and ρ_{os} is the oscillatory component. Our experimental results indicate that the damping is so severe that only the first term in the summation need be considered. This striking attenuation in the amplitudes for decreasing B can be attributed to both thermal broadening and broadening of the Fermi level due to inhomogeneities in the crystal, the latter being the dominant effect. Collision broadening should be very small because of the high mobility.

For B in the [110] direction, two periods would normally be observed. However, because of the damping terms, the component due to the high effective mass will be very small after several periods, leaving predominantly the low-effective-mass oscillations. From the data, plotted in Fig. 2(a), the ratio $\alpha_{110}/\alpha_{100}$ determines a value of K in approximate agreement with the value determined from weak-field magnetoresistance at 300°K. In the [100] case, the phase agrees with that predicted by Adams and Holstein. The phase in the [110] case is somewhat shifted from the predicted value as might be expected from a small admixture of the high-mass period.

The damping term which includes the effect of inhomogeneity and thermal broadening can be written as $\exp[-(\beta_1+\beta_2)/B]$, where $\beta_1 \simeq 200(m_e^*/m_D^*)(\Delta n/n)$ and $\beta_2 = 14.8\, m_e^* T$. The observed damping, $\beta_1 \simeq 7$, indicates a value for $\Delta n/n$ of about 10% which is consistent with the measured inhomogeneity.

To obtain the value of the total damping term [where the effect of inhomogeneity broadening is included in Eq. (1)], $\Delta\rho B^{-1/2}/\rho_0 T$ vs $1/B$ was plotted in Fig. 2(b). ρ in Eq. (1) was taken to be the actual resistivity at a given field, i.e., $\rho \propto \rho_0 B$; however, the actual magnetic field dependence in front of the oscillatory term has only a small effect on the determination of the damping factor. Since the difference in the slope at the two temperatures arises from the change in the value of the β_2 term, an effective mass $m_e^* = 0.035 \pm 0.005$ could be determined. Thus for $K = 4.5$, $m_T^* = 0.025 \pm 0.005$, $m_L^* = 0.12 \pm 0.03$, and $m_D^* = 0.10 \pm 0.02$. Thermoelectric measurements at $T \leq 10$°K have given $m_D^* = 0.08 \pm 0.03$.[2]

More complete results will be published in the future, including studies of longitudinal magnetoresistance and Hall oscillations.

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TRANSVERSE-EVEN VOLTAGE: A HIGH-FIELD GALVANOMAGNETIC EFFECT ASSOCIATED WITH OPEN ORBITS IN METALS

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The magnetoresistance, determined from the voltage measured parallel to the current, represents only one component of the even electric field, the even part of the electric field being that part which is unchanged by a reversal of the magnetic field. The other electric field components lie transverse to the current, and account, for example, for the well-known planar Hall effect. Here, we report observations in copper of such transverse voltages, and demonstrate a significant application of the transverse-even voltage in the determination of open-orbit directions. In addition to providing information about the shape of the Fermi surface, we find that these observations vividly support the concept of open orbits.[1]

When the magnetic field lies in the plane transverse to the current the planar Hall effect vanishes. However, an even voltage usually persists due to the presence of a slight longitudinal misalignment of the Hall contacts to the sample. Such voltages are of one sign because they represent reproductions of the magnetoresistance on a reduced scale, and indeed, they are not true transverse voltages. In contrast, the transverse-even voltages which we have observed in high-purity copper single crystals for high magnetic fields transverse to the current do not scale with the magnetoresistance and characteristically exhibit changes of sign.[2] This new behavior, implicit in the definitive work of Lifshitz and Peschanskii,[3] arises from the existence of a bulk anisotropic magnetoresistance tensor whose principal axes are sensitive to magnetic field orientation because of the presence of open orbits. These new voltages make it possible to determine open-orbit directions with significantly less experimental data than would otherwise be required. In addition, these new voltages are more sensitive to some higher order[4] groups of open orbits than are magnetoresistance data alone. The expected theoretical behavior of these new voltages is discussed under certain simplifying assumptions. Predictions based on these arguments are compared with observed results.

Consider those field directions at high magnetic field strength such that an open Fermi surface leads to a non-negligible number of open orbits whose net direction is along a single direction, say for convenience the x direction. In this case, there is only one dominant even term in the magnetoresistance tensor, $\rho_{xx} = H^2 a$. The positive factor a is, to a first approximation, field insensitive, depending rather on the details of the scattering mechanism in a complicated manner. Let $\vec{\Omega}$ be a unit vector in the direction of the open orbits and $\vec{j}$ be the current density; then

$$\vec{E}_0 = H^2 a \vec{\Omega}(\vec{\Omega}\cdot\vec{j}) \tag{1}$$

expresses the general relationship between the open-orbit direction, the current, and $\vec{E}_0$, the component of the electric field due to open orbits. The longitudinal voltage $V_l \equiv \vec{D}_l \cdot \vec{E}_0$ is proportional to the resistivity, while the two transverse-even voltages are defined by $V_{t_1} = \vec{D}_{t_1}\cdot\vec{E}_0$ and $V_{t_2} = \vec{D}_{t_2}\cdot\vec{E}_0$. In these expressions, the $\vec{D}_i$ are three orthogonal vectors, either parallel ($i=l$) or transverse ($i=t_1, t_2$) to the current, whose lengths are proportional to their respective magnetoresistance- or Hall-probe separations. If, as the orientation of the magnetic field is changed, a new set of open orbits is encountered whose net direction is parallel to $\vec{\Omega}'$, then a new set of voltages V_l', V_{t_1}', and V_{t_2}' become significant. Various open-orbit directions may be encountered for a continuous rotation of the magnetic field. For each encounter the sign of V_l will be the same since $V_l \propto (\vec{D}_l\cdot\vec{\Omega})^2$. On the other hand, the uniformity of sign need not be maintained for the transverse voltage since $V_{t_i} \propto (\vec{D}_{t_i}\cdot\vec{\Omega}) \times(\vec{\Omega}\cdot\vec{D}_l)$, whose sign depends on the relative orientation of $\vec{\Omega}$; this behavior accounts for the changes of sign of the transverse-even voltage that are observed. As a consistency check on the presence of open orbits whose direction has been ascertained from previous measurements, it suffices to compare the predicted transverse-even voltages $V_{t_1}^{\text{pre}}$ and $V_{t_2}^{\text{pre}}$, where

$$V_{t_i}^{\text{pre}} = V_l(\vec{D}_{t_i}\cdot\vec{\Omega})/(\vec{D}_l\cdot\vec{\Omega}), \tag{2}$$

with the corresponding observed values.

The direction $\vec{\Omega}$ of the open orbits, whose presence is signified by a relatively large value of V_l, is determined by

$$\vec{\Omega} = (\text{normalization})[(V_l/D_l), (V_{t_1}/D_{t_1}), (V_{t_2}/D_{t_2})], \tag{3}$$

as expressed in the "sample" coordinate system. This procedure involves considerably less experimental effort than the previous way in which $\vec{\Omega}$ was determined from magnetoresistance data alone, namely, by tracing out a sequence of points of high magnetoresistance in the plane transverse to $\vec{\Omega}$.[5]

One of the strong points of high-field magnetoresistance data is the information they can give regarding suitable topological models of the Fermi surface through the presence of variously directed sets of open orbits which the Fermi surface must support. For an understanding of the complex metals a qualitative model of the Fermi surface should precede the "pinpointing" of certain surface details. The use of the transverse-even voltages should more readily enable the Fermi surface to be determined. We now briefly discuss some of our transverse-even voltage observations in copper.

Figure 1(a) shows the magnetoresistance observed in copper at 18 kgauss in a transverse rotation about an axis near to the $[5\bar{1}1]$ direction. The peaks of magnetoresistance are proportional to H^2 and occur at orientations of the magnetic field for which a sizable number of unidirectional open orbits exist. In Fig. 2 we represent, in a stereographic projection, the data of Fig. 1(a) idealized to either H^2 behavior, represented by short heavy bars, or to saturation. In addition we show the three inequivalent types of principal magnetoresistance "ridges"—great circles on the unit sphere that have been found by earlier measurements to exhibit an H^2 behavior.[5,6] The interpretation of each magnetoresistance ridge requires the existence of open orbits directed normal to the plane of that ridge; thus, open orbits are directed along $\langle 100\rangle$, $\langle 110\rangle$, and $\langle 111\rangle$ axes. Based on this knowledge of the open-orbit

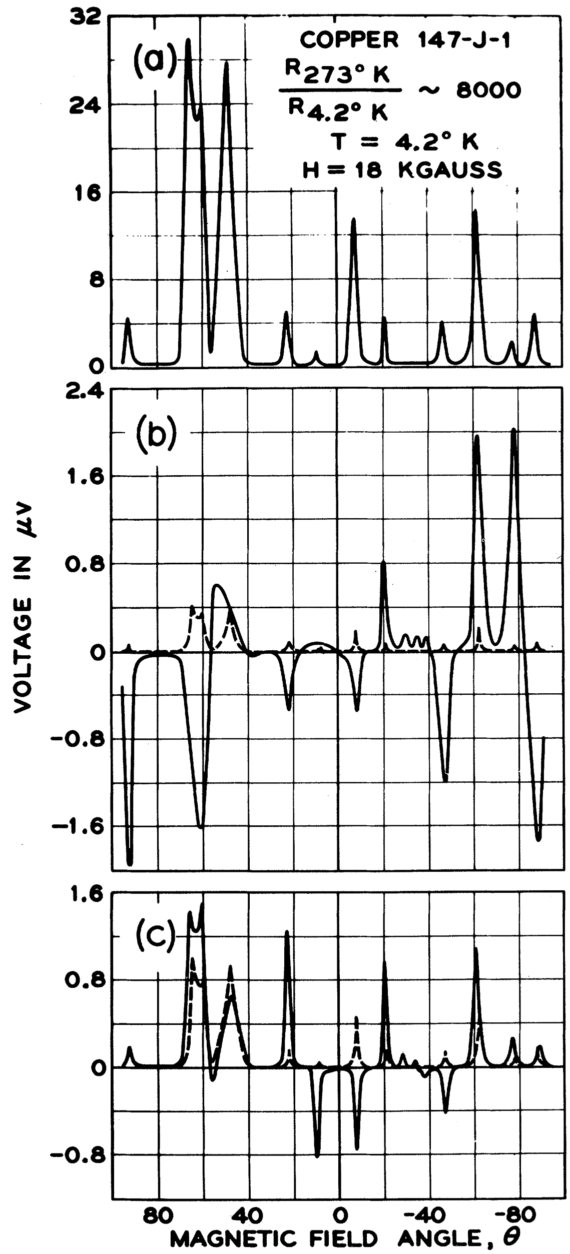

FIG. 1. The longitudinal voltage and the two transverse voltages observed vs angle in a transverse rotation when the current direction is near to a [5$\bar{1}$1] axis. (a) The longitudinal voltage proportional to the magnetoresistance. (b) and (c) Solid curve: the even part (i.e., less the Hall voltage) of the transverse voltages observed across contacts 2-2′ and 3-3′ as shown in Fig. 2. Dashed curve: that portion, similar to Fig. 1(a), of the solid curve which is due to contact misalignment. The difference of the solid and dashed curves represents the transverse-even voltage present because of the anisotropy of the magnetoresistance tensor due to open orbits.

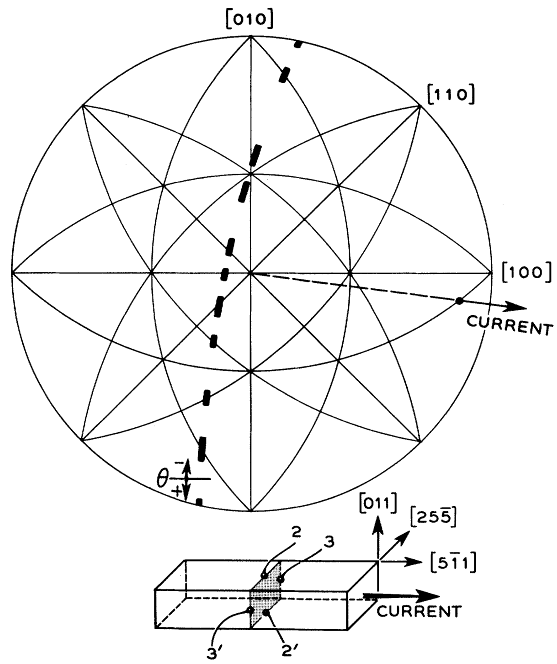

FIG. 2. Stereographic representation of the magnetoresistance data of Fig. 1(a) along with the known contours of H^2 behavior. The sequence of short heavy bars schematically represents the peaks of magnetoresistance observed in the transverse rotation. The great circular lines represent the three inequivalent types of magnetoresistance "ridges" previously observed which indicate the presence of open orbits, along ⟨100⟩, ⟨110⟩, and ⟨111⟩ axes, perpendicular to the ridges of H^2 behavior.

directions for copper, we have calculated the voltage in the two transverse directions that should be observed for each direction of the magnetic field in the transverse plane such that open orbits exist. These results appear in column 3 of Table I.

The solid curves of Figs. 1(b) and 1(c) represent the even part of the observed transverse voltages, while the dashed curves represent that component due to the misalignment voltage, which is a small fixed fraction of Fig. 1(a). In principle, the proportionality constant that relates each dashed curve to Fig. 1(a) can be determined from measurements at zero magnetic field; however, it turned out to be more suitable to determine these factors by a "best fit" to the data. Note the variation in sign; this fact is what first attracted our attention to these voltages. In column 4 of Table I we list the observed transverse voltages at various field angles which,

Table I. Predicted and observed values for the transverse-even voltages at various field angles in a transverse rotation.

Field angle	Open-orbit direction	V_{ti}^{pre} Predicted[a] voltages (μv)	V_{ti}^{obs} Observed[a] voltages (μv)	Open-orbit deviation angle
-88°	⟨110⟩	-1.92 0.15	-1.82 0.03	5°
-78°	⟨100⟩	2.80 0.07	1.97 0.20	9°
-62°	⟨110⟩	2.32 0.42	1.77 0.62	8°
-46°	⟨111⟩	-1.26 -0.49	-1.23 -0.57	4°
-20°	⟨111⟩	0.91 0.83	0.73 0.85	6°
-8°	⟨110⟩	-0.61 -1.27	-0.77 -1.22	2°
10°	⟨100⟩	0 -1.99	0.03 -0.93	12°
22°	⟨110⟩	-0.35 1.10	-0.63 1.09	7°
47°[b]	⟨100⟩, ⟨111⟩	0.5 -0.31	0.5 -0.33	...

[a]The top entry in each case is related to Fig. 1(b), while the bottom entry is related to Fig. 1(c).
[b]The top entry of 0.5 μv in this case was assumed to be correct in order to fix a composite open-orbit direction between [100] and [$1\bar{1}1$]. This direction was then utilized to predict the value -0.31 for the bottom entry by Eq. (2).

according to Fig. 1(a), indicate the presence of open orbits so that Eq. (1) has validity.

In column 5 of Table I we list the angle ψ, where $\cos\psi \equiv \vec{\Omega}^{\text{pre}} \cdot \vec{\Omega}^{\text{true}}$, the deviation between the true open-orbit direction and the direction predicted according to Eq. (3). In nearly every case, the angular deviation ψ does not provide an opportunity for confusion of the open-orbit direction with any other principal crystallographic axis than the correct one. The simple analysis of Eq. (1) ignores additional even terms of lower order in the field; these become significant when $\vec{\Omega}\cdot\vec{j} \approx 0$. The vanishing of $\vec{\Omega}\cdot\vec{j}$ accounts for the absence of any peaks in Fig. 1 when $\theta \simeq -34°$, and its smallness affects the accuracy of the predicted voltages at $\theta = -78°$ and $\theta = 10°$. The transverse voltages appear more sensitive [see field angles between -30° and -40° in Figs. 1(b) and 1(c)] to the "little ridges" which have recently been observed in copper,[5,4] and which may be expected in other metals, whose origins lie in the existence of small bands of higher order open orbits supported by the Fermi surface.

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