tion agrees with experiment. (We assume here that the atomic state existing after K capture from Eu^{152} is that of Sm^{152}. Evidence for this assumption will appear in a study of relaxation times and internal fields in rare earth iron garnets which is in preparation for publication.)

Gd^{154}: Both the parent $Eu^{154}(^7F_0)$ and daughter $Gd^{154}(^8S_{7/2})$ have ground-state configurations that to first order produce vanishing internal magnetic fields. This is in agreement with the fact that the rotations in this case correspond to average fields of 50 kilogauss, while it is of the order of megagauss in Sm and Dy at the same (room) temperature. If one tries to account for this small residual rotation by taking into account higher multiplets, one finds it difficult to explain the observed sign of the rotation. In particular, if one assumes that the low-lying $J=1$ state in Eu (which contributes to the magnetic moment of the Eu ion) is responsible for the hyperfine interaction, a negative rotation is predicted, in disagreement with experiment.

It is clear that in Sm^{152} and Dy^{160}, unlike the case in Fe, the contributions to the internal magnetic field arising from s electrons exchange-coupled to the f-shell electrons are not sufficiently large to change the sign of the observed internal field. In the case of Eu or Gd, however, the main source of field may be the exchange-coupled contribution. If the exchange-coupled contribution does not vary appreciably among the various rare earths, this interpretation is consistent with the results obtained in Sm and Dy. With the further assumption that the exchange-coupled interaction is of the form $A'\vec{I}\cdot\vec{S}$, the sign of A' for Eu or Gd is determined to be negative.

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MAGNETIC FIELD DEPENDENCE OF THE SUPERCONDUCTING ENERGY GAP

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Gor'kov[1] has recently derived the Ginzburg-Landau[2] (GL) phenomenological equations of superconductivity from the microscopic theory in the region where $T_c - T \ll T_c$ and in the London or local limit where the penetration depth $\lambda(T)$ is much greater than the coherence length ξ. His main point was that the effective charge, e^*, in the GL theory was equal to twice the electronic charge; but perhaps of even greater significance is the fact that the order parameter ψ of the GL theory is proportional to the energy gap. Bardeen, Cooper, and Schrieffer[3] (BCS) had previously suggested that the energy gap could be used as an order parameter for nonlinear extensions of their theory.

The success of the nonlinear GL theory has been its ability to explain the behavior of superconductors in strong magnetic fields better than any of the linear theories such as that of London and London, to which it reduces for weak fields. Inasmuch as the BCS theory is capable of handling the application of a magnetic field only as a perturbation, one would expect the solution of their microscopic equations for strong fields to be quite difficult. A weak-field calculation of the field dependence of the energy gap has been done by Gupta and Mathur.[4] On the other hand, the GL equations are relatively easy to solve for strong fields, and in the light of Gor'kov's result the solutions would be expected to have the same rigor as those of the microscopic theory. The restriction of locality, $\lambda \gg \xi$, is perhaps not a serious one, for there is evidence[5,6] that when this condition is not satisfied one may incorporate the nonlocal effects into the local theory by letting the penetration depth be a function of ξ and the mean free path. If one is considering thin films which are prepared by evaporation, then one would expect the coherence distance to be limited by the size of the crystallites. The locality condition would then be $\lambda \gg d$. Thus any superconductor prepared in the form of a very thin evaporated film will satisfy this condition. In regard to the restriction $T_C - T \ll T_C$, Ginzburg[7] has suggested that the order parameter is small even far below T_C. Thus the theory should also be valid at all temperatures below T_C. Bardeen[8] and Ginzburg[7] have considered forms for the full temperature dependence of the equations. However, the question of removing these restrictions is still a moot point, and the application of the equations derived here is only rigorous in the local limit and near T_C.

In the GL theory, ψ is a function of temperature, magnetic field, and coordinates; therefore, so is the energy gap. The question of finding the field dependence of the energy gap reduces to solving the GL equations for ψ.

Consider a plate of thickness d with an external field H_0 applied parallel to the surface. The GL equations have already been solved for this case[9,10] mainly in connection with calculations of the critical field. If $d/\lambda(T) \ll 1/\kappa \approx 10$, where κ is the nonlinear coupling constant of the theory, ψ is independent of coordinates and the two independent equations of condition found by them are

$$H_0^{\,2} = \frac{4\phi_0^{\,2}(\phi_0^{\,2}-1)\cosh^2[\phi_0 d/2\lambda(T)]}{1-[\lambda(T)/\phi_0 d]\sinh[\phi_0 d/\lambda(T)]} H_{cb}^{\,2}, \quad (1)$$

$$H_c^{\,2} = \frac{\phi_c^{\,2}(2-\phi_c^{\,2})}{1-[2\lambda(T)/\phi_c d]\tanh[\phi_c d/2\lambda(T)]} H_{cb}^{\,2}, \quad (2)$$

where $\phi = \psi(T,H)/\psi(T,0)$, $\lambda(T)$ is the temperature-dependent London penetration depth (or perhaps an effective penetration depth), $H_{cb}(T)$ is the bulk critical field, ϕ_0 is the equilibrium value of ϕ in the presence of H_0, and ϕ_C and H_C are the critical values. Gor'kov's result can be stated as

$$\epsilon(T,H)/\epsilon(T,0) = \psi(T,H)/\psi(T,0) = \phi, \quad (3)$$

where $\epsilon(T,H)$ is the energy gap. Equation (1) describes a smooth decrease in the energy gap as the field is increased. As $H_0 \to H_c$ the gap approaches a critical value. For this critical gap, the superconductor is in equilibrium with the normal state. Setting (1) equal to (2) gives ϕ_C, from which one can obtain H_C.

For $d/\lambda(T) \ll 1$, the solutions of Eqs. (1) and (2) are

$$\epsilon(T,H_c)/\epsilon(T,0) = 0, \quad (4)$$

$$H_c^{\,2} = 24[\lambda(T)/d]^2 H_{cb}^{\,2}(T), \quad (5)$$

$$\left[\frac{\epsilon(T,H_0)}{\epsilon(T,0)}\right]^2 = 1 - \frac{1}{24}\left(\frac{d}{\lambda(T)}\right)^2\left(\frac{H_0}{H_{cb}(T)}\right)^2, \quad (6)$$

or, using (5),

$$\left[\frac{\epsilon(T,H_0)}{\epsilon(T,0)}\right]^2 = 1 - \left(\frac{H_0}{H_c(T)}\right)^2. \quad (7)$$

Thus for very thin films the critical gap is zero, and the field dependence of the gap is independent of thickness when the external field is normalized to the critical field of the film. A closer examination of Eqs. (1) and (2) shows that the critical gap remains zero all the way up to $d/\lambda(T) = \sqrt{5}$. Therefore, the superconducting phase transition should be second order for films thinner than this value. This result was previously given by GL.

For small H_0, Eqs. (1) and (2) can be solved to give

$$\frac{\epsilon(T,H_0)}{\epsilon(T,0)} = 1 - \frac{1}{8}\left[\frac{\lambda(T)/d]\sinh[d/\lambda(T)] - 1}{\cosh^2[d/2\lambda(T)]}\right]\left(\frac{H_0}{H_{cb}(T)}\right)^2. \quad (8)$$

This expression is good for all d.[11]

The value of the critical gap has been calculated as a function of $d/\lambda(T)$ and is plotted in Fig. 1.

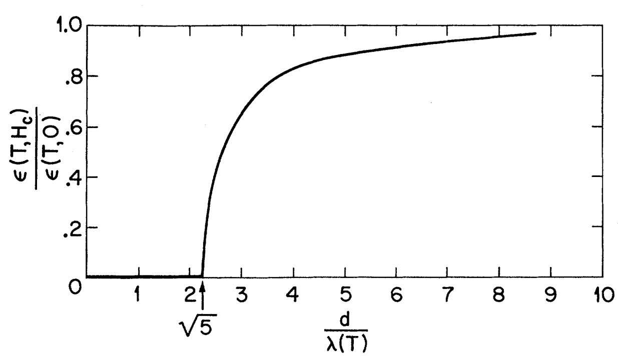

FIG. 1. Energy gap at the critical field vs thickness.

Ginsberg and Tinkham[12] and Richards and Tinkham[13] have looked for a change in the energy gap with field in thin films and in bulk metal, respectively, using infrared techniques. No change in the gap was observed on a 12A film of lead in a field of 8000 gauss. Application of Eq. (6) shows that the change in the gap under these conditions is less than 1%, a change too small for them to measure. Also, no change in the energy gap for bulk superconductors was observable. From Fig. 1 it is seen that the maximum change in the gap approaches zero as the thickness approaches that of bulk metal. Although the expressions derived here are not valid for $d/\lambda(T) > 10$, GL have shown that for bulk superconductors the maximum change in the order parameter (energy gap) with field is of the order of a few percent.

Giaever and Megerle[14] have recently made direct measurements of the energy gap of aluminum as a function of magnetic field. Their results are shown in Fig. 2. Calculating the penetration depth as suggested by Tinkham,[5] one obtains for an Al film of thickness 1600 A at $T = 1.05°$K a value of 2460 A for λ. This makes $d/\lambda \approx 0.65$. Thus from Fig. 1 the transition should be second order and the energy gap should go smoothly to zero. The data indicate that this is true. The field dependence of the energy gap as predicted by Eq. (7) is also shown in Fig. 2, and it is seen that the experimental points all lie below the theoretical curve. Perhaps the reason for the disagreement is that the theoretical model of an ideal homogenous film is too simple and one must consider that the film is really made of small crystallites in order to bring the theorv into closer agreement with experiment.

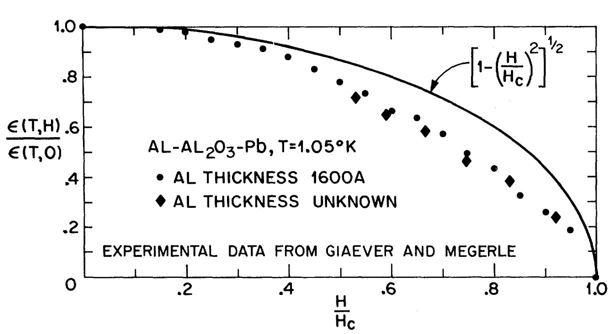

FIG. 2. Energy gap of aluminum vs magnetic field.

The results of the Gupta-Mathur calculation show that for thin samples the gap is always finite in the superconducting state and that the maximum change in the gap becomes smaller as the thickness becomes smaller, which disagrees with the results of this paper. The reason for this divergence is uncertain. Their calculation was done for the Pippard limit, $\xi \gg \lambda$, whereas ours was done for the London limit, $\xi \ll \lambda$; but it is difficult to envision that these different limiting cases would lead to a different thickness dependence. Also it was noticed that in the application of their results, Gupta and Mathur apparently used the bulk critical field instead of allowing the critical field to be a function of thickness; but even with this correction the two calculations are still in qualitative disagreement.

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