}$$

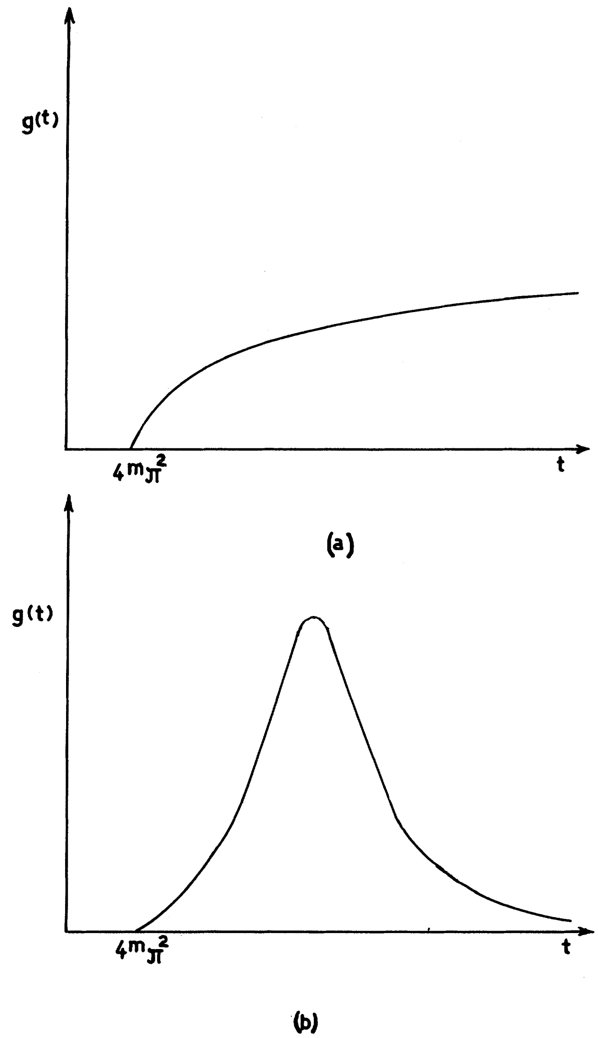

FIG. 1. Schematic representations of $g(t)$ in arbitrary scale. (a) Uncorrelated pions; (b) strong pion-pion resonance.

By taking this attitude, the resonant state at $E_R \simeq 4.7m_\pi$ will be attributed to a $T=1$, $J=1$ two-pion state.

Such a resonant state, at this energy, has been observed in different experiments[7] on pion production by pions, and its parameters, deduced from Eq. (7), have been used[8] to explain satisfactorily the low-energy behavior of the pion-nucleon scattering phase shifts.

The isoscalar charge form factor $G_1^S(t)$ must be of the same order of magnitude of $G_1^V(t)$, in order to give a vanishing neutron charge distribution. This means that we have to expect important low-t contributions to the isoscalar form factors. The isoscalar magnetic form factor is experimentally very small and little is known about its energy variation.

The recent[9] experimental data on electron-proton scattering at larger values of the momentum transfer stress the need of having an important low-mass contribution to the isoscalar charge form factor. It is found that the charge form factor of the proton (Fig. 2) deviates strongly from Eq. (7) and it would be impossible to fit it with any reasonable form with only one pole.

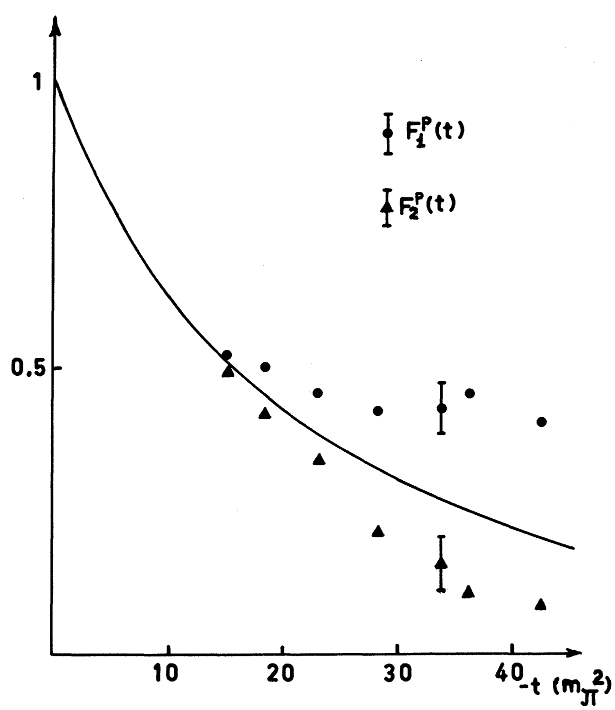

FIG. 2. Proton form factors at high momentum transfer (from reference 9). The solid line represents the Clementel and Villi model [in Eq. (6)].

A reasonable extension of the preceding ideas is to assume that G_1^V is still approximately given by Eq. (7) with the pole at $t_V \simeq 22m_\pi^2$, which is the value suggested by other experimental information. In this manner the flattening of G_1^p will be essentially due to the effect of G_1^S. Thus G_1^S must be composed of an almost constant part and a part which goes to zero much faster than G_1^V. This means that the charge form factor of the neutron will be positive at large values of t. This result seems to be confirmed by recent experiments on the neutron.[10]

If we tentatively attribute an average mass to the decreasing part of G_1^S, we have something like $8m_\pi^2$, certainly less than $22m_\pi^2$. This shows that it would be very difficult to explain the rapid variation of G_1^S on the basis of a simple statistical formula for $g_1^S(t)$, since the spectral integral will only start at $9m_\pi^2$. We thus expect the existence of a $T=0$, $J=1$ three-pion resonance (or bound state if $t_S < 9m_\pi^2$). The magnetic isoscalar part must also have a pole at the same position of t_S, but the present experimental information about it is certainly not enough to allow detection of its effects.

From the preceding discussion we expect expressions for G_1^S which have similar form to those for G_1^V. This leads to the following general form for the nucleon form factors:

$$G_1^V = \frac{e}{2}\left[(1-a_V) + \frac{a_V}{1-(t/t_V)}\right],$$

$$G_1^S = \frac{e}{2}\left[(1-a_S) + \frac{a_S}{1-(t/t_S)}\right],$$

$$G_2^V = \frac{eg_V}{2M}\left[(1-b_V) + \frac{b_V}{1-(t/t_V)}\right],$$

$$G_2^S = \frac{eg_S}{2M}\left[(1-b_S) + \frac{b_S}{1-(t/t_S)}\right], \qquad (8)$$

where t_V and t_S represent the position of the isovector and isoscalar unstable particles, respectively.

The residues of the poles a_V, b_V, a_S, b_S are connected with the constants appearing in similar terms giving the effect of pion-pion interaction in π-N scattering, N-N scattering, and photoproduction.[7,11] The validity of the present model can thus be checked by trying a general fit of many different sets of experimental data using the same phenomenological parameters.

One of the best known experimental properties of the form factors is that the mean charge radius of the neutron is zero. This leads to

$$a_S/t_S \simeq a_V/t_V = a. \quad (9)$$

As a consequence we have

$$G_1^{\,p} = e\left[1+\frac{a}{2}\left(\frac{t_S}{t_S-t}+\frac{t_V}{t_V-t}\right)t\right];$$

$$G_1^{\,n} = -e\,\frac{a}{2}\left(\frac{t_V}{t_V-t}-\frac{t_S}{t_S-t}\right)t. \quad (10)$$

This means that we have in our model five independent parameters (a, t_V, t_S, b_S, and b_V), for one of which (t_V) we know the approximate value from independent experiments.

Some preliminary determinations[10,12] of the parameters contained in Eq. (8), using information from the proton and neutron form factors, confirm the validity of the present model. We wish to stress a very important consequence of the fact that t_S turns out to be smaller than t_V. From Eq. (10) one sees that the outer part of the charge distribution of the neutron is positive in contrast with what one would obtain on the basis of a model without strong pion-pion interaction.[13]

It is not surprising that, in a strong $T=1$ pion-pion interaction model, the three-pion state has a mass lower than the two-pion state. The existence of a resonance in a $T=1$, $J=1$ two-pion state forces the three-pion state to be in a $T=0$, $J=1$ state in which all the three pions are two-by-two resonating in a $T=1$, $J=1$ state.[3] Moreover the $T=0$, $J=1$ three-pion state is a completely saturated unit because if we add an extra pion it will be in a $T=0$ or 2 state with respect to the others. So, if we believe that there is a strong attraction in the $T=1$ state only, we can expect that the four-, five-, etc., pion states are not strongly correlated at low energy.

The possibility of detecting experimentally the $T=0$ unstable particle (which we shall denote by ρ) is discussed in references 2 and 3. A very interesting possibility is to identify it with the $T=0$ particle observed by Abashian et al.,[14] who find $t_S \simeq 5$. The choice $J=1$ for ρ was suggested by Chew[15] in order to explain the small effect of this particle in the decay spectrum of the K meson.

The spin of ρ together with the small available phase space would make the decay probability $K \rightarrow \rho + \pi$ very small but not impossible to detect.

On the other hand, preliminary[10,12] estimates show that a value $t_S \simeq 5$ is not incompatible with the experimental data on the form factors. There also seems to be some indication in favor of $t_S \simeq 5$ coming from the theoretical analysis of the electromagnetic form factor of the α particle.[16]

Another important indication coming from the qualitative determination of the parameters of Eq. (8) is that large constant terms are needed; in other words, the two- and three-pion resonating states are not sufficient to describe the nucleon completely.

One could take two attitudes: The first is that the dispersion relations make sense only in the subtracted version and the necessary constant terms are not linked to any observable effect; the second point of view is that the form factors tend to zero for $t \rightarrow \infty$ and the constants represent contributions of high-energy states. Such contributions, which are approximately constant at low momentum transfer, will show their t dependence as the momentum transfer increases. Being in favor of the second point of view (at least so far as the magnetic form factor is concerned), we are convinced that our knowledge of the nucleon structure is still incomplete and that higher energy electron-nucleon scattering will be of great importance in clarifying the many-particle contributions to the nucleon structure.

We wish to thank Professor R. Hofstadter for having communicated to us his results prior to publication. At the time part of this investigation was carried out, one of us (S.F.) was visiting the Department of Physics of Stanford University. He wishes to express his most sincere thanks for the very kind hospitality extended to him and to acknowledge the very illuminating discussions he had on this subject with Professor L. Schiff and Professor R. Hofstadter.

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[1]W. R. Frazer and J. R. Fulco, Phys. Rev. Letters 2, 365 (1959); Phys. Rev. 117, 1609 (1960).

[2]Y. Nambu, Phys. Rev. 106, 1366 (1957).

[3]G. F. Chew, Phys. Rev. Letters 4, 142 (1960).

[4]This form is valid if no subtractions are needed; otherwise we have

$$G(t) = G(0) + \frac{t}{\pi}\int_0^\infty \frac{g(t')}{t'(t'-t)}\,dt'.$$

[5]S. D. Drell, Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics

(Interscience Publishers, New York, 1957), p. 27.
[6]E. Clementel and C. Villi, Nuovo cimento 4, 1207 (1958).
[7]E. Pickup, F. Ayer, and E. O. Salant, Phys. Rev. Letters 4, 474 (1960); J. G. Rushbrooke and D. Radojičić, Phys. Rev. Letters 5, 567 (1960).
[8]J. Bowcock, W. N. Cottingham, and D. Lurié, Phys. Rev. Letters 5, 386 (1960).
[9]R. Hofstadter, F. Bumiller, and M. Croissiaux, Phys. Rev. Letters 5, 263 (1960).
[10]R. Hofstadter, C. de Vries, and R. Herman, Phys. Rev. Letters 6, 290 (1961); R. Hofstadter and R. Herman, Phys. Rev. Letters 6, 293 (1961).
[11]M. Cini, Proceedings of the Ninth Annual Rochester Conference on High-Energy Nuclear Physics (Interscience Publishers, New York, 1959); F. Fubini, Proceedings of the Conference on Strong Interactions, Berkeley, California, 1960 [Revs. Modern Phys. (to be published)].
[12]S. Bergia and A. Stanghellini (to be published).
[13]L. I. Schiff, Revs. Modern Phys. 30, 462 (1958).
[14]A. Abashian, N. E. Booth, and K. M. Crowe, Phys. Rev. Letters 5, 258 (1960).
[15]G. F. Chew, Proceedings of the Berkeley Conference on Strong Interactions, Berkeley, California, 1960 [Revs. Modern Phys. (to be published)]. We thank Professor M. Baldo-Ceolin and G. Quareni for a very illuminating discussion on this point.
[16]L. Bertocchi, C. Ceolin, and M. Tonin (private communication).

PION-PION INTERACTION IN THE PHOTOPRODUCTION OF NEUTRAL PIONS WITH POLARIZED γ RAYS

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The presence of a possible pion-pion interaction in a simple resonance state $T=1$, $J=1$ has already been investigated by several authors.[1-6] In this Letter we shall see the effect of this interaction in the case of π^0 photoproduction from the reaction $\gamma+p\to\pi^0+p$ with polarized γ. By using the Cini-Fubini[7] approximate version of the Mandelstam representation, one can demonstrate[4] that in order to take into account the pion-pion interaction in the photoproduction to a reasonable approximation, one should simply add to the expression of the scattering amplitude of CGLN[8] a contribution of the form of the Born term that derives from the graph of Fig. 1. The contribution S_β' (β is the

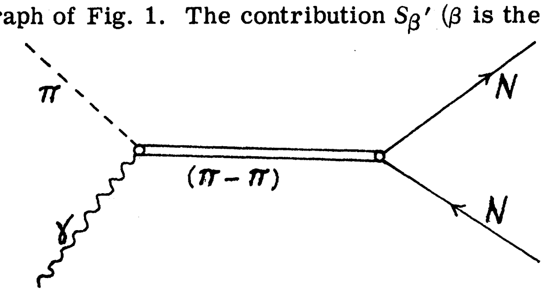

FIG. 1. The first-order perturbation graph which takes into account the π-π interaction. The introduction of the intermediate particle (π-π) simulates the effect of a narrow $J=1$, $T=1$ pion-pion resonance in the intermediate state.

isotopic spin index) to the total S matrix is found to be[6]

$$S_\beta'=i(2\pi)^4\delta_4(p_1+k-p_2-q)\times\frac{1}{(4k_0\omega E_1E_2)^{1/2}}\bar{u}(p_2)Mu(p_1)\tau_\beta, \quad (1)$$

with

$$M=\frac{8\pi\Lambda}{(p_2-p_1)^2+(4\mu)^2}\times\left[M_D-\frac{\mu_p'-\mu_n}{2M}\left[M_A(2q\cdot k+\mu^2)-M_\beta\right]\right], \quad (2)$$

where k, q, p_1, and p_2 are the 4-momenta of the photon, pion, initial nucleon, and final nucleon, respectively; k_0, ω, E_1, and E_2 are the corresponding energies; μ_p' and μ_n are the anomalous magnetic moments of the proton and the neutron; M = nucleon mass; μ = pion mass. The "mass" of the intermediate "particle" is assumed to be $\sim 4\mu$. The constant Λ is proportional to the "strength" of the photon–three-pion interaction and M_A, M_B, M_C, M_D are the invariant terms defined by Chew et al.[8]

In order to carry out the calculation, one simply has to add the coefficients of the invar-