of the deuteron as the z axis, we see that (3) completely forbids the process to proceed from the $m=0$ state for all scattering angles. If this reaction is used to analyze the deuterons coming from some primary reaction, it can serve as a sensitive detector of the angular dependence of $\langle T_{20}\rangle \sim \langle 3S_z^2-2\rangle$.

Although reactions of the above type to low-lying levels of light nuclei are forbidden by isotopic spin conservation, one can break the T selection rule by choosing sufficiently heavy nuclei. An example[9] is the exothermic reaction $Si^{28}(d,\alpha)Al^{26*}$ to the first excited state of Al^{26}.

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$N\bar{N} \longrightarrow \pi\pi$ AMPLITUDE AND THE ELECTROMAGNETIC STRUCTURE OF THE NUCLEON*

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Recently, the effect of a pion-pion P-wave resonance on the $N\bar{N}$ to $\pi\pi$ amplitude and the electromagnetic structure of the nucleon has been investigated by Frazer and Fulco.[1] Their calculation suffers from two deficiencies: (1) The "rescattering cut" they employed produces divergent integrals which necessitate a cutoff. (2) Their "one-pole" effective-range formula corresponds to an extremely short-range pion-pion force and is inconsistent with the crossing symmetry of the pion-pion problem.[2]

The purpose of this Letter is to remedy both of these difficulties. (1) By crossing symmetry, the $N\bar{N}$ to $\pi\pi$ amplitude at zero total energy is identical to the forward pion-nucleon scattering amplitude, which can be calculated directly from experimental information. Having thus determined the $N\bar{N}$ to $\pi\pi$ amplitude at one point, one can calculate both this amplitude and the nucleon form factors more reliably. In fact, a one-subtraction dispersion relation can be formulated which removes the divergence difficulty. (2) Chew and Mandelstam have observed from the crossing relations for pion-pion scattering that a P-wave resonance produces a long-range repulsive force in addition to the shorter range attraction. We represent such an interaction by a "two-pole" P-wave effective-range formula.

To obtain the normalization of the $N\bar{N}\to\pi\pi$ amplitude, we start with the pion-nucleon fixed momentum-transfer dispersion relations in the neighborhood of zero momentum transfer. The projection of the $J=1$, $I=1$ $N\bar{N}\to\pi\pi$ characteristic amplitudes from the dispersion relations yields

$$f_+^{\,1}(t)=\frac{1}{p_-q_-}\left\{4f^2m^3z_0Q_1(z_0)+\frac{1}{4\pi^2}\int_{(m+1)^2}^{\infty}ds\left[\frac{p_-}{q_-}\,\mathrm{Im}A^{(-)}(s,t)+mz\,\mathrm{Im}B^{(-)}(s,t)\right]Q_1(z)\right\}, \tag{1}$$

$$f_-^{\,1}(t)=\frac{\sqrt{2}}{3p_-q_-}\left\{4f^2m^2\left[Q_0(z_0)-Q_2(z_0)\right]+\frac{1}{4\pi^2}\int_{(m+1)^2}^{\infty}ds[\mathrm{Im}B^{(-)}(s,t)][Q_0(z)-Q_2(z)]\right\}, \tag{2}$$

where the notation is that of FF and the Q's are Legendre functions of the second kind.

For sufficiently small t, the functions $A(s,t)$ and $B(s,t)$ can be expressed in terms of pion-nucleon phase-shifts. To suppress the high-energy portion of the integrals in Eqs. (1) and (2), a subtraction can be introduced into the original fixed-t dispersion relations. In the present calculation, we have evaluated the value and the derivative of both f_+ and f_- at $t=0$ in the no-subtraction and the one-subtraction forms. The subtraction was made at the pion-nucleon threshold where the subtraction constants are related to scattering lengths. The results are given in Tables I and II. For convenience these results are given for FF's Γ_1 and Γ_2 which are directly related to the electric and magnetic form factors.

In the evaluation of the integrals, we have expressed ImA (ImB) in terms of combinations of total cross sections and the (3,3) amplitude in such a way that the $J=3/2$ states are taken into account exactly. We believe that such a combination is more accurate than the (3, 3) amplitude alone since the second resonance is probably in the $J=3/2$ D state. The scattering lengths used are taken from the analysis of Barnes et al.[3] and Hamilton and Woolcock.[4] It is clear that $\Gamma_1(0)$ is the most accurately determined normalization. The uncertainty in the one-subtraction values of $\Gamma_2(0)$ and $\Gamma_1'(0)$ comes mainly from the inaccuracy of the small P-wave scattering lengths. If the small P-waves are ignored, we find $\Gamma_2(0)=0.0033$ and $\Gamma_1'(0)=0.0348$. Since the D-wave scattering lengths are yet unknown, we can only estimate the order of magnitude of $\Gamma_2'(0)$ from the no-subtraction formula. It may be possible to improve the accuracy of the normalizations by varying the subtraction energy in the pion-nucleon dispersion relation over the region where pion-nucleon phase-shift analyses have been performed.

Table I. Normalizations with no subtraction.

	Born term	Dispersion integral	Total
$\Gamma_1(0)$	-0.159	0.0186	-0.140
$\Gamma_2(0)$	0.0238	-0.0187	0.0051
$\Gamma_1'(0)$	0.0364	0.00224	0.0386
$\Gamma_2'(0)$	-0.00562	0.00107	-0.00455

To obtain the $\Gamma_{1,2}(t)$, we use the dispersion formula given by FF. The left-hand cut of the Γ is calculated in the same manner as FF except that we terminate the rescattering cut at $t=-26\mu^2$ (where the π-N partial wave expansion diverges) and replace all remaining cuts by a pole whose position and residue are adjusted to give the normalized value and derivative at $t=0$. Of course, this phenomenological pole can also compensate for part of the inaccuracy in the rescattering cut. The D function which appears in the FF formula is now represented by

$$D(\nu)=1-\nu[A_1\nu_1K(\nu_1,-\nu)+A_2\nu_2K(\nu_2,-\nu)],$$
$$\nu=\tfrac{1}{4}(t-4\mu^2), \qquad (3)$$

where ν_1 and ν_2 are the positions of the poles in the pion-pion two-pole effective-range formula, and A_1 and A_2 are the corresponding residues. The $K(\nu',-\nu)$ appearing in Eq. (3) is the kernel defined by CM.[2]

Having obtained the Γ's expressed in terms of the pion-pion parameters, one can then calculate the isovector part of the nucleon form factors. A typical set of parameters which fits the observed value and radius of the magnetic moment form factor is $\nu_1=60$, $\nu_2=4$, $A_1=0.3$, $A_2=-0.32$ (in pion units), corresponding to a resonance at $t=12$. Although we have four pion-pion parameters, the value of ν_2 is closely related to the position of the resonance and therefore can be determined by consistency requirements ($\nu_2\simeq 2\nu_r+1$). For three different values of ν_1 ($\nu_1=25$, 40, 60), the variation of the position and width of the resonance was quite small. The two-pion contribution to the charge in these cases was 20% of the total charge. Although the

Table. II. Normalizations with one subtraction at the pion-nucleon threshold.

	Born term	Scattering lengths S	P	D	Dispersion integral	Total
$\Gamma_1(0)$	-0.1055	-0.0339			-0.0012	-0.141
$\Gamma_2(0)$	0.0238	-0.0002	-0.0319		0.0041	-0.0042
$\Gamma_1'(0)$	0.00535	0.00135	0.0382		-0.0019	0.0430
$\Gamma_2'(0)$	-0.00562	-0.0001	-0.0026	?	-0.0004	?

Γ_1 amplitude is still quite sensitive to the uncertainty in the normalizations due to a cancellation, we believe that our solution for it is qualitatively more reliable than the function given by FF.

While we feel that we have made improvements over the FF solution, we find that the value of A_2 required to fit the magnetic moment is too large to be consistent with the CM crossing relations. This indicates that either the pion-pion effective-range formula in its present form is inadequate, or the higher mass intermediate states are non-negligible for both the charge and magnetic form factors.

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ELECTROMAGNETIC MESON MASS DIFFERENCES

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The π-meson and K-meson electromagnetic mass differences have been qualitatively explained by Matthews and Uretsky, and others,[1,2] in terms of a simple classical model. In considering the quantum mechanical analog of their model, Matthews and Uretsky ignored the two-photon vertex term. This is a divergent term, independent of the meson structure, which is known to dominate the pion case in the conventional gauge, and actually to determine the sign of the mass difference. In disperson theory, the pion mass difference has been calculated by Riazuddin[3] who treats this term only very approximately.

It has been suggested by Feldman[4] that the difficulties associated with this term may be avoided by working in the F.E.M. (finite electromagnetic mass) gauge in which it vanishes identically. The purpose of this note is to show that when this is done the remaining term becomes finite in the "pole" approximation to the dispersion relation, and a rather satisfactory quantum theory of both the pion and the K-meson mass splitting is obtained, in terms of physically reasonable electromagnetic form factors.

We write the self-energy arising from the process of lowest order in e^2 shown graphically in Fig. 1(a) as

$$\delta(m^2)=\frac{i}{4\pi^3}\times\frac{1}{4\pi}\int D_{\mu\nu}(q^2)K(p,q)d^4q, \tag{1}$$

where

$$K(p,q)=i(2E)(2\pi)^3\times\tfrac{1}{2}\{\int d^4y\ \exp(-iqy)(\langle p|T(j_\mu(y)j_\nu^\dagger(0))|p\rangle-\langle 0|T(j_\mu(y)j_\nu^\dagger(0))|0\rangle)$$

$$+\int d^4y\ \delta(y_0)\ \exp(-iqy)\overleftrightarrow{\partial}_{y_0}(\langle p|[\dot{A}_\mu(y),j_\nu^\dagger(0)]|p\rangle-\langle 0|[\dot{A}_\mu(y),j_\nu^\dagger(0)]|0\rangle)\} \tag{2}$$

is the transition matrix element for the scattering of a (real) photon off a boson, and $D_{\mu\nu}(q^2)$ is the virtual photon propagator,

$$D_{\mu\nu}(q^2)=(1/q^2)(g_{\mu\nu}-aq_\mu q_\nu/q^2). \tag{3}$$

In Eq. (2) we ignore the vacuum expectation terms which contribute equally to the self-masses of charged and uncharged mesons. Taking

$$j_\mu=ie[\phi^\dagger\partial_\mu\phi-\partial_\mu\phi^\dagger\phi]-2e^2\phi^\dagger\phi A_\mu, \tag{4}$$