For a further advance, more data on α, $\partial H_0/\partial p$, $\partial^2 H_0/\partial p^2$, and $\partial^2\gamma^*/\partial p^2$ for various directions of stress are required.

Our calculation gives rise to a linear term, A, in $C_{gs} - C_{gn}$. The sign and magnitude of this term depend upon the coefficients of $G(t)$ and $J(t)$ in (5). The experimental data indicate that the coefficient of $J(t)$ in general makes a small positive contribution to A. The coefficient of $G(t)$, which involves $\partial^2\gamma^*/\partial p^2$, is not known, but it might be negative in some cases. If so, and assuming that the parabolic form of $f(t)$ remains valid, formally one deduces an apparent negative specific heat in the superconducting state at the lowest temperatures. It is not clear what modifications or mechanisms should be invoked in order to avoid this conclusion, since existing data do not extend to low enough temperatures. Bryant and Keesom's results indicate $A < 0$, but in their case a positive nuclear quadrupole term keeps the total specific heat from becoming negative.

We should point out that the role of the zero-point energy in causing the observed specific-heat anomalies was realized independently by Schrieffer who has approached the problem from a microscopical point of view.

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OSCILLATIONS IN THE LONGITUDINAL TUNNEL CURRENT OF TUNNEL DIODES

R. R. Haering and P. B. Miller
International Business Machines Research Center, Yorktown Heights, New York
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It has been observed by Chynoweth, Logan, and Wolff[1] that the tunnel current in InSb tunnel diodes displays oscillations at low temperatures and large longitudinal magnetic fields ($H \parallel F$). The field dependence of the periodicity of these oscillations suggests a relation to the de Haas-Schubnikow oscillations in galvanomagnetic properties. These oscillations are due to density of state fluctuations in the highest occupied Landau level. Such oscillations are not to be expected for transverse magnetic fields,[1,2] and none were observed.[1]

If the observed oscillations are of the de Haas-Schubnikow type, their magnitude suggests that a significant portion of the tunneling current is due to electrons with the highest possible magnetic quantum number. This conclusion is surprising, since the tunneling process strongly favors the low quantum numbers. The dependence of the tunneling rate on quantum number is given by[2]

$$\exp[-\tfrac{3}{2}(n\hbar\omega/\epsilon)\lambda],$$

where

$$\lambda = \tfrac{2}{3}(2m\epsilon)^{3/2}/\hbar mF,$$

$$F = [\mu_1 + \mu_2 + \epsilon - qV]/a = \text{average junction field},$$

a = junction width, n = quantum number, $\omega = eH/mc$, ϵ = energy gap, m = electron-hole reduced mass, V = applied voltage. For the particular InSb diode used in reference 1 we have $\lambda \sim 30$. Hence, the ratio of the contributions of the largest and smallest quantum numbers is $\sim \exp(-45\mu_1/\epsilon)$, where μ_1 is the electron Fermi level. It is therefore clear that the observed oscillations are far too large in amplitude and in number to be explained in this manner. Furthermore, the above process only leads to current discontinuities and not oscillations.

We wish to propose an alternative explanation of the observations based on the dependence of the electron Fermi level on magnetic field. In a magnetic field, the electron Fermi level oscillates about its zero-field value. (Fluctuations of the hole Fermi level are much smaller because of the heavy-hole states.) These small Fermi level fluctuations result in a small charging or discharging of the junction capacitance and hence in a small oscillatory change of the average junction field F. Since the tunneling rate is a strong function of F, small fluctuations in F can result in large current oscillations. It is to be noted that whereas the origin of the Fermi level fluctuations is related to density of state fluctuations arising from the highest occupied quantum number, the resulting change in the junction field affects the tunneling rate of all quantum states. We shall show that the above effect accounts for the observed period of the oscillations, as well as for the magnitude and field dependence of the oscillation amplitudes.

For an isotropic conduction band with constant effective mass m_1, the Fermi level $\mu_1(H)$ is determined by the condition:

$$\frac{4}{3}\left[\frac{\mu_1(0)}{\hbar\omega_1}\right]^{3/2} = \sum_{n=0}^{N_+}\left[\frac{\mu_1(H)}{\hbar\omega_1} - n - \tfrac{1}{2} - \tfrac{1}{2}g_1\frac{\beta H}{\hbar\omega_1}\right]^{1/2} + \sum_{n=0}^{N_-}\left[\frac{\mu_1(H)}{\hbar\omega_1} - n - \tfrac{1}{2} + \tfrac{1}{2}g_1\frac{\beta H}{\hbar\omega_1}\right]^{1/2}, \quad (1)$$

where β is the Bohr magneton, g_1 is the conduction band g factor, and $N_\pm$ is the maximum integer $\leq [\mu_1(H)/\hbar\omega_1] - \frac{1}{2} \mp \frac{1}{2}g_1\beta H/\hbar\omega_1$. In the extreme quantum limit ($\mu_1/\hbar\omega_1 < 1$), $\mu_1(H)$ is readily obtained directly from Eq. (1), whereas for $\mu_1/\hbar\omega \gtrsim 1$ we obtain[3]

$$\frac{\mu_1(H)}{\mu_1(0)} \simeq 1 + \frac{1}{6}\left[\frac{\hbar\omega_1}{\mu_1(0)}\right]^{3/2}\left\{E\left[\frac{\mu_1(0)}{\hbar\omega_1} - \tfrac{1}{2} + \tfrac{1}{4}g_1\frac{m_1}{m_0}\right] + E\left[\frac{\mu_1(0)}{\hbar\omega_1} - \tfrac{1}{2} - \tfrac{1}{4}g_1\frac{m_1}{m_0}\right]\right\}. \quad (2)$$

The function $E(x)$ in Eq. (2) has approximately unit amplitude and is periodic with period one. For $0 \leq x \leq 1$, we have

$$E(x) = 2(x+\tfrac{3}{2})^{3/2} - 3(x+1)^{1/2} - 3x^{1/2}. \quad (3)$$

The oscillatory part of Eq. (2) is plotted vs $\mu_1(0)/\hbar\omega_1$ in Fig. 1 for $g_1 = 56$ and $m_1/m_0 = 0.013$.

For a junction for which $a \sim [\mu_1 + \mu_2 - qV + \epsilon]^{1/\alpha}$, a change $\Delta\mu_1$ in the electron Fermi level results in a change ΔF in the average junction field, given by

$$\frac{\Delta F}{F} \simeq \left(\frac{\alpha - 1}{\alpha}\right)\frac{\Delta\mu_1}{(\mu_1 + \mu_2 - qV + \epsilon)}. \quad (4)$$

The number α is normally ~ 2, the value obtained for a step junction. For small biases qV, the tunneling exponential λ is therefore

$$\lambda \simeq \lambda_0\left[1 - \left(\frac{\alpha - 1}{\alpha}\right)\left(\frac{\mu_1}{\mu_1 + \mu_2 + \epsilon}\right)\frac{\Delta\mu_1}{\mu_1}\right]. \quad (5)$$

Since the tunneling current density for small biases is

$$j(H) = \text{const}\; Ve^{-\lambda}\left(\frac{\hbar\omega}{2}\frac{\partial\lambda}{\partial\epsilon}\right)\operatorname{csch}\left(\frac{\hbar\omega}{2}\frac{\partial\lambda}{\partial\epsilon}\right) \times \cosh\left(E_p\frac{\partial\lambda}{\partial\epsilon}\right), \quad (6)$$

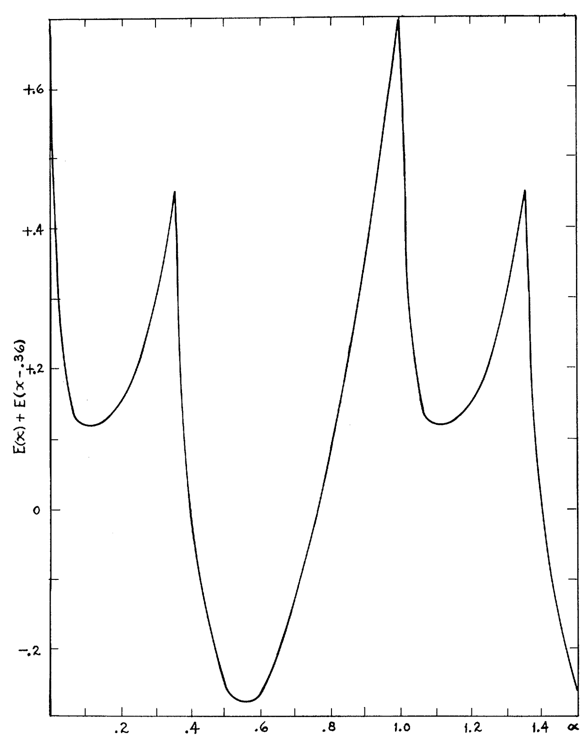

FIG. 1. Plot of oscillatory part of Eq. (2) with $g_1 = 56$, $m_1/m_0 = 0.013$.

where $E_p = \frac{1}{2}(g_1 - g_2)\beta H$, the oscillatory part of the tunneling current is given by

$$\frac{j(H) - j_{av}(H)}{j_{av}(H)} = \left[\lambda_0 + \left(\frac{\hbar\omega}{2}\frac{\partial\lambda_0}{\partial\epsilon}\right)\coth\left(\frac{\hbar\omega}{2}\frac{\partial\lambda_0}{\partial\epsilon}\right) - \left(E_p\frac{\partial\lambda_0}{\partial\epsilon}\right)\tanh\left(E_p\frac{\partial\lambda_0}{\partial\epsilon}\right)\right] \times \left(\frac{\alpha - 1}{\alpha}\right)\left(\frac{\mu_1}{\mu_1 + \mu_2 + \epsilon}\right)\left(\frac{\Delta\mu_1}{\mu_1}\right). \quad (7)$$

Here $\Delta\mu_1/\mu_1$ is to be obtained from (2), and $j_{av}(H)$ is obtained from (6) with λ replaced by λ_0.

Under normal conditions for observation of oscillations, the coth and tanh terms are negligible and we find for InSb (with $\alpha = 2$)

$$\frac{j(H) - j_{av}(H)}{j_{av}(H)} \simeq \frac{\lambda_0}{12}\left(\frac{\mu_1}{\mu_1 + \mu_2 + \epsilon}\right)\left(\frac{\hbar\omega_1}{\mu_1}\right)^{3/2} \times \left[E\left(\frac{\mu_1}{\hbar\omega_1} - 0.32\right) + E\left(\frac{\mu_1}{\hbar\omega_1} - 0.68\right)\right]. \quad (8)$$

For $\mu_1/\hbar\omega_1 \sim 1.5$, $\lambda_0 \sim 30$, $\mu_1/(\mu_1 + \mu_2 + \epsilon) \simeq \frac{1}{4}$, the oscillations are therefore ~30% of the average current. For smaller fields the relative importance of the oscillations is seen to decrease like $H^{3/2}$. Temperature and collision broadening of the sharp peaks in Fig. 1 are ignored in this estimate, which would tend to reduce the amplitudes. This damping effect also accounts for the appearance of only one current peak per cycle. The observations of Chynoweth et al.[1] are in good agreement with the above results.

It should be noted that the above effect is not to be expected for a transverse magnetic field. In this case the discontinuities in the density of electron states for states whose orbit center lies less than one orbit radius from the junction no longer occur at particular values of the magnetic field, but are distributed more or less continuously and hence the average junction field remains unaltered. Density of states fluctuations due to states whose orbit center is far from the junction do result in the appearance of a weak space-charge region on the n side. However, this space-charge region is not in the vicinity of the junction and hence does not affect the junction field.

In previous theories it was customary to ignore the shift of the Fermi level with magnetic field.[2,4] The present analysis indicates that although this shift is small, it plays an important role in the tunneling process.

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STIMULATED SPIN-ECHO MEASUREMENT OF CROSS-RELAXATION IN NEUTRON-IRRADIATED CALCITE*

L. Kent Wanlass and J. Wakabayashi
Electronic Research Laboratory, University of California, Berkeley, California
(Received December 19, 1960; revised manuscript received February 9, 1961)

In recent years, possibly due to the advent of microwave spectroscopy and maser applications, a great deal of research has been conducted on the relaxation times of spin systems. Presently there is a large amount of work being conducted on cross-relaxation decay[1] and its subsequent effects. The majority of these experimental measurements have been conducted by saturation methods or pulse modifications of them, which lead to exponential decay with two or more time constants. The study of electron spin echo as an information storage medium for high-speed carrier-type microwave computers has led the authors to some interesting measurements of relaxation times in neutron-irradiated calcite. Of more significant interest, however, is the use of the stimulated spin-echo technique in cross-relaxation measurements. This method, in cases where the T_1 relaxation time is somewhat larger in magnitude than the cross-relaxation time,