ELECTROMAGNETIC PROPERTIES OF THE PROTON AND NEUTRON*

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Measurements of electron-proton scattering made at Cornell[1] and Stanford[2] below 1000 Mev have shown that F_{1p} and F_{2p}, the electric and magnetic form factors of the proton, are not equal; indeed that F_{2p} is nearly zero for values of q, the momentum-energy transfer, of about 5 fermi^{-1}, while F_{1p} remains in the vicinity of 0.4—all this is indicative of a core of charge of about 0.4 e.

New measurements of scattering cross sections from hydrogen and deuterium at higher values of q appear to permit us to determine the extent and charge of this core of the nucleon and also to deduce similar properties of the surrounding mesonic cloud. The Cornell 1.3-Bev electron synchrotron has been used in extending our earlier measurements in exactly the manner described in Nature.[1] The new results are shown in Table I and Figs. 1 and 3. It is of striking physical significance that the cross section falls so low at 1.2 Bev, i.e., to 10^{-34} cm^2/sr.

Now one of the simplest models of the nucleon has a point core of charge $+\frac{1}{2}e$ surrounded by an extended mesonic cloud of $+\frac{1}{2}e$ for the proton or $-\frac{1}{2}e$ for the neutron. Indeed, some aspects of such a model are reflected in the experimental data, but the model departs significantly from the facts at a number of points. In the first place, from Foldy's interpretation of neutron scattering by atomic electrons[3] we know that the rms radius of the charge distribution of the neutron is zero and this implies that at $q \approx 0$, $F_{1n} \lesssim 0.002q^2$; the above picture implies that $F_{1n} = \frac{1}{6}(0.8)^2q^2$, q^2 being measured in fermi^{-2}. In the second place,

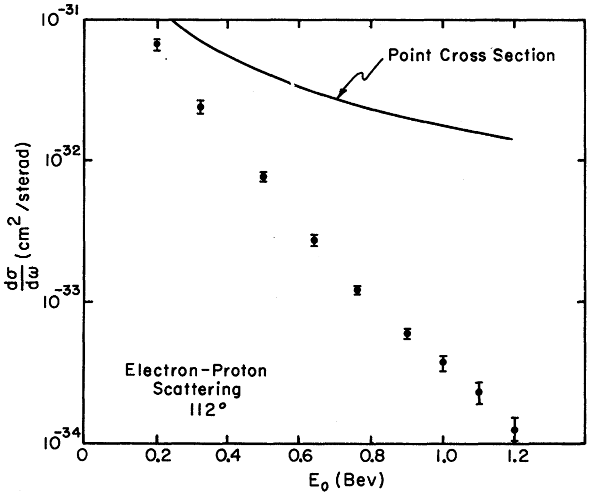

FIG. 1. Experimental results for electron-proton scattering at 112°.

Table I. Experimental results. Columns 2 and 3 give the experimentally determined cross sections and peak cross sections for electrons of incident energy E_0 scattered through an angle of 112°. Radiative corrections have been applied. In order to calculate the integrated deuteron cross sections, allowance must be made for electrons which do not scatter into the detection channel because of internal motion of the nucleons in the deuteron. Calculating the energy spread of scattered electrons using the impulse approximation and including the resolution of the apparatus as determined from electron-proton scattering, we obtain the values for $\Delta E/E$ listed in column 4. Then, subtracting the proton cross section from column 2, we arrive at the integrated cross section for electron-neutron scattering given in the last column.

E_0 (Mev)	$(d\sigma/d\omega)_p$ (10^{-32} cm^2/sr)	$(d^2\sigma/dE\,d\omega)_d$ (10^{-33} cm^2/Mev sr)	$\Delta E/E$	$(d\sigma/d\omega)_d$ (10^{-32} cm^2/sr)	$(d\sigma/d\omega)_n$ (10^{-32} cm^2/sr)
200	6.6 ±0.6	2.30 ±0.33	0.154	5.35 ±0.77	-1.2 ±1.2
325	2.40 ±0.25	0.85 ±0.076	0.133	2.49 ±0.23	0.09 ±0.34
502	0.76 ±0.06	0.250 ±0.016	0.112	0.78 ±0.11	0.15 ±0.12
640	0.270 ±0.023				
761	0.119 ±0.008	0.034 ±0.008	0.093	0.114 ±0.027	0.005 ±0.030
900	0.059 ±0.006				
1000	0.037 ±0.005	0.0158 ±0.0028	0.080	0.051 ±0.009	0.014 ±0.010
1100	0.023 ±0.004				
1200	0.0123 ±0.0025	0.0059 ±0.0028	0.074	0.019 ±0.009	0.007 ±0.009

the form factor F_{1p} should approach 0.5 for large values of q^2; in fact, it seems to be falling below 0.4. Nevertheless, we have interpreted our experiment in the spirit of this very simple model; to fit the facts we have found it necessary to assign a radius of about 0.2 f to the core and to put a small part of the positive charge of the core in an extensive cloud that is the same for neutron and proton.

If we assume that Rosenbluth's formula is still valid at our energies, i.e.,

$$\frac{d\sigma}{d\omega}=\frac{e^4}{4E^2}\,\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\left[1+\left(\frac{2E}{M}\right)\sin^2(\theta/2)\right]^{-1}$$

$$\times\left\{F_1{}^2+\frac{q^2}{4M}\left[2(F_1+kF_2)^2\tan(\theta/2)+k^2F_2{}^2\right]\right\}, \quad (1)$$

where E is the incident electron energy, θ the laboratory angle, M the nucleon mass, and k the anomalous magnetic moment, then we want to find the form factors F_1 and F_2 which are functions of q^2, and for this at least two measurements of $d\sigma/d\omega$ at the same q value but at different angles are necessary. The data have been resolved in this manner for values of $q^2<25$ f^{-2} where cross sections have been measured at 112° and 66°, and the values of F_1 and F_2 that have been obtained are plotted in Fig. 2. At values of $q^2>25$ we have only the data at 112°; at smaller angles the large values of q^2 cannot be reached with our available energy nor can we easily go to larger angles for then the counting rate becomes prohibitively small. We are preparing new experimental equipment, however, to do just this.

On the other hand, by measuring electron-deuteron scattering, we have been able to infer values of F_1 and F_2 for both the proton and neutron as follows: Equation (1) can be rewritten in the form

$$d\sigma/d\omega=(a_{11}F_1{}^2+a_{12}F_1F_2+a_{22}F_2{}^2)\sigma_{ns}, \quad (2)$$

so that we may use the values of the constants and σ_{ns}, the point "no-spin" electric cross section, which have been conveniently tabulated.[4] Thus an ellipse is obtained for a particular value of the cross section when F_2 is plotted against F_1. Such an ellipse is shown in Fig. 2 for the highest energy measured ($q^2=37$). Depending on the value of F_2, F_1 can take on values between plus and minus 0.3. Thus it may be that our

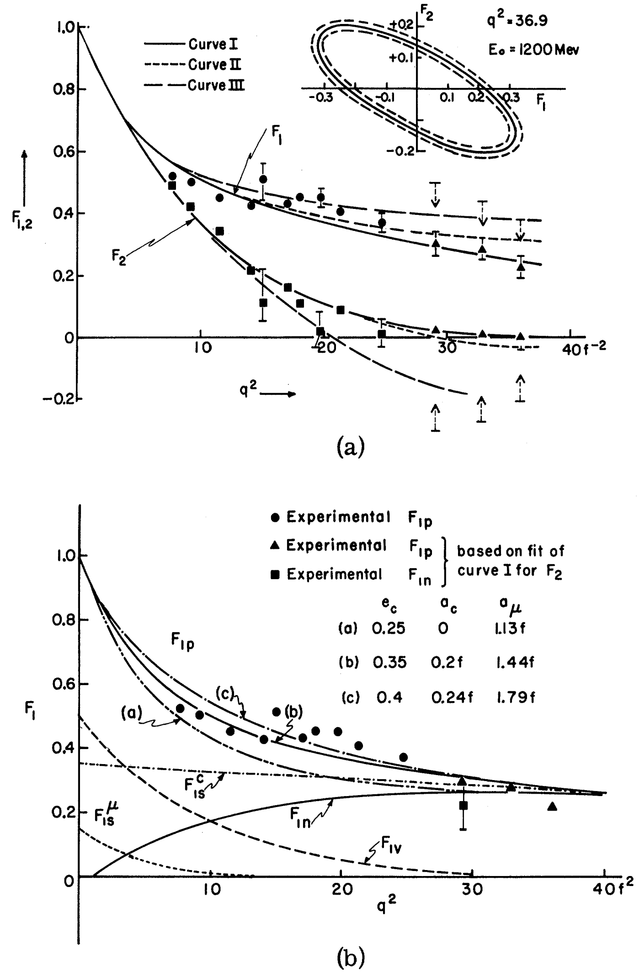

FIG. 2. (a) Experimentally determined form factors for the proton, as functions of the momentum transfer q. Circles (•) are used for the form factor F_{1p} and squares (■) for the magnetic form factor F_{2p} as determined from measurements at two different scattering angles (points with flags: Cornell data; points without flags: Stanford data). For $q^2>25$, only measurements at 112° were performed (Fig. 1). These determine ellipses in the F_1-F_2 plane, as shown in the insertion, from which the extreme values of F_{1p} and F_{2p} can be inferred. These are shown by the arrows but are not, of course, simultaneously realized. Assuming that F_{2p} is given by curves I, II, or III, one can derive respective values for F_{1p}. The results presented in Fig. 3 exclude curves II and III, so that curve I (▲) gives the proper values for F_{1p} and F_{2p}. (b) F_1 can be decomposed into partial form factors according to Eqs. (3) and (4). Three fits using the core charges and radii given in the insertion are shown. The experimental data are best reproduced by case (b). Only for this case are the partial form factors displayed. F_{1n} computed by using them is also included in the figure.

measured cross section, $\sim 10^{-34}$ cm^2/sr, is small because of destructive interference between large values of a positive F_{1p} and a negative F_{2p}. In the case of the neutron, however, the magnetic form factor would be just reversed so that the interference should then be constructive and a large value of the cross section would result. That the magnetic form factors of the proton and neutron are very nearly equal and opposite follows from the fact that the anomalous magnetic moments are equal and opposite to within 5%.[5] The curves of Fig. 3 show the ratio of neutron and proton cross sections based on calculations using expression (2) for the neutron cross section with various assumptions for F_2, but assuming that F_1 is due to a core and is the same for neutron and proton—essentially true for $q^2 > 25$. The experimental points, obtained by replacing our CH_2 targets with CD_2, using the peak value method,[4] and then making the appropriate subtractions, are also shown. They are consistent with curves having values of $F_{2p} = 0 \pm 0.03$. We conclude, therefore, that $F_{2p} \sim 0$ and is, in fact, given as a function of q^2 by Curve I of Fig. 2. This can then be used to obtain the corresponding values of F_{1p} from ellipses similar to that of Fig. 2 but corresponding to measurements at $q^2 = 28.7$ and 32.7. These are plotted in Fig. 2. Curve II of Fig. 3 and the corresponding curve of Fig. 2(a) indicate the accuracy of

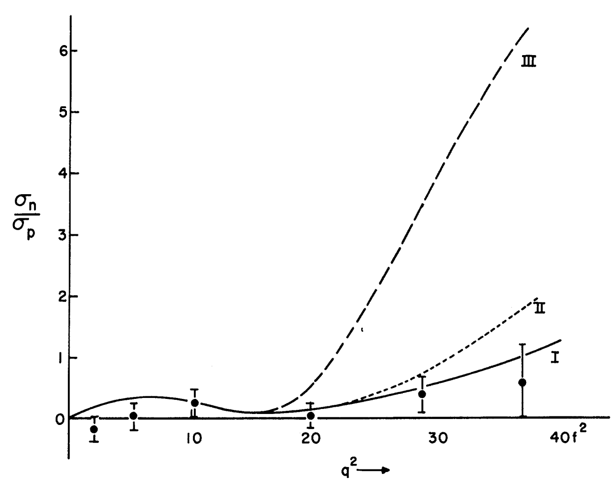

FIG. 3. The ratio of the neutron and proton cross sections as a function of the momentum transfer q. Curves I, II, and III were calculated with form factors F_{1p}, and $F_{2p} = F_{2n}$ given by the curves I, II, and III in Fig. 2(a), and with F_{1n} from Fig. 2(b). The experimental points were computed from the data in Table I.

this procedure, i.e., about ± 0.03 in the value of F_{2p} and ± 0.05 in F_{1p}. Note the pronounced minimum in the curve of electron-neutron scattering at $q^2 \approx 15$ f^{-2}, i.e., about 700 Mev.

Now let us see how the simple core model must be changed to conform with these form factors. We will separate each form factor into an isoscalar and an isovector partial form factor—the vector part changing its sign when we change from a proton to a neutron, the scalar part remaining unchanged. We can write

$$F_{1p} = F_{1S} + F_{1V}, \quad F_{1n} = F_{1S} - F_{1V}, \tag{3}$$

$$F_{2p} = F_{2S} + F_{2V}, \quad F_{2n} = F_{2S} - F_{2V}. \tag{4}$$

As the charges of the proton and the neutron are different, F_1 must consist of an isoscalar and an isovector part. F_2, on the other hand, will be determined predominantly by F_{2V} as the anomalous magnetic moments of proton and neutron agree within 5%. We believe that the experiments are not accurate enough to allow the detection of an F_{2S} contribution of the order of a few percent and therefore we assume $F_{2S} = 0$.[5] Then in order to get a vanishing rms radius for the neutron charge distribution, F_1 has to be decomposed in at least three terms. We shall write F_{1S} as a sum of two terms and shall associate $F_{1S}{}^c$, the term with the smaller radius, with a core, the other one $F_{1S}{}^\mu$ with a cloud. It will be shown that all experimental results can be reproduced without assuming that F_{1V} has a core, too. Again, however, we cannot definitely exclude such a core because of experimental uncertainties. With each form factor we can associate a charge, i.e., the value of the form factor at $q = 0$, and for $F_{1S}{}^c$, $F_{1S}{}^\mu$, and F_{1V} we have, respectively, e_c, e_μ, and e_V. The charge on the proton requires that $e_c + e_\mu + e_V = 1$, and that of the neutron requires that $e_c + e_\mu - e_V = 0$; from this it follows that $e_c + e_\mu = \frac{1}{2}$ and that $e_V = \frac{1}{2}$, a unit charge being that of the electron, of course. We can also write some relations for the mean square radii associated with each of the above charge distributions, i.e.,

$$a_p{}^2 = e_c a_c{}^2 + e_\mu a_\mu{}^2 + e_V a_V{}^2 = (0.8\ \text{f})^2, \tag{5}$$

and

$$a_n{}^2 = e_c a^2 + e_\mu a_\mu{}^2 - e_V a_V{}^2 = 0, \tag{6}$$

where a_p and a_n are the measured rms radii of charge already determined at low q^2. It follows that

$$e_c a_c^{\,2} + e_\mu a_\mu^{\,2} = \tfrac{1}{2} a_p^{\,2}, \tag{7}$$

and hence that $a_V = a_p$.

The core model implies that $a_c^2 \ll a_\mu^2$ in which case (7) gives $a_\mu^2 = a_p^2/2e_\mu$. We are left with the problem of determining e_μ and e_c. This we do by fitting our experimental curve of F_{1p} at very large values of q^2, i.e., between 20 and 37, at which place both $F_{1S}^{\ \mu}$ and F_V have become small compared to $F_{1S}^{\ c}$. For example, let us assume most simply that the core is a point charge; then, quite independently of what we choose for the form of $F_S^{\ \mu}$ or F_V, we obtain $e_c \approx 0.25$ and it follows that $a_\mu = 1.1$. However, such an assumption does not give a good fit of the experimental data. See curve (a) of Fig. 2(b). We can do better if we assume an extended distribution for the core charge—for simplicity a Gaussian form is used. Now although we know that $a_V = a_p = 0.8$ f, we do not know the shape of the charge distribution. We assume rather arbitrarily the same Gaussian shape that fits F_{2V}. This becomes quite negligible for $q^2 > 25$ f^{-2} where only the form factor of the core remains and which we then fit in this region. We could arbitrarily make $e_c \approx 0.5$; then a_c would be 0.37 f, but the fit at intermediate values of q^2 would be poor. The best fit gives $e_c = 0.35 \pm 0.1$ and $a_c = 0.2 \pm 0.1$ f. This choice determines that $e_\mu = 0.15 \pm 0.1$ and $a_\mu = 1.4 \pm 0.4$ f. (See Table II.) The corresponding partial form factors are plotted in Fig. 2(b) together with the total form factor for the neutron and proton given by (3). It should be emphasized that the model leaves only one free parameter, i.e., e_c, if a point core is assumed, and only two parameters for an extended core. The shapes of the distributions can also be considered as free parameters, but we have found by trial that our results are insensitive to whether Gaussian or exponential forms are used. Yukawa-type distributions give somewhat different results, but such distributions have a built-in core. However, the spirit of our approach and underlying it has been the core model; hence, we have avoided, but by no means dismissed, Yukawa distributions.[6]

Table II. Parameters giving the best fit to all experimental data with $F_i = e_i \exp(-a_i^2 q^2/6)$.

$e_c = 0.35 \pm 0.1$	$a_c = 0.2 \pm 0.1$ f
$e_\mu = 0.15 \pm 0.1$	$a_\mu = 1.44 \pm 0.5$ f
$e_{1V} = 0.5$	$a_{1V} = 0.8$ f
$e_{2V} = 1.0$	$a_{2V} = 0.8$ f

The core form factor and its range seem quite reasonable, but the reason for the long-range isoscalar charge form factor, $F_{1S}^{\ \mu}$, is not so obvious. If F_{1V} is due to two-meson processes, then $F_{1S}^{\ \mu}$ may be due to three-meson processes[5] but why, except for a very strong resonance, should this have a longer range? Again, it may have its origin in some obscure one-meson process. Is it possible that $F_{1S}^{\ \mu}$ is of electromagnetic origin, i.e., is the meson cloud polarized by the core charge? The sign of the effect is right, although the Coulomb forces would seem to be too weak.

Taking the Fourier transforms of the form factors, one can compute the charge distributions of the proton and neutron. These are plotted in Fig. 4. Of course, these distributions and the form factors themselves will be changed by relativistic effects which have not been taken into account.[7] Also, by choosing somewhat different models one might obtain somewhat different parameters. We believe, however, that the charge distributions shown in Fig. 4 give a qualitative picture of the nucleons. Its main features are a positive core for both particles with a charge of about 30% and a radius of approxi-

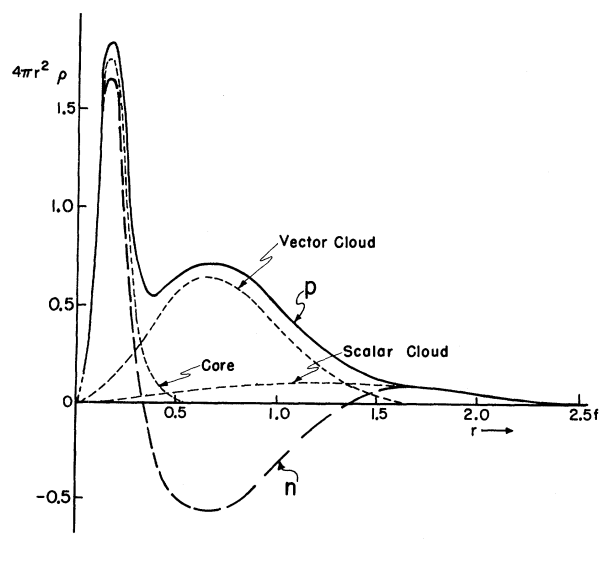

FIG. 4. Charge distribution for the proton and the neutron implied by the form factors shown for the fit (b) in Fig. 2(b).

mately the nucleon Compton wavelength. The proton core is surrounded by a positive cloud, the neutron by a negative one. The neutron has in addition a positive shell at its outside that contains a few percent of the elementary charge. The distributions of the anomalous magnetic moments are spread out with rms radii of about 0.8 f and it is not necessary, with present accuracy, to assign a magnetic core.[8]

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[1]R. Wilson, K. Berkelman, J. Cassels, and D. Olson, Nature 188, 94 (1960).

[2]F. Bumiller, M. Croissiaux, and R. Hofstadter, Phys. Rev. Letters 5, 261 (1960).

[3]L. L. Foldy, Revs. Modern Phys. 30, 471 (1958).

[4]R. Herman and R. Hofstadter, High-Energy Electron Scattering Tables (Stanford University Press, Stanford, California, 1960).

[5]P. Federbush, M. Goldberger, and S. Treiman, Phys. Rev. 112, 642 (1958). They point out that $F_{2S}(0)/F_{2V}(0)=(\mu_p-\mu_n)/(\mu_p+\mu_n)=0.03$.

[6]Unfortunately, there is not much information on the shape of the form factors from meson theory. From dispersion relations one can derive a Yukawa or Clementel-Villi type form factor according to whether one uses unsubtracted or subtracted dispersion relations [J. Bowcock, W. N. Cottingham, and D. Lurié, Phys. Rev. Letters 5, 386 (1960)]. In these calculations the π-π resonance was approximated by a δ function. A. Stanghellini [Nuovo cimento 18, 1258 (1960)] implies that a wider resonance allows for quite different shapes of form factors.

[7]F. Ernst, R. Sachs, and K. Wali, Phys. Rev. 119, 1105 (1960). They suggest that the charge and magnetic moments are determined by $F_{ch}=F_1-(q^2/2M)F_2$ and $F_{mag}=(1/2M)F_1+F_2$.

[8]Our results were presented at the New York meeting of the American Physical Society: R. R. Wilson, Bull. Am. Phys. Soc. 6, 35 (1961); D. N. Olson, H. F. Schopper, and R. R. Wilson, Bull. Am. Phys. Soc. 6, 63 (1961). At the same meeting the Stanford group also presented new measurements of electron-deuteron scattering that allowed the determination of F_{1n} and F_{2n} at $q^2<20$. As nearly as we could determine, their results were quite consistent with our curves of F_{1n} and F_{2n}, but perhaps our interpretations in terms of isoscalar and isovector form factors differed.

DIRAC AND PAULI FORM FACTORS OF THE NEUTRON*

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Recent work on electron scattering at Stanford[1-3] has shown that the electromagnetic form factors of the proton were split apart at large values of the momentum transfer (q) and the detailed behavior of the Dirac (F_{1p}) and Pauli (F_{2p}) form factors was reported. These studies showed also that F_{2p} is approaching zero and that the electron-proton scattering cross section exhibits a diffraction dip at $q^2 \cong 25$ f^{-2} which is associated with the behavior of F_{2p} at that value of the momentum transfer. Some information concerning the proton form factors has also been reported by the Cornell group.[4] The information in references 1-3 was used by Herman and Hofstadter,[5] who deduced values of the Dirac and Pauli form factors of the neutron (F_{1n} and F_{2n}, respectively) from the above data by making the assumption that $F_{2n}=F_{2p}$ which was known from earlier measurements[6,7] to be roughly true at low values of q^2. In this way the work of reference 5 showed that $F_{1n}\neq 0$. Although there is an ambiguity in the sign of F_{1n}, Herman and Hofstadter chose the negative sign because it has been commonly accepted that the charge cloud of the neutron is due primarily to the presence of negative mesons. The chief result of the present communication is the independent experimental determination of the two form factors of the neutron (F_{1n}, F_{2n}) and a verification that $F_{1n}\neq 0$. In another communication[8] we attempt to resolve the ambiguity of sign in F_{1n}.

The above results were obtained by combining measurements of the inelastic electron scattering cross section of the deuteron at two sets of values of energy (E) and angle (θ) of the scattered electron for the same value of q^2. In essence