Dr. M. R. Yearian.

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*This material was presented as a post-deadline paper at the New York Meeting of the American Physical Society, February 1-4, 1961. This work was supported in part by the Office of Naval Research and the U. S. Atomic Energy Commission and by the U. S. Air Force, through the Office of Scientific Research of the Air Research and Development Command.

†On leave from Institute of Nuclear Research, Amsterdam, The Netherlands.

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ELECTRIC AND MAGNETIC STRUCTURE OF THE PROTON AND NEUTRON*

Robert Hofstadter
Stanford University, Stanford, California

and

Robert Herman
Research Laboratories, General Motors Corporation, Warren, Michigan

(Received February 15, 1961)

We attempt to present in this paper a unified interpretation of the presently known experimental data on the electromagnetic form factors of two fundamental particles: the proton and the neutron. As we shall show, this interpretation is fully consistent with the idea that the two particles are two different aspects of a single entity—the nucleon. The third component of the isotopic spin of the nucleon is then used to distinguish between the two fundamental particles. The new experimental material on the neutron form factors,[1] which now completes a block of information on the proton[2] and neutron, has served as the stimulus for the attempted explanation.

We would like to explain the main features of the experimental behavior of the Dirac form factors (F_{1p}, F_{1n}) and Pauli form factors (F_{2p}, F_{2n}) of the proton (p) and neutron (n) as functions of the momentum-transfer invariant (q^2). We propose to do this in a well-known way[3] by expressing each proton and neutron form factor as a sum of a scalar and vector contribution. This decomposition is rooted in the idea that the scalar and vector form factors are simpler and more basic than those of either the proton or neutron. Accordingly we make the following definitions, which are standard except perhaps for the normalizing constants:

$$F_{1S}=F_{1p}+F_{1n}, \qquad (1)$$

$$F_{1V}=F_{1p}-F_{1n}, \qquad (2)$$

$$F_{2S}=[1.79F_{2p}+(-1.91)F_{2n}]/(-0.12), \qquad (3)$$

$$F_{2V}=[1.79F_{2p}-(-1.91)F_{2n}]/(3.70). \qquad (4)$$

This choice of normalization has the advantage that at $q=0$ all four isotopic form factors take on the value of unity.

We shall now attempt to find the four isotopic form factors from the experimental information given in Hofstadter et al.[1,2] for the values of

F_{1p}, F_{1n}, F_{2p}, F_{2n}. A comprehensive, though approximate, fit to all the experimental data can be represented by the following expressions for the fundamental isotopic form factors:

$$F_{1S} = 0.44 + \frac{0.56}{1+0.214q^2}, \quad (5)$$

$$F_{1V} = -0.20 + \frac{1.20}{1+0.10q^2}, \quad (6)$$

$$F_{2V} = -0.20 + \frac{1.20}{1+0.10q^2}, \quad (7)$$

$$F_{2S} = 4.0 + \frac{-3.0}{1+0.214q^2}. \quad (8)$$

These results have very few independent fitting parameters. The independent parameters of Eqs. (5), (6), and (7) are only 0.44, -0.20, and a root-mean-square radius, $a = 0.85\times10^{-13}$ cm. All other numerical values in these equations are determined, once the above choice is made. The rms radius is obtained from the coefficient $(-\frac{1}{6}a^2)$ of q^2 in the expansion of F_{1S}, F_{1V}, and F_{2V} in powers of q^2. The quantity F_{2S} in Eq. (8) requires the additional fitting parameter 4.0. F_{2S} is the least well-known quantity of the set of isotopic form factors, and we regard both its values and its form as somewhat indeterminate at the present time.

Equations (5) to (8) are remarkably simple and have the same fundamental structure, namely, the Clementel-Villi (C-V) form.[4-6] It is very satisfactory that this simple C-V form is also suggested by the dispersion relations idea[7] that the approximate nucleon form factor is just the result of a pole plus a constant representing the core of a nucleon. The Fourier transform of the C-V form factor is a delta function at the center of the distribution ($r=0$) plus a Yukawa cloud. Thus the spatial interpretation of Eqs. (5) to (8) is very clear: Each form factor corresponds to a distribution in space of a simple Yukawa cloud and a point-like core. (Our present experiments are not capable of distinguishing between a point core and a core of radius comparable to a nucleon Compton wavelength.) Though we are aware that the spatial transform is not a completely consistent relativistic concept, we believe that the density distributions so obtained are approximately correct and correspond, at the same time, to dispersion theory models.[3]

We may solve Eqs. (1)-(4) for F_{1p}, F_{1n}, F_{2p}, and F_{2n} and substitute the values of F_{1S}, F_{1V}, F_{2V}, and F_{2S} given in Eqs. (5)-(8). Thus we obtain

$$F_{1p} = 0.12 + \frac{0.28}{1+0.214q^2} + \frac{0.60}{1+0.10q^2}, \quad (9)$$

$$F_{1n} = 0.32 + \frac{0.28}{1+0.214q^2} - \frac{0.60}{1+0.10q^2}, \quad (10)$$

$$F_{2p} = -0.34 + \frac{0.10}{1+0.214q^2} + \frac{1.24}{1+0.10q^2}, \quad (11)$$

$$F_{2n} = -0.068 - \frac{0.094}{1+0.214q^2} + \frac{1.16}{1+0.10q^2}. \quad (12)$$

Graphical representations of these equations are given by the solid lines in Fig. 1. Experimental points of references 1 and 2 are also

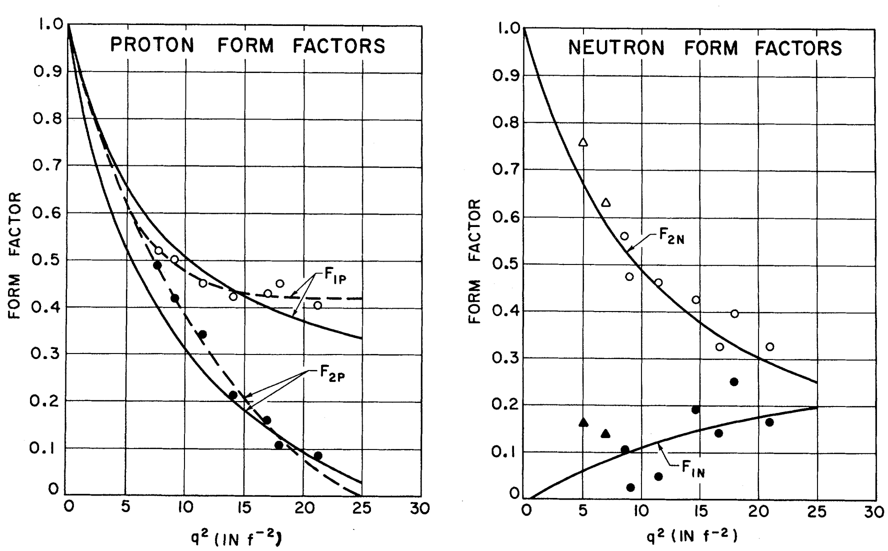

FIG. 1. The experimental points of references 1 and 2 together with theoretical curves (solid lines) representing Eqs. (9)-(12). Below $q^2 \cong 7$ no points are shown but the theoretical curves are in good agreement with previously published cross sections.[5] The dashed lines refer to the empirical curves given in reference 2. An improved fitting of the data at $q^2 \geq 7$ by the method of least squares is now in progress and can be carried through with slight adjustments of the constants in Eqs. (5)-(8). The solid and hollow triangles refer to the neutron data of Sobottka.[8]

given in the figures. Even though a best adjustment of the free parameters has not yet been made, the fit between experiment and theory is already satisfactory over the entire range of values of q^2 and embraces all the measurable quantities F_{1p}, F_{1n}, F_{2p}, and F_{2n}. The largest departures from the curves correspond to the points of Sobottka[8] which we have analyzed, but which were measured at an earlier time when our spectrometer was not stabilized by magnetic flux-coil methods.

The coefficients of q^2 in the expressions (9)-(12) give immediately the rms radii of the Dirac and Pauli charge and magnetic moment distributions in the proton and neutron. We find that

$$a_{1p} = 0.85 \text{ f}; \quad a_{1n} = 0.00 \text{ f};$$
$$a_{2p} = 0.94 \text{ f}; \quad a_{2n} = 0.76 \text{ f}. \tag{13}$$

These radii are consistent with known facts about these distributions. We note the important point that the root-mean-square radius of the neutron is zero, in agreement with the measurements of Fermi, Rabi, Hughes, Havens, and their collaborators on the neutron-electron interaction.[9] The rms magnetic radius of the neutron is nearly the same as that of the proton. One of the conditions employed in finding the parameters of Eqs. (5)-(8) was that $a_{1n} = 0$. Thus the long-standing problem of a small or zero neutron charge radius and a normal magnetic radius seems to be resolved.

The splitting of F_{1p} and F_{2p} at small values of q^2 is perfectly in accord with known data on the proton cross sections, as may easily be verified by substituting such form factors into the Rosenbluth formula.

The choice of positive sign for values of F_{1n} was required for the above set of isotopic form factors. If negative values of F_{1n} are taken for the intersections of reference 1, a different set of isotopic form factors is obtained which seems difficult to understand in any simple way.[7,10] It is possible, in principle, to find the sign of F_{1n} relative to F_{1p} by making elastic scattering measurements in the deuteron. The present experimental evidence[11,12] is not definitive on the question. If the relativistic correction of Blankenbecler[13] is employed together with the results of references 11 or 12, the choice of F_{1n} should be positive, as we have suggested. But the experimental errors do not permit a definite decision on this point.

We now find the Fourier spatial transforms of

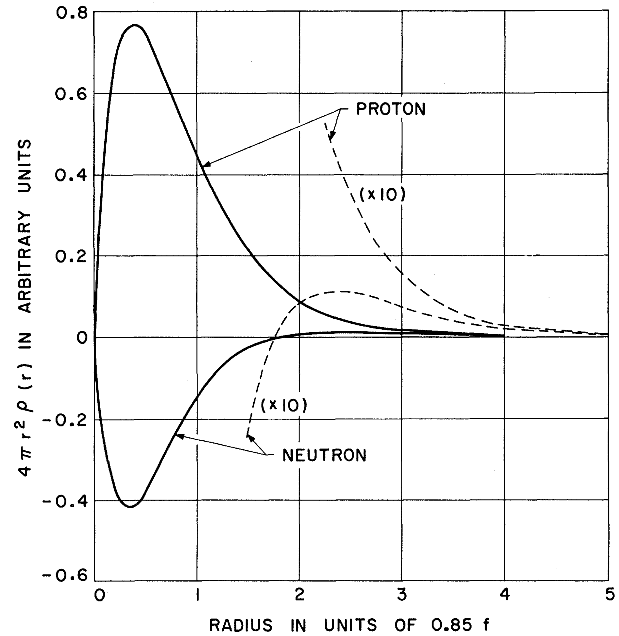

FIG. 2. The proton and neutron charge density distributions given by the Fourier transforms of Eqs. (9) and (10). The expressions for the spatial transforms are given in reference 6. The delta functions at the origin are omitted in this figure.

Eqs. (9) and (10) and present the results graphically in Fig. 2. It may be seen from Eqs. (9) and (10) that the neutron charge distribution is obtained from that of the proton essentially by flipping over one of the two Yukawa clouds. Thus the neutron and proton charge clouds are in a partial sense mirror images of each other. The fact that the cores are different (0.12 for the proton, 0.32 for the neutron) is probably a consequence of the inexact nature of our approximation. It seems quite likely that the higher order terms in Eqs. (9) and (10), which are omitted in our analysis, might account for the actual differences between F_{1V} and F_{2V} which we ignored in our approximate choice of isotopic form factors. Such higher order terms may well restore full symmetry between neutron and proton.

The magnetic moments of proton and neutron are found from the combinations $(F_{1p} + 1.79F_{2p})$ and $(F_{1n} - 1.91F_{2n})$, and have the approximate mirror symmetry expected of them. The details of the magnetic moment clouds will be presented in a subsequent communication.

We call attention particularly to the prediction that the neutron charge cloud has a positive outer fringe.[14] The positive sign of F_{1n} is connected with the positive outer cloud. It would be interesting to seek other experimental evidence on the sign of the outer cloud.

We also note the fact that the ranges of the component Yukawa charge clouds in the proton (or in the neutron) are different. The vector cloud has a range of approximately 0.32 fermi and the scalar cloud a range of approximately 0.47 fermi. Thus, this evidence indicates that the three-pion resonance has a lower energy than the two-pion resonance.[7,15] An improved adjustment of the constants in Eqs. (5)-(8) might change this quantitative relationship between the two resonances.

If the above considerations prove to be true, the scheme of construction of proton and neutron is simpler than might have been expected. Furthermore, the internal consistency of the results suggests that the techniques of quantum electrodynamics are still valid at distances whose values lie between a nucleon Compton wavelength and a pion Compton wavelength.

We would like to offer our special thanks to Professor S. Fubini of the University of Padua for his important suggestion to choose positive F_{1n} values and for his illuminating comments on the dispersion-theoretic aspects of the C-V form factors. We also appreciate his gracious encouragement during the course of analysis of the form factors. We wish to thank Professor L. I. Schiff also for similar suggestions and for his warm and constant encouragement during the course of this work. We are grateful to Professor G. Breit for his kind critical comments. Finally we wish to express our appreciation to Mrs. Penny Bakey Seligman and Dr. Denos Gazis for their generous assistance with some of the calculations.

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*This material was presented as a post-deadline paper at the New York Meeting of the American Physical Society, February 1-4, 1961. This work was supported in part by the Office of Naval Research and the U. S. Atomic Energy Commission and by the U. S. Air Force, through the Office of Scientific Research of the Air Research and Development Command.

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