PION-PION INTERACTION AND HIGH-ENERGY $p+d$ COLLISIONS

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(Received February 10, 1961)

A possible explanation of the anomaly found in meson production in proton-deuteron collisions[1] has recently been put forward by Tubis and Uretsky.[2] They suggest that if a final state P-wave pion-pion interaction, described in the scattering length approximation, is added to the statistical model of Abashian et al.,[1] a P-wave scattering length of magnitude 2.5 pion Compton wavelengths explains the anomaly. We wish to present evidence here that does not agree with such a large value for this scattering length. This evidence comes from equations relating the three scattering lengths as derived from crossing symmetry, and from the experimental evidence on the asymmetry of the pion momentum spectrum in τ decay.

We will use the crossing symmetry equations[3] for pion-pion scattering,

$$\tfrac{1}{3}(A_0-A_2)(\nu,\cos\theta)=\tfrac{1}{2}(A_1+A_2)(\nu_c,\cos\theta_c), \quad (1)$$

where A_I is the pion-pion scattering amplitude with isotopic spin I, ν is the squared momentum in units of the pion mass, and θ the scattering angle, both in the center-of-mass system, while ν_c, θ_c are the crossed variables $\nu_c=\frac{1}{2}\nu(1+\cos\theta)-(\nu+1)$, $\nu_c\cos\theta_c=\frac{1}{2}\nu(1+\cos\theta)+(\nu+1)$. We will approximate $A_I(\nu,\cos\theta)$ by keeping only the S and P waves, which should be a good approximation in the low-energy region. We take ν, ν_c near the symmetry point $-2/3$, and $\cos\theta$, $\cos\theta_c$ near zero. We assume[3] that the partial-wave amplitudes $A_I(\nu)$ are analytic in the cut ν plane with cuts from $-\infty$ to -1 and 0 to ∞, so that with $\nu_c=\nu+x$, Eq. (1) becomes

$$\begin{aligned}\tfrac{1}{3}(x+\nu)\int[a_0(\nu')-a_2(\nu')](\nu'-\nu)^{-1}d\nu' \\ -\tfrac{1}{2}(x+\nu)\int a_2(\nu')(\nu'-\nu-x)^{-1}d\nu' \\ =\tfrac{3}{2}(x+3\nu+2)\int a_1(\nu')(\nu'-\nu-x)^{-1}d\nu'. \quad (2)\end{aligned}$$

In Eq. (2) the integrals are along the real axis except for the interval $(-1, 0)$, and the real functions $a_I(\nu)$ are the absorptive parts of the $A_I(\nu)$. Equation (2) is true for x in a small neighborhood of 0 and for ν near $-\frac{2}{3}$, so we may equate coefficients of powers of x in the power series expansion in x on each side of Eq. (2) to obtain

$$\begin{aligned}\nu\int[\tfrac{1}{3}a_0(\nu')-\tfrac{5}{6}a_2(\nu')](\nu'-\nu)^{-1}d\nu' \\ =\tfrac{3}{2}(2+3\nu)\int a_1(\nu')(\nu'-\nu)^{-1}d\nu'. \quad (3)\end{aligned}$$

Each side of Eq. (3) is an analytic function of ν in the cut ν plane and these functions are equal in a real interval near $\nu=-2/3$, so they are equal everywhere. In particular

$$\frac{1}{3}a_0(\nu)-\frac{5}{6}a_2(\nu)=\left(\frac{3}{\nu}+\frac{9}{2}\right)a_1(\nu),$$

$$\frac{1}{3}A_0(\nu)-\frac{5}{6}A_2(\nu)=\left(\frac{3}{\nu}+\frac{9}{2}\right)A_1(\nu). \quad (4)$$

In the scattering length approximation $A_I(\nu)=a_I$, $a_I(\nu)=a_I^2\nu^{1/2}(I=0,\ 2)$, $A_1(\nu)=a_1\nu$, $a_1(\nu)=a_1^2\nu^{5/2}$, then in the limit of small ν, Eqs. (4) become

$$a_0-\tfrac{5}{2}a_2=9a_1,$$
$$a_0^2=\tfrac{5}{2}a_2^2. \quad (5)$$

We use also the result of the analysis of the asymmetry of the pion momentum spectrum in τ decay[4,5] that[6]

$$a_2-a_0\approx 0.7. \quad (6)$$

This gives two possible solutions to Eqs. (5) and (6):

$$(a_0, a_1, a_2)=(-1.9, +0.1, -1.2)$$
$$\text{or}\quad (-0.4, -0.1, +0.3). \quad (7)$$

Both these solutions have $|a_1|\sim 0.1$, which is two orders of magnitude smaller than the value $|a_1|\sim 15$ required by Tubis and Uretsky. In order to obtain this larger value of $|a_1|$ it is necessary to choose $(a_0, a_2)=\pm(-15, 50)$ or $\pm(740, 240)$. For such large values of the scattering lengths the τ-decay analysis leading to Eq. (6) is no longer good, and it may be possible to explain the existing asymmetry in the pion spectrum in terms of such large scattering lengths. However, the low-energy pion-pion scattering cross section then comes to above 600 barns. This is inconsistent with the values of $\lesssim 0.5$ barn found[7] in the extrapolation procedure applied to the reaction $\pi^-+p\to\pi^-+\pi^0+p$. Such a strong interaction should certainly have shown itself in the extrapolation procedure. It also should give a very pronounced peaking at low momentum in the nucleon recoil momentum spectrum, which is definitely not present.[8] Thus we conclude that $|a_1| \sim 15$ is inconsistent with present data and the crossing equations (5).

We note that in the approximation of keeping only S and P waves and applying the properties of crossing, analyticity, and unitarity,[3] it may be shown[9] that the inner regions of the $I=0$ and $I=1$ potentials are attractive and that of the $I=2$ is repulsive. The scattering length solutions of Eq. (7) will both give an $I=0$ resonance,[10] the second solution only will give both an $I=1$ and $I=2$ resonances, while the first solution will give neither an $I=1$ or an $I=2$ resonance.

This is in agreement with the further results obtained at Berkeley on p - d collisions,[11] where a very low energy $I=0$ pion-pion resonance seems to be present. If the anomaly[1] in $p+d \rightarrow He^3+\pi^0+\pi^0$ is interpreted in terms of a resonance, then this will single out the second pion-pion scattering length solution as the correct one. This assignment is also supported by evidence of strong $I=0$ and $I=1$ pion-pion interactions from pion-nucleon scattering[12] and a strong $I=1$ or $I=2$ pion-pion interaction in the final state of the reaction $\pi^-+p \rightarrow \pi^-+\pi^0+p$.[13,14]

The data of Pickup et al.[14] are not completely in accord with the second solution, since peaking in the Q value of two pions only appears for backward nucleons for $(\pi^-\pi^0)$ and not for forward nucleons for this pair or forward or backward nucleons for $(\pi^-\pi^+)$. The absence of peaking for forward nucleons is to be expected due to the vanishing of the nucleon-pion vertex, but the absence of peaking in $(\pi^-\pi^+)$ for backward nucleons is disturbing, since $I=0$, 1, and 2 resonances should all contribute, if they exist.

The positions of the predicted resonances are known only very approximately at present.[9] For the first scattering length solution the resonance in $I=0$ is at $\nu_R \approx 1.5$, and for the second solution in $I=0$ is at $\nu_R \approx 2.7$ and in $I=2$ is at $\nu_R \approx 5.0$. The $I=1$ resonance position in the latter case cannot be predicted even approximately, since the value of b is not known. The resonance values for the $I=0$ state are not in agreement with the value[11] $\nu_R \approx 0.2$ found at Berkeley, but we hope to obtain better agreement by machine solution of the S- and P-wave pion-pion equations using the second scattering length solution.

The author would like to thank Professor Lévy and Dr. Motchane for the kind hospitality extended to him while part of this work was being carried out.

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