ANALYSIS OF THE ANOMALY IN DOUBLE MESON PRODUCTION IN $p+d$ COLLISIONS AND THE S-WAVE PION-PION INTERACTION*

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The purpose of this note is to show how the anomaly in double pion production in $p+d$ collisions observed by Abashian, Booth, and Crowe[1] may be explained by a nonresonant final-state interaction of the produced pions in the isospin state $T=0$, instead of assuming the existence of a neutral ω^0 particle.

The separation of the production process into two separate mechanisms, that of the primary interaction in which the particles are produced, and the final-state interaction in which the produced particles interact with each other, is well known.[2] It is assumed in this note that the final-state interaction is responsible for any deviation from simple phase-space predictions. In the following calculation we neglect the effects of the deuteron and He^3 wave functions and the pion-nucleon final-state interaction, since they tend to smear out the spectrum and therefore are unlikely to produce a sharp peak. We limit ourselves to an S-wave ($T=0$) π-π interaction[3]; the next angular momentum D state, which is permitted by π-π isospin $T=0$, is very unlikely to contribute.[4] A straightforward analysis based on charge independence yields

$$\sigma(p+d\to He^3+\pi^++\pi^-)+\sigma(p+d\to He^3+\pi^0+\pi^0)=\tfrac{1}{2}|f_1|^2+|f_0|^2, \quad (1)$$

$$\sigma(p+d\to H^3+\pi^++\pi^0)=|f_1|^2, \quad (2)$$

where f_T is the production amplitude in a state in which the two pion charge states couple together to make an isospin T. Experimentally (2) is much smaller than (1), which is what one expects for the production of the two pions in the nonresonant P state at this energy.[4] In the following analysis we neglect the contribution of the f_1 production amplitude to (1). The cross section is proportional to

$$\int \frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\frac{d^3p_3}{2E_3}\delta(\vec{p}_1+\vec{p}_2+\vec{p}_3-\vec{p}_T)\times\delta(E_1+E_2+E_3-E_T)|T_{fi}(q/2)|^2. \quad (3)$$

The subscripts 1, 2, and 3 refer, respectively, to the two pions and the He^3; $q/2$ is the momentum of each pion in their own center-of-mass system. We have put the primary interaction matrix to be constant, and $T_{fi}(q/2)$ is the part which is due to the final-state interaction. After carrying out the integration in (3), we have

$$d^2\sigma/dp_3d\Omega_3=c(p_3^2/\omega_3)(q/\omega_q)|T_{fi}|^2, \quad (4)$$

where $\omega_q^2=q^2+4m_\pi^2$ and the He^3 dynamical quantities are evaluated in the laboratory system; $(p_3^2/\omega_3)(q/\omega_q)$ is the phase-space volume element. To obtain $|T_{fi}|^2$ empirically, we divide the measured spectrum by the normalized phase space. Using the data given in reference 1, the plot of $|T_{fi}|^2$ is shown in Fig. 1. For a very small radius of the primary interaction, it is the square of the ratio of the pion-pion wave function with and without π-π interaction taken at the origin, and is nothing but the usual enhancement factor.[5] We use an exponential potential well of range $d=\frac{1}{2}\hbar/\mu c$ to calculate this enhancement factor; we also use an asymptotic π-π wave function with exponential cutoff for the range of the primary interaction to obtain the energy dependence of the matrix element, and

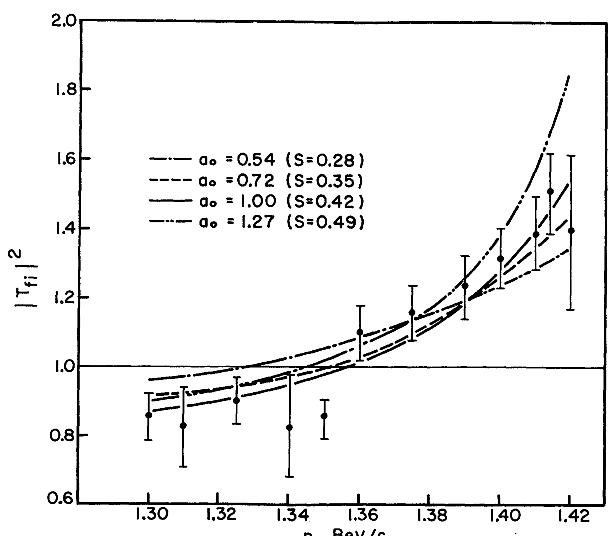

FIG. 1. The empirical probability density function $|T_{fi}|^2$ as a function of the He^3 momentum. Experimental data are taken from reference 1 at incident proton energy of 743 Mev. Fits to the data by the theory of pion-pion final-state interactions are shown.

find that to fit the data one needs a larger scattering length. A reasonable fit to the data is obtained with the well depth parameter $s=0.3$ to 0.5 corresponding to a scattering length $a_0 \approx \frac{1}{2}\hbar/\mu c$ to $\frac{3}{2}\hbar/\mu c$ (attractive), a_0 being the scattering length for $T=0$. Since we take into account only the effect of π-π interaction, the result should only be regarded as evidence for the $T=0$ π-π interaction being attractive and for its scattering length being not too large. It is interesting to note that this conclusion is in agreement with that obtained by Ishida et al.[6] and Efremov et al.[7] in their work on the δ_{31} and δ_{13} phase shifts in pion-nucleon scattering.

Mitra found that it is possible to fit the τ-decay data spectrum with either or both $T=0$ and $T=2$ resonances, and with the position of the $T=0$ resonance at $\omega_{\pi\pi}^2 \approx 12m_\pi^2$ which is quite far away from the energy region we are considering.[8] A resonance at $\omega_{\pi\pi}^2 = 5m_\pi^2$ would not fit the τ-decay data. A low-energy $T=0$ π-π resonance would also imply a too large inelastic cross section for the process $\pi + N \rightarrow 2\pi + N$ near the pion-production threshold.[9] The analysis of the τ decay by Lomon, Morris, Irwin, and Truong[5] shows that the π-π interaction is attractive for the state $T=2$ and probably repulsive for the state $T=0$; with both $T=2$ and $T=0$ attractive, and $T=2$ more attractive than $T=0$, a fit to the data was also obtained although it was not as good.

It is worthwhile noticing that the previous remark is not in disagreement with the results obtained by the dispersion relation methods of Khuri and Treiman.[10] Combining the present analysis of the anomaly in the $p+d$ experiment and the τ-decay data, it may be possible to infer that the S-wave pion-pion interaction in the states $T=2$ and $T=0$ are both attractive with $T=2$ more attractive.

We would like to mention that our analysis does not exclude the possibility of the existence of the ω^0 particle. Accurate experimental data at different incident proton energies and He^3 angles, could be used to tell whether the observed peak is due to the final-state interaction of the two pions or to the existence of the ω^0 particle.

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[4]From the usual centrifugal barrier argument, one can see that the production of two pions in P state (relative to each other) should be much less than in S state, i.e., the H^3 cross section should be much less than the He^3 cross section at this energy. The probability of the two pions being emitted in the relative D state should be negligible.

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[10]N. N. Khuri and S. B. Treiman, Phys. Rev. 119, 1115 (1960). To fit the τ-decay data, Khuri and Treiman obtained the relation $a_2 - a_0 = 0.7\hbar/\mu c$ (with the convention $a > 0$ for an attractive interaction). This relation could be satisfied by taking both $T=2$ and $T=0$ repulsive with $T=0$ more repulsive, $a_0 < a_2 < 0$; or $T=2$ weakly attractive and $T=0$ repulsive, $a_2 > 0$, $a_0 < 0$; or $T=2$ more attractive than $T=0$, $0 < a_0 < a_2$. If it was required that $a_0 > 0$, then only the last solution holds. These solutions could be understood qualitatively from the enhancement factor argument. A similar calculation to that in reference 5 is being carried out under less severe restriction on the curvature.