EXPERIMENTAL RESULTS ON THE π-π CROSS SECTION*

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At the 1960 Rochester Conference,[1] we reported our first experimental results on the π-π cross section, which we obtained by applying the Chew-Low extrapolation method[2] to about 700 π-p inelastic scatterings. These events were analyzed in photographs taken with the Alvarez 72-inch hydrogen bubble chamber, which was exposed to a 1.03-Bev/c π^- beam at the Bevatron. We now have a total of 1275 inelastic events with the proton stopping in the chamber. In this paper we will use the notation, the selection and evaluation criteria, and the extrapolation procedures which have been discussed in reference 1.

Figure 1 shows our experimental distributions,

$$F(p^2, \omega^2) = (p^2+1)^2 \partial^2\sigma(p^2, \omega^2)/\partial p^2 \partial\omega^2,$$

as functions of the four-momentum transfer p^2

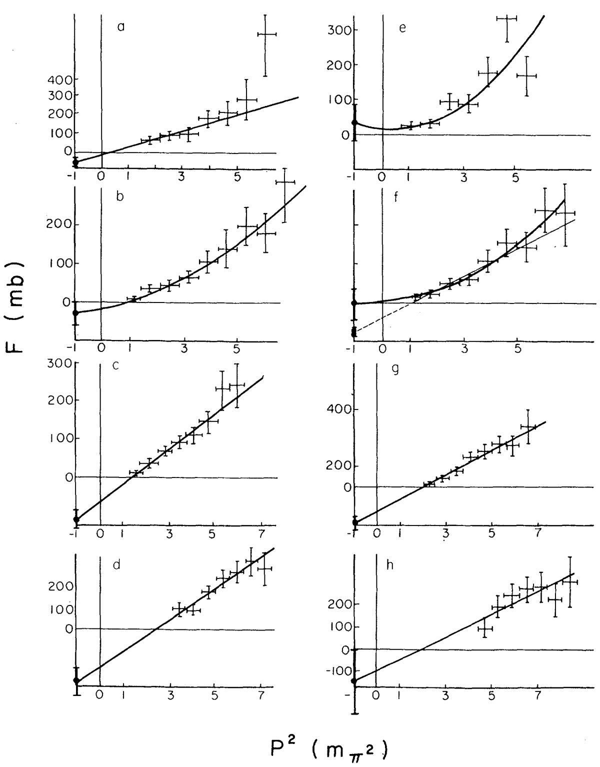

FIG. 1. Extrapolation curves $F(p^2, \omega^2)$ at fixed ω^2. (a) $\omega^2 = 5$ to $8.2\ m_\pi^2$; (b) $\omega^2 = 11$ to 13.7; (c) $\omega^2 = 16.5$ to 19.2; (d) $\omega^2 = 22$ to 24.7; (e) $\omega^2 = 8.2$ to 11; (f) $\omega^2 = 13.7$ to 16.5; (g) $\omega^2 = 19.2$ to 22; (h) $\omega^2 = 24.7$ to 27.5.

for eight different intervals of ω^2, the invariant mass of the di-pion. We express p^2 and ω^2 in units of the pion mass squared. Fitting polynomials in (p^2+1) of several orders through these data, we obtain results as follows.

1. Whereas at the time of the Rochester Conference a first-order polynomial adequately fitted the data for all the intervals of ω^2, we find that a χ^2 test now reveals the necessity of a quadratic fit for the low-ω^2 intervals. In the higher ω^2 intervals, although we have more events than at the lower ω^2 intervals, the linear fits are still exceedingly good. We therefore take the linear values for the extrapolation π-π cross section at the higher energy.

2. Also at the time of the Rochester Conference some of the fitted curves had the unphysical feature that they went negative in the physical region. This was possible because we do not have events right up to the edge of the physical region. Since we now require quadratic fits for the lower ω^2 intervals, this difficulty has been removed in all cases except for the lowest ω^2 interval (5.5 to 7.8 m_π^2). For this interval only we had to apply a constraint forcing the fitted curve to go through the end of the physical region. We then find that a linear curve with this added constraint gives a good fit. In all other intervals the fitted curve remains positive in the physical region without this constraint.

In order to get some information about the π-π interaction from a very small number of events, Bonsignori and Selleri[3-5] have made the assumption that formula (3.13) of reference 2 (which is exactly correct only at the pole $p^2 = -1$) approximately describes the p^2 dependence at the beginning of the physical region. With this assumption an average π-π cross section has been determined. The assumption means that all other contributions to the amplitude for single-pion production (p^2 dependence of the vertex functions of the single-pion exchange diagram, exchanges of 3, 5, or more pions, and production by collision of the pion with the nucleon core rather than with the cloud) have been neglected in comparison with the single-pion exchange at the pole. With our present statistics we are able to check the assumption of Bonsignori and Selleri. If it were justified, it should be possible [as one can see from (3.12) in reference 2] to fit the experimental distributions (or the first part of them) of Fig. 1 by a straight line passing through zero for $p^2=0$. Our data show that this is not possible. Thus, nonpole terms are important and an extrapolation requiring a large number of events is necessary to extract the pole term.

Frazer and Fulco[6] could explain the vector part of the electromagnetic structure of the nucleon assuming a strong π-π resonance in the $T=J=1$ state at $\omega^2=11$. The π^--π^0 amplitude is 50% isotopic spin 1 and 50% isotopic spin 2. We find no evidence for a π-π resonance of the width and location predicted in reference 6.

Our data do show a rise in the π-π cross section starting at $\omega^2=17$, reaching a value of the order of 200 mb at $\omega^2=20$ to 22. (See Fig. 2.)

However, one must remember that it is just in this region of ω^2 that our extrapolation distance begins to get larger, making the extrapolation procedure less conclusive. We are now scanning film obtained at 1.275 Bev/c incident momentum in order to reduce the extrapolation distance in this region. Also, for ω^2 greater than 9, we have a background of events coming from the reaction $\pi^-+p\rightarrow\pi^-+p+2\pi^0$. These events do not have a pole at $p^2=-1$, but come from a branch cut. We are eliminating this background by means of a kinematical fit.

On the other hand, Bowcock et al.[7] found on a later analysis of the nucleon electromagnetic structure and the low-energy pion-nucleon phase shifts that the Frazer-Fulco resonance should be shifted to about $\omega_{\mathrm{res}}^2=22$. This is consistent with our present results. If we assume that our data peak at $\omega^2=20$ to 22 (our incident energy is insufficient to examine the high-energy side of the peak), then the height is in accord with $(2J+1)4\pi\lambda\!\!\!^{-2}\sin^2\delta_1$ for a p-state resonance—that is, $\sin^2\delta_1=1$. Our half-width (obtained from the low-energy side) is approximately $5m_\pi^2$. Of course, our data do not rule out a nonresonant rise in the cross section composed of s, p, d, f, $\cdots$ states which just happens to satisfy $12\pi\lambda\!\!\!^{-2}$ at $\omega^2=20$ to 22. We are currently evaluating the differential cross section in the region $\omega^2=20$. We hope to resolve this experimental ambiguity in the very near future.

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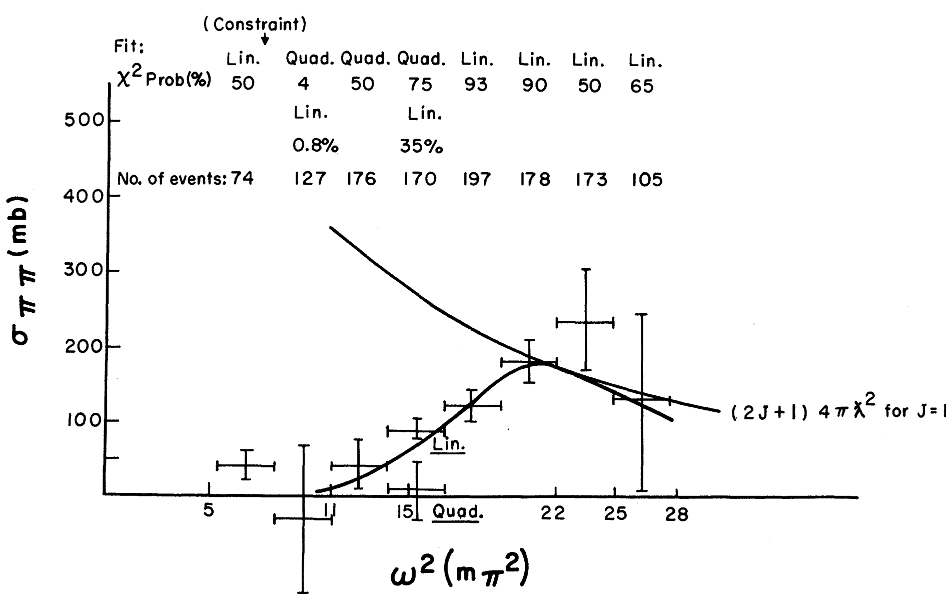

FIG. 2. The π^--π^0 cross section as a function of the total di-pion mass squared as determined by the Chew-Low method. Also shown are the maximum height of a p-state resonance and the shape of the Frazer-Fulco resonance [W. R. Frazer and J. R. Fulco, Phys. Rev. Letters 2, 367 (1959), Eq. (10)], assuming the parameters $v_r=3.5$, $\Gamma=0.3$.

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[1] J. A. Anderson, P. G. Burke, D. D. Carmony, and N. Schmitz, in Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 58.
[2] G. F. Chew and F. E. Low, Phys. Rev. 113, 1640 (1959).
[3] V. Alles-Borelli, S. Bergio, E. Perez-Ferreira, and P. Waloschek, Nuovo cimento 14, 211 (1959).
[4] I. Derado and N. Schmitz, Phys. Rev. 118, 309 (1960).
[5] F. Bonsignori and F. Selleri, Nuovo cimento 15, 465 (1960).
[6] W. R. Frazer and J. R. Fulco, Phys. Rev. 117, 1609 (1960).
[7] F. J. Bowcock, W. N. Cottingham, and D. Lurié, Phys. Rev. Letters 5, 386 (1960).

ELECTROMAGNETIC FORM FACTORS OF THE NUCLEON AND PION-PION INTERACTION

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We wish to propose a simple model for the electromagnetic structure of the nucleon, based on dispersion theory and on a strong pion-pion interaction. The model is a synthesis of several theoretical ideas proposed by Frazer and Fulco,[1] Nambu,[2] and Chew.[3]

Let us first of all summarize some general properties of the nucleon form factors. We write the interaction of the nucleon with the electromagnetic field in the form:

$$\langle p'|j_\mu|p\rangle A_\mu = i\bar{u}(p')[G_1(t)\gamma_\mu + G_2(t)\sigma_{\mu\nu}k_\nu]u(p)A_\mu, \quad (1)$$

where p', p, and k are the four-momenta of the final nucleon, initial nucleon, and photon, respectively, and $t=k^2=(p'-p)^2$. The G_i still are operators in the isospin space:

$$G_i = G_i^S + G_i^V\tau_3,$$

and so

$$G_i^p = G_i^S + G_i^V; \quad G_i^n = G_i^S - G_i^V.$$

As is well known, the separation into the isoscalar and the isovector current is very useful because only an even number of pions contribute to G^V and an odd number to G^S. At $t=0$ the G_i functions tend to the static charge and magnetic moment of the nucleon:

$$G_1^p(0)=e, \quad G_1^n(0)=0,$$
$$G_2^p(0)=\mu_p=eg_p/2M, \quad G_2^n(0)=\mu_n=eg_n/2M,$$
$$G_1^S(0)=G_1^V(0)=e/2,$$
$$G_2^S(0)=(\mu_p+\mu_n)/2=eg_S/2M,$$
$$G_2^V(0)=(\mu_p-\mu_n)/2=eg_V/2M,$$
$$g_p=1.79, \quad g_n=-1.91,$$
$$g_S=-0.06, \quad g_V=1.85, \quad (2)$$

The functions $G(t)$ are related to the usual Hofstadter form factors $F(t)$ by the following definitions:

$$G_i^{p,n}(t)=G_i^{p,n}(0)F_i^{p,n}(t). \quad (3)$$

Dispersion theory allows one to write the different functions $G(t)$ in the following form[4]:

$$G(t)=\frac{1}{\pi}\int_0^\infty \frac{g(t')}{t'-t}dt'. \quad (4)$$