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RELAXATION MECHANISMS IN FERROMAGNETIC RESONANCE

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Up to the present, the agreement between theory and experiment in ferromagnets for the relaxation time, τ_0, of spin waves with zero wave vector (uniform precession) has not been satisfactory, especially in highly polished samples with narrow linewidths.[1] The purpose of this Letter is to report the most recent observations of τ_0 in yttrium iron garnet (YIG), which has the narrowest known ferromagnetic resonance linewidth, and to outline the essential points of a theory which adequately accounts for these observations. The theory is based on the effects of single-ion anisotropy, or local magnetostriction, in a simplified two-sublattice ferrimagnetic model.

The character of τ_0^{-1} in YIG along the [111] crystal axis is as follows:

(i) The magnitude of τ_0^{-1} is 2.4×10^6 sec^{-1} at room temperature and a frequency of 5.7 kMc/sec.

(ii) τ_0^{-1} is proportional to T^n, where $1<n<2$ in the range 150°K to 400°K with the larger values of n corresponding to higher temperatures.[2] (The behavior or τ_0^{-1} below 150°K in the present samples appears to be determined by additional processes other than those considered here, and will be reported on by E. G. Spencer.)

(iii) τ_0^{-1} at room temperature is proportional to frequency, at least for $\nu \geq 3$ kMc/sec.[3]

(iv) τ_0^{-1} at room temperature is nearly proportional to M_S^{-1}, where M_S is the saturation magnetization. This relation was determined by doping YIG with gallium and aluminum, which substitute primarily on tetrahedral sites, and indium, which substitutes on octahedral sites.[4]

The above observations of τ_0^{-1} have been made possible primarily by the following three developments: (1) elimination of the effects of surface roughness, (2) elimination of rare earth impurities,[5] and (3) separation of spin-spin from spin-lattice relaxation effects. These and other related developments will be discussed in detail in a forthcoming paper. At this time we will briefly describe the technique which was used for most of the data.

Schlömann and Morganthaler have shown[6] that growing pairs of spin waves of equal and opposite wave number and frequency $\omega_p/2$ may be excited when an rf field of frequency ω_p with sufficient magnitude is applied parallel to the dc magnetic field, H_{dc}. The threshold rf field is

$$h_{crit} = \omega_p/(\gamma^2\tau_k 4\pi M_s \sin^2\theta_k), \qquad (1)$$

where θ_k is the angle between $\vec{k}$ and H_{dc}. Thus the threshold is lowest for $\theta_k = \pi/2$. By measuring h_{crit} as a function of H_{dc}, and using the familiar dispersion relation for spin waves, together with the measured value[7] of the exchange constant D, one can obtain a plot of τ_k^{-1} vs k as shown in Fig. 1. A conventional microwave spectrometer with a critically coupled reflection cavity was used for these experiments.

The data in Fig. 1 can be described by

$$1/\tau_k = (1/\tau_0) + Bk, \qquad (2)$$

from $k = 0.35\times10^5$ to 1.55×10^5 cm^{-1}. These data have several important features: (1) τ_0^{-1} obtained by this technique is essentially independent of surface roughness, which is known to affect strongly the ordinary uniform-precession linewidth. Hence τ_0^{-1} must be a property of the bulk material. (2) τ_0^{-1} obtained here is essentially the same as T_{10}^{-1}, the inverse spin-lattice relaxation time of the uniform precession, as measured on the same sample by the frequency modulation technique.[1] (3) For the k numbers involved here, Sparks and Kittel[8] have shown that the k-dependent part of τ_k^{-1} should be linear in k. The observed value of B in Fig. 1 is in approximate agreement with their theory. (The latter point will be considered in more detail separately.)

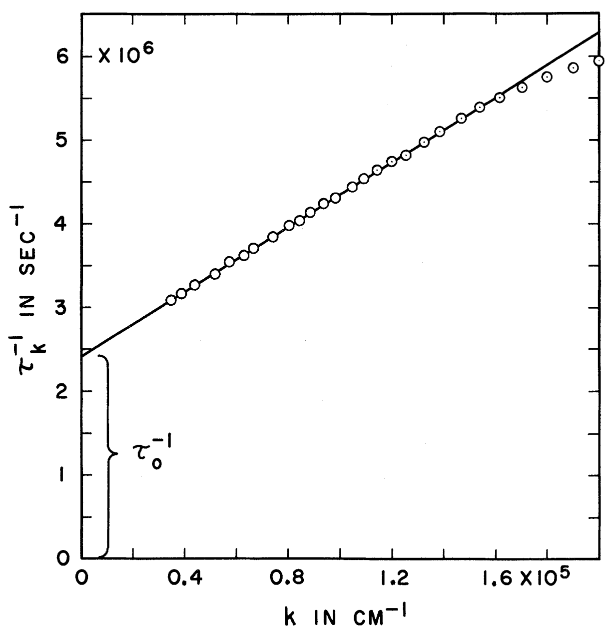

FIG. 1. Room temperature values of τ_k^{-1} vs wave number k for single-crystal YIG along the [111] crystal axis, using the parallel-pump technique and small spherical samples. The pump frequency is 11.4 kMc/sec and the spin-wave frequencies are 5.7 kMc/sec. The departure of the data from the straight line of Eq. (2) is probably due to the assumption $Dk^2 \ll \hbar\omega$, in reference 8, no longer being satisfied.

To explain the above results we have considered the following interactions: dipole, pseudodipole, and single-ion anisotropy, including both uniaxial and cubic terms as found by Geschwind.[9] As mechanisms of relaxation we have considered the following: three magnons, four magnons, one magnon—one phonon, two magnons—one phonon, and one magnon—two phonons. Therefore, there are twenty mechanisms when combined with the above four interaction Hamiltonians.

In this note we cannot give the theoretical details. Our purpose is to describe the important differences between our ferrimagnetic model and previously used ferromagnetic models, and to identify the mechanisms which have been found in detailed calculations to be in order of magnitude agreement with the experiments.

We have used a simplified ferrimagnetic model for YIG with two equivalent sublattices having different quantities of spin in each sublattice, namely $S_A = 5/2$ and $S_B = 5/3$. A ferrimagnetic model was used for the following reasons. Firstly, the spin-wave spectrum, $E(k)$, is considerably different in a ferrimagnet from that in a ferromagnet, with ferrimagnets having both acoustic and optical branches. For the acoustic spin-wave modes, $E(k)$ for the low-lying states is written as Jk^2 in both cases. But for smaller values of $S_A - S_B$, J becomes larger and the range of equivalence becomes smaller. In the larger k range, $E(k)$ should then be replaced by $\hbar u_s k - \Delta$ as in an antiferromagnet, where u_s is the spin-wave velocity and Δ is associated with the energy gap between the acoustic and optical branches. This implies that the state density becomes larger than would be expected by extrapolation from the smaller k range.

In the second place, the amplitude of the spin waves becomes larger than in a simple ferromagnetic model. Also in a ferrimagnetic model a spin operator means one spin quantum up or down in total, but when we consider each sublattice the amplitude becomes larger, for example in the small k range by the factor $[(S_A \text{ or } S_B)/(S_A - S_B)]^{1/2}$. (When $S_A - S_B$ becomes very small, this factor is determined by the anisotropy.) This factor is very important in explaining the M_s^{-1} dependence of τ_0^{-1}. We are considering here relaxation processes of small k spin waves. Thus in cases of long-range mutual spin interactions, such as the dipole type, and interactions with other small k quanta, this fine structure does not have an important effect. Here there is no essential difference between ferro- and ferrimagnetism. But for interactions with large k quanta, the fine structure has an essential effect and the interaction increases by the above factor. In the case of single-spin interactions or short-range interactions this factor does not vanish in interactions with either small or large k spin waves.

Among the twenty mechanisms originally considered, the important interactions are three-magnon processes by dipole, uniaxial, and cubic anisotropies; and two-magnon—one-phonon processes by dipole and uniaxial anisotropy (hereafter referred to as processes I, II, III, IV, and V, respectively). Processes I and III are available to τ_k^{-1} $(k \neq 0)$ and II, IV, and V are available to τ_0^{-1}. Concerning τ_k^{-1}, process I was calculated by Sparks and Kittel[8] in a simple ferromagnetic model. We would like to mention briefly the difference obtained with our simple ferrimagnetic model. For k larger than 10^4 cm^{-1} in YIG, interactions with smaller k spin waves are important and the result is the same as that of Sparks and Kittel. For $k < 10^4$ cm^{-1}, however, the fine structure becomes important and τ_k^{-1} is multiplied by the factor $S_A^2/(S_A - S_B)^2$. Process

III is also important for τ_k^{-1}. This process gives the same T, ν, and k dependence as I and also the same order of magnitude, but the M_S dependence is very different, namely $\tau_k^{-1} \sim M_S^{-3}J^{-1}$. Therefore, III would be important in materials with smaller M_S.

To explain the experimental results for τ_0^{-1} in YIG, we have considered in detail processes II, IV, and V. Process IV depends rather sensitively on the crystal structure and energy spectrum, and in some cases (not unreasonable in YIG) it gives the correct T, ν, and M_S^{-1} dependence. However, in order of magnitude it is more than two orders too small. A simple body-centered cubic lattice was assumed for the calculation of IV and although the exact crystal structure is an important factor, we do not expect an increase of two orders of magnitude in other crystal structures. Concerning II and V, the particular character of the uniaxial anisotropy in YIG is highly important in both cases. As is well known, YIG is cubic, and the large local uniaxial anisotropy term when averaged over the unit cell vanishes. The macroscopic magnetostriction is also observed to be small. It is logical then to assume that a large local magnetostriction effect exists which is important in process V.

Because of the above, the only interactions available to process II are those in which the total wave vector of the spin waves and phonons changes by a finite value K; or some types of interband transitions. In process V both types of interactions, $K=0$ and $K\neq 0$, are available, with the $K=0$ type probably more important. The $K=0$ type includes interactions with both acoustic and optical phonons, and it is possible that the interaction with optical phonons is more important than with acoustic phonons.

Detailed calculations show that in process II, τ_0^{-1} is proportional to ν, and nearly M_S^{-1} and T^2. The order of magnitude is also in good agreement. In process V with $K=0$, τ_0^{-1} is proportional to ν, T, and nearly M_S^{-1}. In order of magnitude, the agreement is good if we take a value of 5 cm^{-1} for the anisotropy constant per unit uniaxial distortion. This value is not unreasonable when compared with values obtained from other experiments. We are led to conclude that processes II and V are probably the dominant processes for the relaxation of τ_0 in YIG at intermediate temperatures, with V being dominant at lower temperatures and II at higher temperatures. The details will be given in full papers.

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